



Statistics

On estimation in a spatial functional linear regression model with derivatives



Estimation dans un modèle de régression fonctionnelle spatiale avec dérivées

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ARTICLE INFO

Article history:

Received 5 December 2016

Accepted after revision 23 February 2018

Available online 30 March 2018

Presented by Paul Deheuvels

ABSTRACT

This paper deals with functional linear regression for spatial data. We study the asymptotic properties of an estimator of a linear model where a spatial scalar response variable is related to a spatial functional explanatory variable and to its derivative. Convergence results with rate of this estimator are derived.

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R É S U M É

Cet article aborde l'estimation de la régression linéaire fonctionnelle dans un cadre spatial. Nous étudions les propriétés asymptotiques de l'estimateur d'un modèle où une variable réponse réelle est liée à une variable dépendante fonctionnelle et sa dérivée. Nous établissons des résultats de convergence pour cet estimateur, et des vitesses de convergence sont données.

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Version française abrégée

Nous considérons le modèle défini par

$$Y_{\mathbf{i}} = \langle \beta, X_{\mathbf{i}} \rangle_E + \langle \gamma, X'_{\mathbf{i}} \rangle_F + \epsilon_{\mathbf{i}}, \quad \mathbf{i} \in D \subset \mathbb{Z}^d, \quad d \geq 2,$$

où E est l'espace de Sobolev d'ordre $(2, 1)$ ($f \in E$ signifie que f et f' appartiennent à $F := L^2[0, 1]$), β et γ sont des fonctions inconnues appartenant respectivement à E et F , $X_{\mathbf{i}}$ et $\epsilon_{\mathbf{i}}$ sont des variables aléatoires centrées et indépendantes à valeurs dans E et \mathbb{R} , respectivement, et $X'_{\mathbf{i}}$ désigne la dérivée première de $X_{\mathbf{i}}$. Nous supposons que $(Y_{\mathbf{i}}, X_{\mathbf{i}})$ a

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<https://doi.org/10.1016/j.crma.2018.02.013>

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la même distribution qu'un vecteur aléatoire (Y, X) , et que le processus est observé dans une région $\mathcal{I}_n = \{1, 2, \dots, n\}^d$ où $\mathbf{n} = (n, \dots, n)$, $n \in \mathbb{N}^*$. Nous nous intéressons à l'estimation des paramètres β et γ , puis à la prédiction à un site non visité. L'estimateur de (β, γ) est $(\hat{\beta}_n, \hat{\gamma}_n)$ défini, comme dans [9], sur la base d'estimateurs empiriques donnés de (2) à (5), par $\hat{\beta}_n = (S_{n,\beta} + \psi_n I)^{-1} u_{n,\beta}$ et $\hat{\gamma}_n = (S_{n,\gamma} + \psi_n I)^{-1} u_{n,\gamma}$. L'opérateur T_n (resp. T) étant l'un des opérateurs suivants : $\Gamma_n, \Gamma'_n, \Gamma_n^{*/}$ et Γ_n^{**} (resp. $\Gamma, \Gamma', \Gamma^{*/}$, Γ^{**}), on a le théorème suivant.

Théorème 3.1. *Sous les Hypothèses 3.1–3.5 avec $\alpha_{1,\infty}(t) = O(t^{-\theta})$, $\theta \geq d + 1$, on a :*

$$\mathbb{E} \left(\|T_n - T\|_\infty^2 \right) = O \left(n^{-d} \log n \right).$$

Notant \mathcal{HS} l'espace des opérateurs de Hilbert–Schmidt et considérant les semi-normes $\|\cdot\|_\Gamma := \|\Gamma^{1/2}(\cdot)\|_E$ et $\|\cdot\|_{\Gamma^{**}} := \|\Gamma^{**/2}(\cdot)\|_F$, on en déduit le corollaire suivant.

Corollaire 3.1. *Soit $(v_j)_{j \geq 1}$ une base orthonormée de E constituée de vecteurs propres associés aux valeurs propres $(\lambda_j)_{j \geq 1}$ de Γ avec $\lambda_j = O(r^j)$, $0 < r < 1$, $j \geq 1$. Alors, sous les hypothèses du Théorème 3.1, on a :*

- (i) $\|T_n - T\|_{L^2(\mathcal{HS})} = O(n^{-d/2} \log n)$.
- (ii)

$$\|\beta - \hat{\beta}_n\|_\Gamma^2 = O_p \left(\frac{\psi_n^2}{\phi_n^2} \right) + O_p \left(\frac{(\log n)^2}{\phi_n^2 \psi_n^2 n^d} \right) \quad \text{et} \quad \|\gamma - \hat{\gamma}_n\|_{\Gamma^{**}}^2 = O_p \left(\frac{\psi_n^2}{\phi_n^2} \right) + O_p \left(\frac{(\log n)^2}{\phi_n^2 \psi_n^2 n^d} \right).$$

Posant $\mathcal{I}_{n+1_d} = \{1, 2, \dots, n+1\}^d$, le prédicteur et sa version « théorique » à un site non visité $\mathbf{j} \in \mathcal{I}_{n+1_d} \setminus \mathcal{I}_n$ sont respectivement définis par $\hat{Y}_j = \langle \hat{\beta}_n, X_j \rangle_E + \langle \hat{\gamma}_n, X'_j \rangle_F$ et $Y_j^* = \langle \beta, X_j \rangle_E + \langle \gamma, X'_j \rangle_F$. On a alors ce qui suit.

Corollaire 3.2. *Sous les hypothèses du Corollaire 3.1, on a, pour tout $\mathbf{j} \in \mathcal{I}_{n+1_d} \setminus \mathcal{I}_n$:*

$$\mathbb{E} \left[\left(\hat{Y}_j - Y_j^* \right)^2 \right] = O \left(\frac{\psi_n^2}{\phi_n^2} \right) + O \left(\frac{(\log n)^2}{\phi_n^2 \psi_n^2 n^d} \right).$$

1. Introduction

Several types of functional linear models for independent data have been developed over the years, serving different purposes. The most studied is perhaps the functional linear model for scalar response, originally introduced by [6]. Estimation and prediction problems for this model and some of its generalizations have been tackled mainly for independent data (see, e.g., [1], [2], [8], [9]). Some works exist on functional spatial linear prediction using kriging methods (see, e.g., [3], [4], [7], [10]). They highlight the interest of considering spatial linear functional models. In this paper, the results obtained in [9] on estimation and prediction in the functional linear model with derivatives for independent data are extended to the spatial case. Namely, we consider the model given by:

$$Y_{\mathbf{i}} = \langle \beta, X_{\mathbf{i}} \rangle_E + \langle \gamma, X'_{\mathbf{i}} \rangle_F + \epsilon_{\mathbf{i}}, \quad \mathbf{i} \in D \subset \mathbb{Z}^d, \quad d \geq 2, \tag{1}$$

where E is the Sobolev space of order $(2, 1)$ ($f \in E$ means that f and f' belong to $F := L^2[0, 1]$), β and γ are unknown functions belonging to E and F , respectively, $X_{\mathbf{i}}$ and $\epsilon_{\mathbf{i}}$ are centered and independent random variables defined on a probability space (Ω, \mathcal{A}, P) and valued into E and \mathbb{R} respectively, and $X'_{\mathbf{i}}$ stands for the first derivative of $X_{\mathbf{i}}$. We assume that $(Y_{\mathbf{i}}, X_{\mathbf{i}})$ has the same distribution as a random vector (Y, X) , and that the process is observed in a region $\mathcal{I}_n = \{1, 2, \dots, n\}^d$ where $\mathbf{n} = (n, \dots, n)$, $n \in \mathbb{N}^*$. In Section 2, estimators of β and γ are introduced, as well as a predictor at a non-visited site. Section 3 gives assumptions and establishes asymptotic results, namely convergences with rate of the estimates. Finally, Section 4 is devoted to some indications for proving the theoretical results presented in this note.

2. Estimation and prediction

We propose estimators of β and γ by using the same approach than [9]. Denoting by \otimes_E (resp. \otimes_F) the tensor product defined by $(u \otimes_E v)(h) = \langle u, h \rangle_E v$ (resp. $(u \otimes_F v)(h) = \langle u, h \rangle_F v$), we consider the covariance and cross-covariance operators defined by

$$\Gamma = \mathbb{E}(X \otimes_E X), \quad \Gamma^{*/} = \mathbb{E}(X \otimes_E X'), \quad \Gamma' = \mathbb{E}(X' \otimes_F X), \quad \Gamma^{**} = \mathbb{E}(X' \otimes_F X')$$

and we set:

$$\Delta = \mathbb{E}(YX) \in E, \quad \Delta' = \mathbb{E}(YX') \in F, \quad S_\beta = \Gamma - \Gamma' \Gamma''^{-1} \Gamma'^*, \quad S_\gamma = \Gamma'' - \Gamma'^* \Gamma^{-1} \Gamma'.$$

Empirical versions of these operators and functions are given by:

$$\Gamma_n = \frac{1}{n^d} \sum_{\mathbf{i} \in \mathcal{I}_n} X_{\mathbf{i}} \otimes_E X_{\mathbf{i}}, \quad \Gamma'_n = \frac{1}{n^d} \sum_{\mathbf{i} \in \mathcal{I}_n} X'_{\mathbf{i}} \otimes_F X_{\mathbf{i}}, \quad \Delta_n = \frac{1}{(n-1)^d} \sum_{\mathbf{i} \in \mathcal{I}_n^1} Y_{\mathbf{i}} X_{\mathbf{i}}, \tag{2}$$

$$\Gamma_n^* = \frac{1}{n^d} \sum_{\mathbf{i} \in \mathcal{I}_n} X_{\mathbf{i}} \otimes_E X'_{\mathbf{i}}, \quad \Gamma''_n = \frac{1}{n^d} \sum_{\mathbf{i} \in \mathcal{I}_n} X'_{\mathbf{i}} \otimes_F X'_{\mathbf{i}}, \quad \Delta'_n = \frac{1}{(n-1)^d} \sum_{\mathbf{i} \in \mathcal{I}_n^1} Y_{\mathbf{i}} X'_{\mathbf{i}}. \tag{3}$$

Then, we consider the operators

$$\tilde{\Gamma}_n^{-1} = (\Gamma_n + \phi_n I)^{-1}, \quad \tilde{\Gamma}_n''^{-1} = (\Gamma''_n + \phi_n I)^{-1}, \quad S_{n,\beta} = \Gamma_n - \Gamma'_n (\tilde{\Gamma}_n''^{-1}) \Gamma_n^*, \quad S_{n,\gamma} = \Gamma''_n - \Gamma_n^* (\tilde{\Gamma}_n^{-1}) \Gamma'_n, \tag{4}$$

where ϕ_n and ψ_n are strictly positive real-sequences defined on \mathbb{N}^d that tend to zero as $n \rightarrow +\infty$, and I is the identity operator, and we put

$$u_{n,\beta} = \Delta_n - \Gamma'_n (\tilde{\Gamma}_n''^{-1}) \Delta'_n, \quad u_{n,\gamma} = \Delta'_n - \Gamma_n^* (\tilde{\Gamma}_n^{-1}) \Delta_n. \tag{5}$$

The estimate of the pair (β, γ) is $(\hat{\beta}_n, \hat{\gamma}_n)$ defined by:

$$\hat{\beta}_n = (S_{n,\beta} + \psi_n I)^{-1} u_{n,\beta}, \quad \hat{\gamma}_n = (S_{n,\gamma} + \psi_n I)^{-1} u_{n,\gamma}.$$

Let $\mathcal{I}_{n+1_d} = \{1, 2, \dots, n+1\}^d$, the predictor at a non-visited site $\mathbf{j} \in \mathcal{I}_{n+1_d} \setminus \mathcal{I}_n$ is defined as

$$\hat{Y}_{\mathbf{j}} = \langle \hat{\beta}_n, X_{\mathbf{j}} \rangle_E + \langle \hat{\gamma}_n, X'_{\mathbf{j}} \rangle_F.$$

3. Assumptions and results

In order to establish the asymptotic results, the following assumptions will be considered.

Assumption 3.1. $\text{Ker } \Gamma = \text{Ker } \Gamma'' = \{0\}$, where, for any operator A , $\text{Ker } A = \{x : Ax = 0\}$.

Assumption 3.2. $(\beta, \gamma) \notin \mathcal{N}$, where $\mathcal{N} = \{(\beta, \gamma) \in E \times F : \beta + D^* \gamma = 0\}$ with D the ordinary differential operator and D^* its adjoint.

Assumption 3.3. $\|\Gamma^{-1/2} \beta\|_E < \infty$, $\|(\Gamma'')^{-1/2} \gamma\|_F < \infty$.

Assumption 3.4. The process $\{Z_{\mathbf{i}} = (X_{\mathbf{i}}, Y_{\mathbf{i}}, X'_{\mathbf{i}}), \mathbf{i} \in \mathbb{Z}^d\}$ is strongly mixing, that is $\lim_{n \rightarrow \infty} \alpha_{1,\infty}(n) = 0$, where

$$\alpha_{1,\infty}(n) = \sup\{\alpha(\sigma(Z_{\mathbf{i}}), F_G), \mathbf{i} \in \mathbb{Z}^d, G \subset \mathbb{Z}^d, \rho(G, \{\mathbf{i}\}) \geq n\}, \tag{6}$$

α being the α -mixing coefficient defined, for two sub σ -algebras \mathcal{U} and \mathcal{V} , by $\alpha(\mathcal{U}, \mathcal{V}) = \sup\{|\mathbb{P}(A \cap B) - \mathbb{P}(A)\mathbb{P}(B)|, A \in \mathcal{U}, B \in \mathcal{V}\}$, $F_G = \sigma(Z_{\mathbf{j}}; \mathbf{j} \in G)$ and the distance ρ is defined for any subsets G_1 and G_2 of \mathbb{Z}^d by $\rho(G_1, G_2) = \min\{\|\mathbf{i} - \mathbf{j}\|, \mathbf{i} \in G_1, \mathbf{j} \in G_2\}$ with $\|\mathbf{i} - \mathbf{j}\| = \max_{1 \leq s \leq d} |i_s - j_s|$ for any \mathbf{i} and \mathbf{j} in \mathbb{Z}^d .

Assumption 3.5. $\|X_{\mathbf{i}}\|_E \leq M$ a.s. where M is a strictly positive constant.

Assumptions 3.1–3.3 are technical conditions, which are also considered in [9]. Assumption 3.1 is needed for identifying the model defined in (1) and Assumption 3.2 permits to obtain estimators of the parameters β and γ . Examples of spatial processes that satisfy Assumption 3.4 can be found in [5]. It is usual to assume that $\alpha_{1,\infty}(i)$ tends to zero with polynomial rate, or $\alpha_{1,\infty}(i) \leq C \exp(-si)$, for some $C, s > 0$ i.e. $\alpha_{1,\infty}(i)$ tends to zero with exponential rate. We will consider the polynomial case; however our results can be proved under the exponential case.

In what follows, let the norms $\|\cdot\|_\infty$ and $\|\cdot\|_{L^2(\mathcal{HS})}$ be defined by $\|T\|_\infty = \sup_{x \in E} \|Tx\|_E / \|x\|_E$ and $\|T\|_{L^2(\mathcal{HS})} = (\mathbb{E}(\|T\|_{\mathcal{HS}}^2))^{1/2}$ where \mathcal{HS} is the space of Hilbert–Schmidt operators endowed with the inner product $\langle T, S \rangle_{\mathcal{HS}} = \sum_{i=1}^{+\infty} \langle T(u_i), S(u_i) \rangle_E$ where $(u_i)_{i \geq 1}$ is a basis of E . Let T_n (resp. T) be one of the following: $\Gamma_n, \Gamma'_n, \Gamma_n^*$ and Γ''_n (resp. $\Gamma, \Gamma', \Gamma'^*, \Gamma''$). The following result establishes the almost sure convergence of T_n to T with respect to $\|\cdot\|_\infty$.

Theorem 3.1. Under Assumptions 3.1–3.5 with $\alpha_{1,\infty}(t) = O(t^{-\theta})$, $\theta \geq d + 1$, we have:

$$\mathbb{E}(\|T_n - T\|_\infty^2) = O(n^{-d} \log n).$$

A rate of convergence of $T_{\mathbf{n}}$ with respect to norm $\|\cdot\|_{L^2(\mathcal{H}_S)}$ appears in the following corollary, as well as those of $\widehat{\beta}_{\mathbf{n}}$ and $\widehat{\gamma}_{\mathbf{n}}$ with respect to semi-norms $\|\cdot\|_{\Gamma} := \|\Gamma^{1/2}(\cdot)\|_E$ and $\|\cdot\|_{\Gamma''} := \|\Gamma''^{1/2}(\cdot)\|_F$ respectively.

Corollary 3.1. *Let $(v_j)_{j \geq 1}$ be a sequence of orthonormal eigenfunctions associated with a sequence of eigenvalues $(\lambda_j)_{j \geq 1}$ of the operator Γ with $\lambda_j = O(r^j)$, $0 < r < 1$, $j \geq 1$. Then, under assumptions of Theorem 3.1, we have:*

(i)

$$\|T_{\mathbf{n}} - T\|_{L^2(\mathcal{H}_S)} = O\left(n^{-d/2} \log n\right).$$

(ii)

$$\|\beta - \widehat{\beta}_{\mathbf{n}}\|_{\Gamma}^2 = O_p\left(\frac{\psi_{\mathbf{n}}^2}{\phi_{\mathbf{n}}^2}\right) + O_p\left(\frac{(\log n)^2}{\phi_{\mathbf{n}}^2 \psi_{\mathbf{n}}^2 n^d}\right) \text{ and } \|\gamma - \widehat{\gamma}_{\mathbf{n}}\|_{\Gamma''}^2 = O_p\left(\frac{\psi_{\mathbf{n}}^2}{\phi_{\mathbf{n}}^2}\right) + O_p\left(\frac{(\log n)^2}{\phi_{\mathbf{n}}^2 \psi_{\mathbf{n}}^2 n^d}\right).$$

The following corollary gives a bound of the prediction error of the predictor $\widehat{Y}_{\mathbf{j}}$ of $Y_{\mathbf{j}}^* = \langle \beta, X_{\mathbf{j}} \rangle_E + \langle \gamma, X'_{\mathbf{j}} \rangle_F$ at a non-visited site $\mathbf{j} \in \mathcal{I}_{\mathbf{n}+1_d} \setminus \mathcal{I}_{\mathbf{n}}$.

Corollary 3.2. *Assume that assumptions of Corollary 3.1 hold. Then, for each $\mathbf{j} \in \mathcal{I}_{\mathbf{n}+1_d} \setminus \mathcal{I}_{\mathbf{n}}$, we have*

$$\mathbb{E}\left[\left(\widehat{Y}_{\mathbf{j}} - Y_{\mathbf{j}}^*\right)^2\right] = O\left(\frac{\psi_{\mathbf{n}}^2}{\phi_{\mathbf{n}}^2}\right) + O\left(\frac{(\log n)^2}{\phi_{\mathbf{n}}^2 \psi_{\mathbf{n}}^2 n^d}\right). \tag{7}$$

4. Brief outline of proofs

For the sake of simplicity, we only give the proof of the empirical operator $\Gamma_{\mathbf{n}}$, since for other estimators similar arguments can be applied.

Proof of Theorem 3.1. Let $T_{\mathbf{n}} = \Gamma_{\mathbf{n}} = \frac{1}{n^d} \sum_{\mathbf{i} \in \mathcal{I}_{\mathbf{n}}} X_{\mathbf{i}} \otimes_E X_{\mathbf{i}}$ and $T = \mathbb{E}(X \otimes_E X)$. Recall that

$$\mathbb{E}\left[\|T_{\mathbf{n}} - T\|_{\infty}^2\right] = \sup_{x \in E} \frac{\mathbb{E}\left(\|T_{\mathbf{n}}x - Tx\|_E^2\right)}{\|x\|_E^2}.$$

Let $L_{\mathbf{ij}} = \langle \langle X_{\mathbf{i}}, x \rangle_E X_{\mathbf{i}} - \mathbb{E}\langle \langle X, x \rangle_E X \rangle, \langle X_{\mathbf{j}}, x \rangle_E X_{\mathbf{j}} - \mathbb{E}\langle \langle X, x \rangle_E X \rangle \rangle_E$, then we have

$$\mathbb{E}\left(\|T_{\mathbf{n}}x - Tx\|_E^2\right) = \frac{1}{n^{2d}} \sum_{\mathbf{i} \in \mathcal{I}_{\mathbf{n}}} \mathbb{E}\left(\|\langle X_{\mathbf{i}}, x \rangle_E X_{\mathbf{i}} - \mathbb{E}\langle \langle X, x \rangle_E X \rangle\|_E^2\right) + \frac{1}{n^{2d}} \sum_{\mathbf{i} \neq \mathbf{j}} \mathbb{E}(L_{\mathbf{ij}}) := A + B,$$

where

$$A = \frac{1}{n^{2d}} \sum_{\mathbf{i} \in \mathcal{I}_{\mathbf{n}}} \mathbb{E}\left(\|\langle X_{\mathbf{i}}, x \rangle_E X_{\mathbf{i}} - \mathbb{E}\langle \langle X, x \rangle_E X \rangle\|_E^2\right) \leq \frac{4M^4 \|x\|_E^2}{n^d} = c_1 \frac{\|x\|_E^2}{n^d}$$

with c_1 a positive constant, and

$$B = \frac{1}{n^{2d}} \sum_{0 < |\mathbf{i} - \mathbf{j}| \leq C_n} \mathbb{E}(L_{\mathbf{ij}}) + \frac{1}{n^{2d}} \sum_{|\mathbf{i} - \mathbf{j}| > C_n} \mathbb{E}(L_{\mathbf{ij}}) := B_1 + B_2.$$

It is easy to show that $\mathbb{E}(L_{\mathbf{ij}}) \leq \mathbb{E}(\sqrt{L_{\mathbf{ii}}}\sqrt{L_{\mathbf{jj}}}) \leq 4M^4 \|x\|_E^2$. Then, we obtain

$$|B_1| \leq \frac{c_2 \|x\|_E^2 \log n}{n^d}$$

when $C_n = \lfloor (\log n)^{1/d} \rfloor$ (where $\lfloor x \rfloor$ stands for the integer part of x) and c_2 is a positive constant. By stationarity, boundedness of X ($\|X\|_E < M$ a.s.), and Lemma 2.1 (ii) in [11], we have

$$|\mathbb{E}(L_{\mathbf{ij}})| \leq C \|x\|_E^2 M^4 \alpha_{1,\infty}(|\mathbf{i} - \mathbf{j}|),$$

where C is a positive constant. Since $\alpha_{1,\infty}(t) = O(t^{-\theta})$ with $\theta \geq d + 1$, we have

$$|B_2| \leq \frac{C \|x\|_E^2 M^4}{n^d} \sum_{t=1}^{\infty} t^{d-1-\theta} \leq \frac{c_3 \|x\|_E^2}{n^d},$$

with c_3 a positive constant. Therefore, we deduce that $\mathbb{E}\left[\|T_{\mathbf{n}} - T\|_{\infty}^2\right] = O\left(n^{-d} \log n\right)$. \square

Proof of Corollary 3.1. (i) Let $T_{\mathbf{n}} = \Gamma_{\mathbf{n}} = \frac{1}{n^d} \sum_{\mathbf{i} \in \mathcal{I}_{\mathbf{n}}} X_{\mathbf{i}} \otimes_E X_{\mathbf{i}}$ and $T = \mathbb{E}(X \otimes_E X)$. By definition, we have

$$\|T_{\mathbf{n}} - T\|_{L^2(\mathcal{H}_S)} = \left\{ \mathbb{E} \left[\|T_{\mathbf{n}} - T\|_{\mathcal{H}_S}^2 \right] \right\}^{1/2}.$$

Since $(v_j)_{j \geq 1}$ is an orthonormal basis of E , we have

$$\mathbb{E} \left[\|T_{\mathbf{n}} - T\|_{\mathcal{H}_S}^2 \right] = \sum_{i=1}^Q \mathbb{E} \left[\|T_{\mathbf{n}}(v_i) - T(v_i)\|_E^2 \right] + \sum_{i>Q} \mathbb{E} \left[\|T_{\mathbf{n}}(v_i) - T(v_i)\|_E^2 \right] := A + B.$$

Let us first treat the second term B :

$$B = \sum_{i>Q} \mathbb{E} \left[\sum_{j=1}^{+\infty} \langle T_{\mathbf{n}}(v_i) - T(v_i), v_j \rangle_E^2 \right] \leq 4 \sum_{i>Q} \left[\mathbb{E} \left(\langle X, v_i \rangle_E^4 \right) \right]^{1/2} \sum_{j=1}^{+\infty} \left[\mathbb{E} \left(\langle X, v_j \rangle_E^4 \right) \right]^{1/2}.$$

Since $\langle X, v_j \rangle_E^4 \leq \|X\|_E^2 \|v_j\|_E^2 \langle X, v_j \rangle_E^2 < M^2 \langle X, v_j \rangle_E^2$ a.s. and $\mathbb{E} \left(\langle X, v_j \rangle_E^2 \right) = \lambda_j$ with $\lambda_j = O(r^j)$, $0 < r < 1$, $j \geq 1$, if $Q = \lfloor K \log n \rfloor$ with $K = \frac{3d}{\log(\frac{1}{r})}$, then $B \leq C_1 \exp(-K(\log n)(\log(1/r))/2) = \frac{C_1}{n^{3d/2}}$, where C_1 is a positive constant. Applying Theorem 3.1 and taking $Q = \lfloor K \log n \rfloor$ allow us to obtain the inequality $A \leq \mathbb{E} \left[\|T_{\mathbf{n}} - T\|_{\infty}^2 \right] \sum_{i=1}^Q \|v_i\|_E^2 \leq Cn^{-d}(\log n)^2$, where C is a positive constant. This finishes the proof of Corollary 3.1 (i).

The proofs of Corollary 3.1 (ii) and of Corollary 3.2 are derived from that of Corollary 3.1 (i) arguing as in [9]. \square

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