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Higher-order and secondary Hochschild cohomology

Cohomologie de Hochschild d'ordre supérieur et secondaire

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ABSTRACT

In this note we give a generalization for the higher-order Hochschild cohomology and show that the secondary Hochschild cohomology is a particular case of this new construction.

RÉSUMÉ

Nous généralisons dans cette Note la cohomologie de Hochschild d'ordre supérieur et nous démontrons que la cohomologie de Hochschild secondaire est un cas particulier de cette nouvelle construction.

1. Introduction

Hochschild cohomology is a useful tool for studying deformation theory and it was studied extensively over the years (for example see [3–5,10,11,15]).

Higher-order Hochschild (co)homology was introduced by Pirashvili in [12] (see also [1] and [6]). It associated with a simplicial set $X_\bullet$, a commutative k-algebra A and an A-bimodule M, the higher-order Hochschild cohomology groups $H^n_{X_\bullet}(A,M)$. When $X_\bullet$ is the standard simplicial set associated with the sphere S^1, one recovers the usual Hochschild cohomology. One important feature of these cohomology groups is that they depend only on the homotopy type of the geometric realization of the simplicial set $X_\bullet$. For more recent results about higher-order Hochschild cohomology see [2] and [8].

Secondary Hochschild cohomology was introduced in [13] where it was used to study B-algebra structures on the algebra $A[[t]]$. It associates with a triple (A,B,ε) (where ε gives the B-algebra structure on A), and an A-bimodule M that is B-symmetric (where the B-module structure on M is induced by ε), the secondary Hochschild cohomology groups $H^n((A,B,\varepsilon),M)$. For more recent results about the secondary cohomology see [9] and [14].

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Our main goal in this paper is to show that secondary Hochschild cohomology is a certain version of higher-order Hochschild cohomology. More precisely, we consider a simplicial pair $(X_\bullet, Y_\bullet)$ (where $Y_\bullet$ is a simplicial set and $X_\bullet$ is simplicial subset of $Y_\bullet$), a triple (A, B, ε) (where A and B are commutative k-algebras, and $\varepsilon : B \to A$ is a morphism of k-algebras) and M a symmetric A-bimodule. With this setting we associate the groups $H^n_{(X_\bullet,Y_\bullet)}((A, B, \varepsilon), M)$. When $X_\bullet = Y_\bullet$ we recover the higher-order Hochschild cohomology $H^n_{X_\bullet}(A, M)$. When $(X_\bullet, Y_\bullet) = (S^1, D^2)$ with the natural simplicial structure, we recover the secondary Hochschild cohomology $H^n((A, B, \varepsilon), M)$.

2. Preliminaries

In this paper we fix a field k and denote $\otimes_k$ by $\otimes$. We assume that the reader is familiar with Hochschild cohomology, but provide some details for the discussion of higher order and secondary Hochschild cohomology. We also assume familiarity with simplicial sets.

2.1. Higher-order Hochschild cohomology

We follow the description in [6] (see also [12]). Assume that A is a commutative k-algebra and M is a symmetric A-bimodule.

Let V be a finite pointed set such that $|V| = v + 1$ (we identify it with $v_+ = \{0, 1, ..., v\}$ with 0 the fixed element) and define $\mathcal{H}(A, M)(V) = \mathcal{H}(A, M)(v_+) = \mathrm{Hom}_k(A^{\otimes v}, M)$. For $\phi : V = v_+ \to W = w_+$ we define

$$\mathcal{H}(A, M)(\phi) : \mathcal{H}(A, M)(w_+) \to \mathcal{H}(A, M)(v_+)$$

determined as follows: if $f \in \mathcal{H}(A, M)(w_+)$ then

$$\mathcal{H}(A, M)(\phi)(f)(a_1 \otimes ... \otimes a_v) = b_0 f(b_1 \otimes ... \otimes b_w),$$

where

$$b_i = \prod_{\{j\in V \mid j\neq 0, \phi(j)=i\}} a_j.$$

Take $X_\bullet$ to be a pointed simplicial set. Suppose that $|X_n| = s_n + 1$, we identify the set X_n with $(s_n)_+ = \{0, 1, ..., s_n\}$ then define

$$C^n_{X_\bullet} = \mathcal{H}(A, M)(X_n) = \mathrm{Hom}_k(A^{\otimes s_n}, M).$$

For each $d_i : X_{n+1} \to X_n$ we define $d_i^* = \mathcal{H}(A, M)(d_i) : C^n_{X_\bullet} \to C^{n+1}_{X_\bullet}$ and take $\partial_n : C^n_{X_\bullet} \to C^{n+1}_{X_\bullet}$ defined as $\partial_n = \sum_{i=0}^{n+1}(-1)^i(d_i)^*$.

The homology of this complex is denoted by $H^n_{X_\bullet}(A, M)$ and is called the higher-order Hochschild cohomology group. One interesting fact is that these groups depend only on the homotopy type of the geometric realization of the simplicial set $X_\bullet$.

Remark 2.1. When $X = S^1$ with the usual simplicial structure one recovers the complex that defines Hochschild cohomology. When $X = S^2$ with the usual simplicial structure, one has $C^n_{S^2_\bullet} = \mathrm{Hom}_k(A^{\otimes \frac{n(n-1)}{2}}, M)$ (see [6] for more details).

2.2. Secondary Hochschild cohomology

We recall the construction from [13]. Let A be a k-algebra, B a commutative k-algebra, $\varepsilon : B \to A$ a morphism of k-algebras such that $\varepsilon(B) \subset \mathcal{Z}(A)$ and M an A-bimodule that is B-symmetric.

We define $C^n((A, B, \varepsilon); M) := \mathrm{Hom}_k(A^{\otimes n} \otimes B^{\otimes \frac{n(n-1)}{2}}, M)$ and

$$\delta^\varepsilon_{n-1} : C^{n-1}((A, B, \varepsilon); M) \to C^n((A, B, \varepsilon); M)$$

such that for $f \in C^{n-1}((A, B, \varepsilon); M)$, we have:

$$\delta^\varepsilon_{n-1}(f)\left(\otimes\begin{pmatrix} a_1 & \alpha_{1,2} & \dots & \alpha_{1,n-1} & \alpha_{1,n} \\ 1 & a_2 & \dots & \alpha_{2,n-1} & \alpha_{2,n} \\ . & . & \dots & . & . \\ 1 & 1 & \dots & a_{n-1} & \alpha_{n-1,n} \\ 1 & 1 & \dots & 1 & a_n \end{pmatrix}\right) = \tag{2.1}$$

$$a_1\varepsilon(\alpha_{1,2}\alpha_{1,3}...\alpha_{1,n})f\left(\otimes\begin{pmatrix} a_2 & \alpha_{2,3} & ... & \alpha_{2,n-1} & \alpha_{2,n} \\ 1 & a_3 & ... & \alpha_{3,n-1} & \alpha_{3,n} \\ . & . & ... & . & . \\ 1 & 1 & ... & a_{n-1} & \alpha_{n-1,n} \\ 1 & 1 & ... & 1 & a_n \end{pmatrix}\right)+$$

$$\sum_{i=1}^{n-1}(-1)^i f\left(\otimes\begin{pmatrix} a_1 & \alpha_{1,2} & ... & \alpha_{1,i}\alpha_{1,i+1} & ... & \alpha_{1,n-1} & \alpha_{1,n} \\ 1 & a_2 & ... & \alpha_{2,i}\alpha_{2,i+1} & ... & \alpha_{2,n-1} & \alpha_{2,n} \\ . & . & ... & . & ... & . & . \\ 1 & 1 & ... & a_i a_{i+1}\varepsilon(\alpha_{i,i+1}) & ... & \alpha_{i,n-1}\alpha_{i+1,n-1} & \alpha_{i,n}\alpha_{i+1,n} \\ . & . & ... & . & ... & . & . \\ 1 & 1 & ... & . & ... & a_{n-1} & \alpha_{n-1,n} \\ 1 & 1 & ... & . & ... & 1 & a_n \end{pmatrix}\right)+$$

$$(-1)^n f\left(\otimes\begin{pmatrix} a_1 & \alpha_{1,2} & ... & \alpha_{1,n-2} & \alpha_{1,n-1} \\ 1 & a_2 & ... & \alpha_{2,n-2} & \alpha_{2,n-1} \\ . & . & ... & . & . \\ 1 & 1 & ... & a_{n-2} & \alpha_{n-2,n-1} \\ 1 & 1 & ... & 1 & a_{n-1} \end{pmatrix}\right)a_n\varepsilon(\alpha_{1,n}\alpha_{2,n}...\alpha_{n-1,n}),$$

where an element in $A^n\otimes B^{\frac{n(n-1)}{2}}$ is represented by a tensor matrix with elements $a_i\in A$ arranged on the diagonal, elements $\alpha_{i,j}\in B$ above the diagonal and $1\in k$ below the diagonal. It was proved in [13] that $(C^n((A,B,\varepsilon);M),\delta_n^{\varepsilon})$ is a complex. The homology of this complex is called the secondary Hochschild cohomology and is denoted by $H^n((A,B,\varepsilon);M)$. The homology and the cyclic version of this theory were discussed in [9].

3. Main construction

In this section we introduce a new cohomology associated with a simplicial pair $(X_\bullet,Y_\bullet)$ and a triple (A,B,ε) (where A and B are commutative k-algebra and $\varepsilon:B\to A$ is a morphism of k-algebras). We start with a few notations.

Definition 3.1. We consider Γ_2 to be the category whose objects are pairs (U,V), where V is a finite pointed set with base point $*$, and U is a pointed subset of V. A morphism $f\in \mathrm{Hom}_{\Gamma_2}((U_1,V_1),(U_2,V_2))$ is a map of pointed sets $f:V_1\to V_2$ such that $f(U_1)\subset U_2$.

Remark 3.2. The category of finite pointed sets Γ can be see as a full subcategory of Γ_2 in two different ways. First we can take the inclusion given by $V\to(V,V)$, second we can take the inclusion $V\to(\{*\},V)$.

Definition 3.3. A Γ_2-module is a functor from Γ_2^{op} to k-modules.

Example 3.4. Let A and B be two commutative k-algebras, $\varepsilon:B\to A$ a morphism of k-algebras and M a symmetric A-bimodule. We construct

$$\mathcal{L}((A,B,\varepsilon);M):\Gamma_2^{\mathrm{op}}\to k-mod$$

to be the Γ_2-module determined as follows. For $(U,V)\in\Gamma_2$ such that $|U|=1+m$ and $|V|=1+m+n$ define

$$\mathcal{L}((A,B,\varepsilon);M)((U,V))=\mathrm{Hom}_k(A^{\otimes m}\otimes B^{\otimes n},M).$$

If $f:(U_1,V_1)\to(U_2,V_2)$ is a morphism in Γ_2, we define

$$\mathcal{L}((A,B,\varepsilon);M)(f):\mathrm{Hom}_k(A^{\otimes m_2}\otimes B^{\otimes n_2},M)\to\mathrm{Hom}_k(A^{\otimes m_1}\otimes B^{\otimes n_1},M),$$

for $\psi\in\mathrm{Hom}_k(A^{\otimes m_2}\otimes B^{\otimes n_2},M)$, then
$\mathcal{L}((A,B,\varepsilon);M)(f)(\psi)(a_1\otimes...\otimes a_{m_1}\otimes\alpha_1\otimes...\otimes\alpha_{n_1})=b_0\cdot\psi(b_1\otimes...\otimes b_{m_2}\otimes\beta_1\otimes...\otimes\beta_{n_2})$ where for $i\in U_2$, we have

$$b_i=\prod_{\{j\in U_1|j\neq *,\ f(j)=i\}}a_j\prod_{\{k\in V_1\setminus U_1|k\neq *,\ f(k)=i\}}\varepsilon(\alpha_k)\in A,\tag{3.1}$$

and for $p\in V_2\setminus U_2$, we have

$$\beta_p=\prod_{\{q\in V_1\setminus U_1|q\neq *,\ f(q)=p\}}\alpha_q\in B.$$

With the convention that if the product is taken over the empty set then we put $b_i=1\in A$ and $\beta_p=1\in B$.

We say that a pair $(X_\bullet, Y_\bullet)$ is a simplicial pair if $Y_\bullet$ is a simplicial set and $X_\bullet$ a simplicial subset of $Y_\bullet$. In other words we have a functor

$$(X_\bullet, Y_\bullet) : \Delta^{\mathrm{op}} \to \Gamma_2^{\mathrm{op}}.$$

For a simplicial pair $(X_\bullet, Y_\bullet)$ we define the higher-order Hochschild cohomology associated with the triple (A, B, ε) and a symmetric A-bimodule M, to be the homology of the complex defined as follows. For every $q \in \mathbb{N}$ we consider $(X_q, Y_q) \in \Gamma_2^{\mathrm{op}}$ and take $C^q_{(X_\bullet,Y_\bullet)} = \mathcal{L}((A, B, \varepsilon); M)((X_q, Y_q))$. We construct a complex by taking the differential induced by the simplicial structure on $(X_\bullet, Y_\bullet)$. More precisely if $d_i : Y_{q+1} \to Y_q$ then we define

$$\delta^i = \mathcal{L}((A, B, \varepsilon); M)(d_i) : C^q_{(X_\bullet,Y_\bullet)} \to C^{q+1}_{(X_\bullet,Y_\bullet)}$$

and take $\partial_{(X_\bullet,Y_\bullet)} : C^q_{(X_\bullet,Y_\bullet)} \to C^{q+1}_{(X_\bullet,Y_\bullet)}$,

$$\partial_{(X_\bullet,Y_\bullet)} = \sum_{i=0}^{q+1} (-1)^i \delta^i. \tag{3.2}$$

Definition 3.5. The homology of the above complex is called the higher-order Hochschild cohomology associated with the simplicial pair $(X_\bullet, Y_\bullet)$, of the triple (A, B, ε) with coefficients in M and is denoted by $H^q_{(X_\bullet,Y_\bullet)}((A, B, \varepsilon); M)$.

Remark 3.6. In the event that $X_\bullet = Y_\bullet$, this definition agrees with the definition of higher-order Hochschild cohomology $H^q_{X_\bullet}(A, M)$.

4. Secondary cohomology as a higher-order cohomology

In this section, we show that when A is commutative and M is a symmetric A-bimodule, then the secondary Hochschild cohomology $H^n((A, B, \varepsilon); M)$ is a particular case of the construction from the previous section.

Consider the simplicial pair $(X_\bullet, Y_\bullet) = (S^1, D^2)$, where the sphere S^1 is a obtained from the interval $I = [01]$ by identifying the ends of the interval, and the disk D^2 is obtained from the 2-simplex $\Delta = [012]$ by collapsing the edges [01] and [12] (i.e. the boundary of D^2 is the edge [02]).

More precisely, we take $X_\bullet$ to be the simplicial set where the only nondegenerate 1-simplex is $I = [02]$. We denote by $*_n$ the base point in dimension n, and by I^a_b the simplex in dimension $n = a+b+1$, where we iterate the [0] vertex a times, and the [2] vertex b times. For example, I^0_0 is the interval I with $d_0(I^0_0) = d_1(I^0_0) = *_0$, and I^1_0 is a 2-simplex [002] such that $d_0(I^1_0) = d_1(I^1_0) = I^0_0$ and $d_2(I^1_0) = *_1$.

For $Y_\bullet$, besides the above simplices, we also have a nondegenerate 2-simplex $\Delta = [012]$. Denote it by ${}^0\Delta^0_0$ and take $d_0({}^0\Delta^0_0) = d_2({}^0\Delta^0_0) = *_1$ and $d_1({}^0\Delta^0_0) = I^0_0$. More generally, take ${}^a\Delta^b_c$ the $a + b + c + 2$-dimensional simplex obtained by iterating the [0] vertex a times, the [1] vertex b times, and the [2] vertex c times. For example ${}^1\Delta^0_0$ is a 3-simplex [0012] with $d_0({}^1\Delta^0_0) = d_1({}^1\Delta^0_0) = {}^0\Delta^0_0$, $d_2({}^1\Delta^0_0) = I^1_0$, and $d_3({}^1\Delta^0_0) = *_2$.

In general we have $X^n = \{*_n\} \cup \{I^a_b | a, b \in \mathbb{N},\ a + b = n - 1\}$ and $Y^n = X^n \cup \{{}^a\Delta^b_c | a, b, c \in \mathbb{N},\ a + b + c = n - 2\}$. The $d_i : Y^n \to Y^{n-1}$ are defined as follows:

$$d_i(*_n) = *_{n-1}, \tag{4.1}$$

$$d_i(I^a_b) = \begin{cases} *_{a+b} & \text{if } a = 0 \text{ and } i = 0 \\ I^{a-1}_b & \text{if } a \neq 0 \text{ and } i \le a \\ I^a_{b-1} & \text{if } b \neq 0 \text{ and } i > a \\ *_{a+b} & \text{if } b = 0 \text{ and } i = n = a+1, \end{cases} \tag{4.2}$$

$$d_i({}^a\Delta^b_c) = \begin{cases} *_{a+b+c+1} & \text{if } a = 0 \text{ and } i = 0 \\ {}^{a-1}\Delta^b_c & \text{if } a \neq 0 \text{ and } i \le a \\ I^a_c & \text{if } b = 0 \text{ and } i = a+1 \\ {}^a\Delta^{b-1}_c & \text{if } b \neq 0 \text{ and } a < i \le a+b+1 \\ *_{a+b+c+1} & \text{if } c = 0 \text{ and } i = n = a+b+2 \\ {}^a\Delta^b_{c-1} & \text{if } c \neq 0 \text{ and } i \ge a+b+2. \end{cases} \tag{4.3}$$

Notice that I^a_b is degenerate if $a + b > 0$, and ${}^a\Delta^b_c$ is degenerate if $a + b + c > 0$. Also, we have that $|X^n| = 1 + n$ and $|Y^n| = 1 + n + \frac{n(n-1)}{2}$. In particular, we get that

$$\mathcal{L}((A, B, \varepsilon); M)((X^n, Y^n)) = \mathrm{Hom}_k(A^{\otimes n} \otimes B^{\otimes \frac{n(n-1)}{2}}, M).$$

Next we need to make the identification with the notation from [13]. First recall that an element in $A^{\otimes n} \otimes B^{\otimes \frac{n(n-1)}{2}}$ was represented by a tensor matrix

$$T = \otimes \begin{pmatrix} a_{1,1} & \alpha_{1,2} & \dots & \alpha_{1,n-1} & \alpha_{1,n} \\ 1 & a_{2,2} & \dots & \alpha_{2,n-1} & \alpha_{2,n} \\ . & . & \dots & . & . \\ 1 & 1 & \dots & a_{n-1,n-1} & \alpha_{n-1,n} \\ 1 & 1 & \dots & 1 & a_{n,n} \end{pmatrix},$$

where $a_{i,i} \in A$ and $\alpha_{i,j} \in B$.

For $a, b \in \mathbb{N}$ with $a+b+1 = n$ the element $I_b^a \in Y^n$ corresponds to the position $(a+1, a+1)$ in the tensor matrix. For $a, b, c \in \mathbb{N}$ with $a+b+c+2 = n$ the element ${}^a\Delta_c^b \in Y^n$ corresponds to the position $(a+1, a+b+2) = (a+1, n-c)$ in the tensor matrix. We also add the symbol $(0,0)$ to correspond to $*_n$.

With the above identifications the formulas (4.2) and (4.3) become:

$$d_i((a+1, a+1)) = \begin{cases} (0,0) & \text{if } a = 0 \text{ and } i = 0 \\ (a,a) & \text{if } a \neq 0 \text{ and } i \leq a \\ (a+1, a+1) & \text{if } b \neq 0 \text{ and } i > a \\ (0,0) & \text{if } b = 0 \text{ and } i = n = a+1, \end{cases}$$

$$d_i(a+1, a+b+2) = \begin{cases} (0,0) & \text{if } a = 0 \text{ and } i = 0 \\ (a, a+b+1) & \text{if } a \neq 0 \text{ and } i \leq a \\ (a+1, a+1) & \text{if } b = 0 \text{ and } i = a+1 \\ (a+1, a+b+1) & \text{if } b \neq 0 \text{ and } a < i \leq a+b+1 \\ (0,0) & \text{if } c = 0 \text{ and } i = n = a+b+2 \\ (a+1, a+b+2) & \text{if } c \neq 0 \text{ and } i \geq a+b+2. \end{cases}$$

If we use the above identification, the formula for $\partial_{(S^1,D^2)}$ from equation (3.2) is the same as the formula for differential $\delta_{n-1}^{\varepsilon}$ from equation (2.1). To summarize we have the following result.

Theorem 4.1. *Let A and B be commutative k-algebras, $\varepsilon : B \to A$ a morphism of k-algebras and M a symmetric A-bimodule, then we have*

$$H^q((A, B, \varepsilon); M) \simeq H^q_{(S^1,D^2)}((A, B, \varepsilon); M).$$

5. Some remarks

One can see that $H^q_{(X_\bullet,Y_\bullet)}((A, B, \varepsilon); M)$ is functorial with respect to all of its entries. More precisely, let $(A_1, B_1, \varepsilon_1)$ and $(A_2, B_2, \varepsilon_2)$ be two triples, M a symmetric A_2-bimodule, and $f : A_1 \to A_2$ a morphism of k-algebras such that $f(B_1) \subseteq B_2$ and $f\varepsilon_1(b) = \varepsilon_2(f(b))$, then we have the natural morphism

$$f^* : H^q_{(X_\bullet,Y_\bullet)}((A_2, B_2, \varepsilon_2); M) \to H^q_{(X_\bullet,Y_\bullet)}((A_1, B_1, \varepsilon_1); M),$$

where the A_1-bimodule structure on M is induced by f. Also, if $g : M \to N$ is a morphism of symmetric A-bimodules then

$$g_* : H^q_{(X_\bullet,Y_\bullet)}((A, B, \varepsilon); M) \to H^q_{(X_\bullet,Y_\bullet)}((A, B, \varepsilon); N).$$

Moreover if $h : (X_\bullet, Y_\bullet) \to (Z_\bullet, T_\bullet)$ is a morphism of simplicial sets then

$$h^* : H^q_{(Z_\bullet,T_\bullet)}((A, B, \varepsilon); M) \to H^q_{(X_\bullet,Y_\bullet)}((A, B, \varepsilon); M).$$

If we take the natural inclusion of simplicial pairs $i : (S^1, S^1) \to (S^1, D^2)$ with the simplicial structure discussed in the previous section, then

$$i^n : H^n_{(S^1,D^2)}((A, B, \varepsilon); M) \to H^n_{(S^1,S^1)}((A, B, \varepsilon); M)$$

is nothing else but the morphism $\Phi_n : H^n((A, B, \varepsilon); M) \to H^n(A, M)$ discussed in [14].

One natural question is whether the construction in this paper depends only of the homotopy type of the geometric realization of the simplicial pair $(X_\bullet, Y_\bullet)$ (or maybe invariant under a certain equivalence relation among simplicial pairs). We explored this problem but we were not able to prove any interesting result. The main issue is finding an equivalence relation among simplicial pairs that is manageable at the algebraic level.

It was pointed out to us by the referee that this construction is connected to stratified factorization homology (see [7] and [8]). This relation could be useful in studying homotopy invariance of our construction with respect to the maps above.

Furthermore, in the case where $Y = D^n$ is the n-dimensional disk and $X = S^{n-1}$ is its boundary, the construction could be related to a model for deformation complexes of higher swiss cheese algebras and thus of n-shifted coisotropic structures in derived geometry.

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References

[1] D.W. Anderson, Chain functors and homology theories, in: Symposium on Algebraic Topology, Battelle Seattle Res. Center, Seattle, Wash., 1971, in: Lect. Notes Math., vol. 249, Springer, Berlin, 1971, pp. 1–12.
[2] B.R. Corrigan-Salter, Coefficients for higher order Hochschild cohomology, Homol. Homotopy Appl. 17 (1) (2015) 111–120.
[3] M. Gerstenhaber, The cohomology structure of an associative ring, Ann. of Math. (2) 78 (1963) 267–288.
[4] M. Gerstenhaber, On the deformation of rings and algebras, Ann. of Math. (2) 79 (1964) 59–103.
[5] M. Gerstenhaber, S.D. Schack, Algebraic cohomology and deformation theory, in: Deformation Theory of Algebras and Structures and Applications, Il Ciocco, 1986, in: NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., vol. 247, Kluwer Acad. Publ., Dordrecht, 1988, pp. 11–264.
[6] G. Ginot, Higher order Hochschild cohomology, C. R. Math. Acad. Sci. Paris 346 (1–2) (2008) 5–10.
[7] G. Ginot, Notes on factorization algebras, factorization homology and applications, in: Mathematical Aspects of Quantum Field Theories, in: Math. Phys. Stud., Springer, Cham, 2015, pp. 429–552.
[8] G. Ginot, T. Tradler, M. Zeinalian, Higher Hochschild homology, topological chiral homology and factorization algebras, Commun. Math. Phys. 326 (3) (2014) 635–686.
[9] J. Laubacher, M.D. Staic, A. Stancu, Bar simplicial modules and secondary cyclic (co)homology, arXiv e-prints, May 2016.
[10] O.A. Laudal, Formal Moduli of Algebraic Structures, Lect. Notes Math., vol. 754, Springer, Berlin, 1979.
[11] J.-L. Loday, Cyclic Homology, Grundlehren Math. Wiss. (Fundamental Principles of Mathematical Sciences), vol. 301, Springer-Verlag, Berlin, 1992, appendix E by María O. Ronco.
[12] T. Pirashvili, Hodge decomposition for higher order Hochschild homology, Ann. Sci. Éc. Norm. Supér. (4) 33 (2) (2000) 151–179.
[13] M.D. Staic, Secondary Hochschild cohomology, Algebr. Represent. Theory 19 (1) (2016) 47–56.
[14] M.D. Staic, A. Stancu, Operations on the secondary Hochschild cohomology, Homol. Homotopy Appl. 17 (1) (2015) 129–146.
[15] D. Yau, Deformation theory of modules, Commun. Algebra 33 (7) (2005) 2351–2359.