C. R. Acad. Sci. Paris, Ser. I 356 (2018) 725–731

Contents lists available at ScienceDirect

C. R. Acad. Sci. Paris, Ser. I

www.sciencedirect.com

Mathematical analysis/Partial differential equations

The quenching behavior of a quasilinear parabolic equation with double singular sources

Le comportement désactivant d'une équation parabolique quasi linéaire avec deux sources singulières

Liping Zhu

College of Science, Xi'an University of Architecture & Technology, Xi'an, 710055, China

A R T I C L E I N F O

Article history:
Received 9 April 2018
Accepted after revision 23 May 2018
Available online 30 May 2018

Presented by the Editorial Board

A B S T R A C T

In this paper, we study the quenching behavior for a one-dimensional quasilinear parabolic equation with singular reaction term and singular boundary flux. Under certain conditions on the initial data, we show that quenching occurs only on the boundary in finite time. Moreover, we derive some lower and upper bounds of the quenching rate and get some estimates for the quenching time.

R É S U M É

Nous étudions ici le comportement désactivant d'une équation parabolique quasi-linéaire avec un terme de réaction singulier et un flux au bord singulier. Sous certaines conditions sur les données initiales, nous montrons que la désactivation intervient seulement au bord en temps fini. De plus, nous obtenons des bornes inférieure et supérieure du taux de désactivation ainsi que des estimations du temps de désactivation.

1. Introduction

In this paper, we mainly study the following problem with double singular sources

$$\begin{cases} u_t = \left(|u_x|^{p-2}u_x\right)_x - u^{-r}, & 0 < x < 1,\ t > 0, \\ u_x(0,t) = u^{-q}(0,t),\ \ u_x(1,t) = 0, & t > 0, \\ u(x,0) = u_0(x), & 0 \le x \le 1, \end{cases} \tag{1.1}$$

E-mail address: 78184385@qq.com.

https://doi.org/10.1016/j.crma.2018.05.013
1631-073X/

where $p>1$, $r,q>0$, and u_0 satisfies the second-order compatibility conditions. Equation (1.1) models a generalized electrostatic Micro-Electro-Mechanical-System (MEMS) device consisting of a thin dielectric elastic membrane. In this model where $p=2$, the dynamic solution u characterizes the dynamic deflection of the elastic membrane; we refer the reader to [3,9] and the references therein. If quenching occurs in finite time, we denote by T the quenching time, or else $T=\infty$. Many authors have studied quenching problems with various nonlinear source terms and boundary conditions, we refer to [1,4–6,8,10–12,17] and references therein. Zhao [17] considered the problem

$$\begin{cases} u_t=\Delta u+u^p, & x\in\Omega,\ t>0,\\ \dfrac{\partial u}{\partial \nu}=-u^{-q}, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x), & x\in\Omega. \end{cases} \tag{1.2}$$

Under certain conditions on initial data, Zhao not only showed that quenching occurs only on the boundary, but also derived the quenching rate;

$$\min_{x\in\partial\Omega} u(x,t)\sim (T-t)^{\frac{1}{2(q+1)}},\quad t\to T^-.$$

In [18,19], Zhi and Mu studied the following semilinear equation

$$\begin{cases} u_t=u_{xx}+f(x)(1-u)^{-p}, & 0<x<1,\ t>0,\\ u_x(0,t)=u^{-q}(0,t),\ u_x(1,t)=0, & t>0,\\ u(x,0)=u_0(x), & 0\le x\le 1. \end{cases} \tag{1.3}$$

They proved that quenching occurs only at $x=0$, and that the quenching rate satisfies $u(0,t)\sim (T-t)^{1/[2(q+1)]}$, as $t\to T^-$. While $f(x)\equiv 1$ and the boundary flux becomes $u_x(1,t)=-u^{-q}(1,t)$, Selcuk and Ozalp [11] showed that the lower bound of the quenching rate is $u(0,t)\ge 1-C(T-t)^{1/(p+1)}$ for t sufficiently close to T. Furthermore, Ozalp and Selcuk [8] studied the semilinear equation with singular reaction term and singular boundary flux

$$\begin{cases} u_t=u_{xx}+(1-u)^{-p}, & 0<x<1,\ t>0,\\ u_x(0,t)=0,\ u_x(1,t)=(1-u(1,t))^{-q}, & t>0,\\ u(x,0)=u_0(x), & 0\le x\le 1. \end{cases} \tag{1.4}$$

Under some assumptions on initial data, they proved that quenching occurs in finite time and $x=1$ is the only quenching point. Moreover, the lower bound of the quenching rate was estimated, i.e. $u(1,t)\ge 1-C(T-t)^{1/(p+1)}$ if $p>2q+1$ and $u(1,t)\ge 1-C(T-t)^{1/[2(q+1)]}$ if $q\le p\le 2q+1$, as $t\to T^-$. However, they did not show the upper bound of the quenching rate.

To the best of our knowledge, very few works are concerned with the quenching rate of the quasilinear equations of p-Laplacian type, except for [13]. More precise, based on the work [2], Yang, Yin and Jin studied the p-Laplacian problem

$$\begin{cases} u_t=\left(|u_x|^{p-2}u_x\right)_x, & 0<x<1,\ t>0,\\ u_x(0,t)=0,\ u_x(1,t)=-g(u(1,t)), & t>0,\\ u(x,0)=u_0(x), & 0\le x\le 1, \end{cases} \tag{1.5}$$

where $\lim_{s\to 0^+} g(s)=+\infty$ and $g(s)>0,\ g'(s)<0$ for $s>0$. They showed that $x=1$ is the unique quenching point, and gave the quenching rate

$$\int_0^{u(1,t)} \frac{ds}{-g^{p-1}(s)g'(s)}\sim C(T-t),\quad t\to T^-.$$

Later, Yang, Yin and Jin [14] studied the positive radial solutions to (1.5) in higher dimensional space and got the similar results to [13]. Besides, there are also some other singular properties for nonlinear parabolic equations such as L^∞ blowup and gradient blowup, see the latest papers [7,15,16,20,21] for examples and the references therein.

Motivated by the works [8,11,13,18], in this paper, we will study the quenching phenomenon of the more generalized equation (1.1). We prove that quenching occurs only at $x=0$. Moreover, we give the bounds of the quenching rate and time. Our results are based on the ingenious construction of auxiliary functions. From our results, we know that the quenching rate of Problem (1.1) is really affected by both the reaction u^{-r} and the boundary flux u^{-q}.

Throughout this paper, we assume that the initial function u_0 satisfies

$$(|(u_0)_x|^{p-2}(u_0)_x)_x-u_0^{-r}\le 0,\ \text{but} \not\equiv 0,\ \ 0\le x\le 1. \tag{1.6}$$

$$u_0>0,\ \ (u_0)_x\ge 0 \text{ and } (u_0)_{xx}\le 0,\ \ 0\le x\le 1. \tag{1.7}$$

Here, we note that the assumptions (1.6)–(1.7) are proper, since we can easily find such u_0 satisfying (1.6)–(1.7) and compatibility conditions. For example, $u_0(x) = A^{-1/q} + Ax - \frac{A}{2}x^2$ for $A > 0$ large enough.

In the following, we use C to denote various generic positive constants if there is no confusion.

2. Quenching on the boundary

In this section, we prove that the solution u to Problem (1.1) quenches in finite time. In general, for the degeneracy, u may not be the classical solution and it is only the weak one. However, we here study smooth solutions for simplicity, since we may consider the corresponding equation to Problem (1.1) with the approximated initial boundary data. Especially, we can choose

$$\begin{aligned} &u_x(0,t) = u^{-q}(0,t), \quad u_x(1,t) = \varepsilon, \quad t > 0, \ \varepsilon > 0, \\ &u(x,0) = u_0(x) + \frac{\varepsilon}{2}x^2, \quad 0 \le x \le 1. \end{aligned}$$

Definition 2.1. The solution $u(x,t)$ to Problem (1.1) quenches in the finite time T, if there exists $0 < T < \infty$ such that

$$\lim_{t \to T^-} \min_{0 \le x \le 1} u(x,t) = 0. \tag{2.1}$$

Lemma 2.1. *Let (1.6)–(1.7) be in force, and the solution u to Problem (1.1) exists in $(0,T)$ for some $T > 0$. Then $u_x(x,t) > 0$ and $u_t(x,t) < 0$ in $[0,1) \times (0,T)$.*

Proof. First, we know that $u(x,t) \ge c > 0$ in $[0,1] \times [0,\tau]$ for any fixed $\tau \in (0,T)$. Let $v = u_x$. Then v satisfies

$$\begin{cases} v_t = \left(|v|^{p-2}v\right)_x + ru^{-r-1}v, & 0 < x < 1, \ 0 < t < \tau, \\ v(0,t) = u^{-q}(0,t), \ v(1,t) = 0, & 0 < t < \tau, \\ v(x,0) = (u_0)_x, & 0 \le x \le 1, \end{cases}$$

By the maximum principle, we know that $v > 0$, and thus $u_x(x,t) > 0$ in $[0,1) \times (0,\tau)$. Similarly, letting $V = u_t$, we get

$$\begin{cases} V_t = (p-1)\left(|u_x|^{p-2}V_x\right)_x + ru^{-r-1}V, & 0 < x < 1, \ 0 < t < \tau, \\ V_x(0,t) = -qu^{-q-1}(0,t)V, \ V_x(1,t) = 0, & 0 < t < \tau, \\ V(x,0) = (|(u_0)_x|^{p-2}(u_0)_x)_x + u_0^{-r}, & 0 \le x \le 1, \end{cases}$$

Using the maximum principle again, we obtain $u_t(x,t) < 0$ in $[0,1) \times (0,\tau)$. Obviously, we see that the solution u to Problem (1.1) is in fact classical, i.e. $u \in C^{2,1}([0,1) \times (0,T))$, and $u_x(x,t) > 0$ and $u_t(x,t) < 0$ in $[0,1) \times (0,T)$. $\square$

Theorem 2.1. *Let (1.6)–(1.7) be in force. Then every solution to Problem (1.1) quenches in finite time, and $x = 0$ is the only quenching point.*

Proof. Define

$$E(t) = \int_0^1 u(x,t)\,\mathrm{d}x, \quad \delta = u_0^{-(p-1)q}(0) + \int_0^1 u_0^{-r}(x)\,\mathrm{d}x.$$

We find that

$$\begin{aligned} E'(t) = \int_0^1 u_t(x,t)\,\mathrm{d}x &= \int_0^1 \left[\left(|u_x|^{p-2}u_x\right)_x - u^{-r}\right]\mathrm{d}x \\ &= -u^{-(p-1)q}(0,t) - \int_0^1 u^{-r}\mathrm{d}x \\ &\le -u_0^{-(p-1)q}(0) - \int_0^1 u_0^{-r}\mathrm{d}x \\ &= -\delta, \end{aligned} \tag{2.2}$$

where we use the condition of $u_t < 0$. Since

$$(|(u_0)_x|^{p-2}(u_0)_x)_x - u_0^{-r} \le 0, \text{ but not equal to 0 identically,} \tag{2.3}$$

then

$$\int_0^1 \left[(|(u_0)_x|^{p-2}(u_0)_x)_x - u_0^{-r}\right] \mathrm{d}x < 0. \tag{2.4}$$

Thus

$$\begin{aligned} &|(u_0)_x|^{p-2}(u_0)_x\Big|_0^1 - \int_0^1 u_0^{-r} \mathrm{d}x \\ =& -u_0^{-(p-1)q}(0) - \int_0^1 u_0^{-r} \mathrm{d}x \\ =& -\delta < 0. \end{aligned} \tag{2.5}$$

We obtain

$$E(t) \le E(0) - \delta t,$$

which can not hold for all time, and $u_x > 0$ for $x \in [0, 1)$, so there exists a finite time T such that $\lim_{t\to T^-} u(0,t) = 0$.

Next, we will prove $x = 0$ is the unique quenching point. Obviously, we only need to show that, for some $\eta \in (0, T)$, quenching can not occur in $\{(x,t)|\ x \in (0, 1/2),\ t \in (\eta, T)\}$. Without loss of generality, we assume $\min_{\eta\le t<T} u(0,t) = m > 0$. Define $J(x,t) = (u_x)^{p-1} - \varepsilon\,(3/4 - x)\,m^{-(p-1)q}$. By a simple calculation, we obtain

$$\begin{aligned} J_t =&(p-1)(u_x)^{p-2}u_{xt} \\ =&(p-1)(u_x)^{p-2}\left[((u_x)^{p-1})_{xx} + ru^{-r-1}u_x\right] \\ =&(p-1)(u_x)^{p-2}J_{xx} + (p-1)ru^{-r-1}(u_x)^{p-1}, \end{aligned} \tag{2.6}$$

so $J_t - (p-1)(u_x)^{p-2}J_{xx} \ge 0$ for $(x,t) \in (0, 3/4) \times (\eta, T)$. On the parabolic boundary, $J(3/4, t) = (u_x(3/4, t))^{p-1} \ge 0$ and $J(0,t) \ge (1 - 3\varepsilon/4)m^{-(p-1)q} \ge 0$, $t \in [\eta, T)$, if ε is small enough. Moreover, we have $J(x,\eta) \ge (u_x(x,\eta))^{p-1} - 3\varepsilon m^{-(p-1)q}/4 \ge 0$, $x \in [0, 3/4]$, if ε is small enough, where we use the fact $u_x > 0$.

Thus, we obtain by the maximum principle that $J(x,t) \ge 0$ in $(0, 3/4) \times (\eta, T)$, i.e.

$$u_x \ge \left[\varepsilon\left(\frac{3}{4} - x\right)m^{-(p-1)q}\right]^{\frac{1}{p-1}}.$$

Integrating from 0 to x, so

$$u(x,t) \ge u(0,t) + \int_0^x \left[\varepsilon\left(\frac{3}{4} - x\right)m^{-(p-1)q}\right]^{\frac{1}{p-1}} \mathrm{d}x > 0,$$

which means that $u(x,t) > 0$, if $x > 0$. So we complete the proof of Theorem 2.1. □

Theorem 2.2. *u_t blows up at the quenching point $x = 0$.*

Proof. Suppose that u_t is bounded in $[0,1] \times [0,T)$. Since $u_t \le 0$ by Lemma 2.1, there exists a constant $M > 0$ such that $u_t > -M$. We have $(|u_x|^{p-2}u_x)_x - u^{-r} > -M$, and thus $(|u_x|^{p-2}u_x)_x > -M$. Integrating it with respect to x first on $[0, x]$, and then on $[0, 1]$, we derive

$$u(1,t) - u(0,t) > \int_0^1 (-Mx + u^{-(p-1)q}(0,t))^{\frac{1}{p-1}} \mathrm{d}x.$$

As $t \to T^-$, the left is bounded, while the right tends to infinity. This contradiction implies that u_t will blow up at $x = 0$. □

3. Lower and upper bounds of the quenching rate

In the following, we shall derive the estimates of the lower and upper bounds of the quenching rate. In this section, we assume that u_0 satisfies

$$(u_0)_x \geq (1-x)^{\frac{1}{p-1}} u_0^{-q}, \quad 0<x<1, \tag{3.1}$$

and u satisfies, for $x=0$,

$$u_t(0,t) = (|u_x|^{p-2}u_x)_x(0,t) - u^{-r}(0,t), \quad 0<t<T. \tag{3.2}$$

Theorem 3.1. *Let (1.6)–(1.7), (3.1), and (3.2) be in force. Then there exist positive constants C_1 and C_2 such that*

$$\begin{cases} u(0,t) \leq C_1(T-t)^{\frac{1}{r+1}}, & \text{if } r>pq+1, \\ u(0,t) \leq C_2(T-t)^{\frac{1}{pq+2}}, & \text{if } q \leq r \leq pq+1, \end{cases} \tag{3.3}$$

as $t \to T^-$.

Proof. Let $G(x,t) = u_x^{p-1} - (1-x)u^{-(p-1)q}$ in $(0,1)\times(0,T)$. By a simple calculation, we know that $G(x,t)$ satisfies

$$\begin{aligned} & G_t - (p-1)u_x^{p-2}G_{xx} - (p-1)ru^{-r-1}G \\ = & (p-1)(r-q)(1-x)u^{-(p-1)q-r-1} \\ & +2(p-1)^2 q u^{-(p-1)q-1}u_x^{p-1} \\ & +(p-1)^2 q[(p-1)q+1](1-x)u^{-(p-1)q-2}u_x^p. \end{aligned}$$

Due to $r \geq q$ and $u_x>0$, we know from the maximum principle that $G(x,t)\geq 0$ in $(0,1)\times(0,T)$. Also, $G(x,0)\geq 0$ by (3.1) and $G(0,t)=G(1,t)=0$ for $0<t<T$. Therefore, we get

$$G_x(0,t) = \lim_{s\to 0^+}\frac{G(s,t)-G(0,t)}{s} = \lim_{s\to 0^+}\frac{G(s,t)}{s} \geq 0.$$

From (3.2), we get

$$\begin{aligned} G_x(0,t) &= \left[(|u_x|^{p-2}u_x)_x + (p-1)q(1-x)u^{-(p-1)q-1}u_x + u^{-(p-1)q} \right]\Big|_{x=0} \\ &= u_t(0,t) + u^{-r}(0,t) + u^{-(p-1)q}(0,t) + (p-1)qu^{-pq-1}(0,t). \end{aligned}$$

Hence, we have

$$\begin{cases} u_t(0,t) \geq -[(p-1)q+2]u^{-r}(0,t), & \text{if } r>pq+1, \\ u_t(0,t) \geq -[(p-1)q+2]u^{-pq-1}(0,t), & \text{if } q\leq r\leq pq+1, \end{cases}$$

Integrating with respect to t on (t,T), we derive

$$\begin{cases} u(0,t) \leq C_1(T-t)^{\frac{1}{r+1}}, & \text{if } r>pq+1, \\ u(0,t) \leq C_2(T-t)^{\frac{1}{pq+2}}, & \text{if } q \leq r \leq pq+1, \end{cases} \tag{3.4}$$

where $C_1 = \{(r+1)[(p-1)q+2]\}^{1/(r+1)}$ and $C_2 = \{(pq+2)[(p-1)q+2]\}^{1/(pq+2)}$. $\square$

Remark 3.1. From Theorem 3.1, we can get the lower bounds of the quenching time, which are as follows

$$\begin{cases} T \geq u_0^{r+1}(0)/\{(r+1)[(p-1)q+2]\}, & \text{if } r>pq+1, \\ T \geq u_0^{pq+2}(0)/\{(pq+2)[(p-1)q+2]\}, & \text{if } q\leq r\leq pq+1. \end{cases}$$

Motivated by [2,13], we will prove the lower bound of the quenching rate.

Let $d(u) = -qu^{-(p-1)(\sigma-1)q-q-1}$, and it is easy to check that there exists a value of σ satisfying

$$-\infty < \sigma < 1 - \frac{q+1}{(p-1)q}, \tag{3.5}$$

such that $(p-1)(\sigma-1)q+q+1<0$.

Theorem 3.2. *Let the hypotheses of Theorem 3.1 be in force, and (3.5) holds. Then, there exist constants $C_3, C_4 > 0$, such that the following inequalities hold*

$$\begin{cases} u(0,t) \geq C_3(T-t)^{\frac{1}{r+1}}, & \textit{if } r > pq+1, \\ u(0,t) \geq C_4(T-t)^{\frac{1}{pq+2}}, & \textit{if } q \leq r \leq pq+1, \end{cases} \tag{3.6}$$

as $t \to T^-$.

Proof. Choose $\tau \in (0,T)$ such that $T-\tau>0$ and $\kappa>0$ are small, and define $F(x,t) = u_t - d(u)(\varepsilon_1 u_x^\alpha + \varepsilon_2 u_x^\beta + \varepsilon_3 u_x^\gamma)$ in $\{(x,t)|\ x \in (0,\kappa),\ t \in (\tau,T)\}$, with the constants $\varepsilon_1, \varepsilon_2, \varepsilon_3 > 0$ and $\alpha = (p-1)(2-\sigma)$, $\beta = -[(p-1)(\sigma-1)q+1]/q$, $\gamma = -[(p-1)(\sigma-1)q+q+1-r]/q$. By a careful and lengthy computation, we get

$$\begin{aligned} &F_t - (p-1)u_x^{p-2}F_{xx} - (p-1)(p-2)u_x^{p-3}u_{xx}F_x \\ &= ru^{-r-1}(u_t - u^{-r}) + c(x,t), \end{aligned} \tag{3.7}$$

where

$$\begin{aligned} c(x,t) =& [(p-1)d''(u)u_x^p - d'(u)u_t](\varepsilon_1 u_x^\alpha + \varepsilon_2 u_x^\beta + \varepsilon_3 u_x^\gamma) \\ & - ru^{-r-1}d(u)(\varepsilon_1 \alpha u_x^\alpha + \varepsilon_2 \beta u_x^\beta + \varepsilon_3 \gamma u_x^\gamma) \\ & + (p-1)d(u)[\varepsilon_1\alpha(\alpha-1)u_x^{\alpha-2} + \varepsilon_2\beta(\beta-1)u_x^{\beta-2} + \varepsilon_3\gamma(\gamma-1)u_x^{\gamma-2}]u_x^{p-2}u_{xx}^2 \\ & + (p-1)d'(u)[\varepsilon_1(2\alpha+p-1)u_x^\alpha + \varepsilon_2(2\beta+p-1)u_x^\beta + \varepsilon_3(2\gamma+p-1)u_x^\gamma] \\ & \cdot u_x^{p-2}u_{xx}. \end{aligned}$$

Since $u_t < 0$, we know that $ru^{-r-1}(u_t - u^{-r}) + c(x,t) < 0$ if $\varepsilon_1, \varepsilon_2$ and ε_3 are small enough. Thus we have

$$F_t - (p-1)u_x^{p-2}F_{xx} - (p-1)(p-2)u_x^{p-3}u_{xx}F_x < 0, \tag{3.8}$$

$(x,t) \in (0,\kappa) \times (\tau,T)$. Also, since $x=0$ is the unique quenching point, then $F(\kappa,t)$ and $F(x,\tau)$ are non-positive for $\varepsilon_1, \varepsilon_2, \varepsilon_3$ sufficiently small. At $x=0$, by (3.5) and $u_t<0$, we obtain

$$\begin{aligned} F_x(0,t) =& -q[(p-1)(\sigma-1)q+q+1]u^{-q-1}(0,t)F(0,t) \\ & + [(p-1)(\sigma-1)q+1]u^{-q-1}(0,t)u_t(0,t) \\ & - d(u)(\varepsilon_1\alpha u_x^{\alpha-1} + \varepsilon_2\beta u_x^{\beta-1} + \varepsilon_3\gamma u_x^{\gamma-1})u_{xx} \\ \geq & -q[(p-1)(\sigma-1)q+q+1]u^{-q-1}(0,t)F(0,t), \end{aligned}$$

for $\varepsilon_1, \varepsilon_2, \varepsilon_3$ sufficiently small. Using the maximum principle, we have $F(x,t) \leq 0$ in $\{(x,t)|\ x \in [0,\kappa],\ t \in [\tau,T)\}$. More precisely, $F(0,t) \leq 0$, which implies that

$$\begin{aligned} u_t(0,t) &\leq d(u(0,t))(\varepsilon_1 u_x^\alpha + \varepsilon_2 u_x^\beta + \varepsilon_3 u_x^\gamma)(0,t) \\ &= -q\varepsilon_1 u^{-pq-1}(0,t) - q\varepsilon_2 u^{-q}(0,t) - q\varepsilon_3 u^{-r}(0,t). \end{aligned}$$

Therefore, we have

$$\begin{cases} u_t(0,t) \leq -q(\varepsilon_1+\varepsilon_2+\varepsilon_3)u^{-r}(0,t), & \text{if } r > pq+1, \\ u_t(0,t) \leq -q(\varepsilon_1+\varepsilon_2+\varepsilon_3)u^{-pq-1}(0,t), & \text{if } q \leq r \leq pq+1, \end{cases}$$

Integrating with respect to t on (t,T), we derive

$$\begin{cases} u(0,t) \geq C_3(T-t)^{\frac{1}{r+1}}, & \text{if } r > pq+1, \\ u(0,t) \geq C_4(T-t)^{\frac{1}{pq+2}}, & \text{if } q \leq r \leq pq+1, \end{cases}$$

where $C_3 = \{q(r+1)(\varepsilon_1+\varepsilon_2+\varepsilon_3)\}^{1/(r+1)}$ and $C_4 = \{q(pq+2)(\varepsilon_1+\varepsilon_2+\varepsilon_3)\}^{1/(pq+2)}$. So the proof of Theorem 3.2 is completed. □

Remark 3.2. From Theorem 3.2, we can get the upper bounds of the quenching time, which are as follows

$$\begin{cases} T \leq u_0^{r+1}(0)/\{q(r+1)(\varepsilon_1+\varepsilon_2+\varepsilon_3)\}, & \text{if } r > pq+1, \\ T \leq u_0^{pq+2}(0)/\{q(pq+2)(\varepsilon_1+\varepsilon_2+\varepsilon_3)\}, & \text{if } q \leq r \leq pq+1. \end{cases}$$

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Nos. 11401458, 11702206) and the grant of China Scholarship Council (No. 201607835015).

References

[1] M. Chipot, F.B. Weissler, Some blowup results for a nonlinear parabolic equation with a gradient term, SIAM J. Math. Anal. 20 (1989) 886–907.
[2] K. Deng, M.X. Xu, Quenching for a nonlinear diffusion equation with a singular boundary condition, Z. Angew. Math. Phys. 50 (1999) 564–584.
[3] P. Esposito, N. Ghoussoub, Y. Guo, Mathematical analysis of partial differential equations modeling electrostatic MEMS, Courant Lect. Notes Math. 20 (2010).
[4] E. Feireisl, H. Petzeltová, F. Simondon, Admissible solutions for a class of nonlinear parabolic problems with non-negative data, Proc. R. Soc. Edinb., Sect. A 131 (2001) 857–883.
[5] J.S. Guo, On the quenching behavior of the solution of a semilinear parabolic equation, J. Math. Anal. Appl. 151 (1990) 58–79.
[6] R.F. Li, L.P. Zhu, Z.C. Zhang, Quenching time for a semilinear heat equation with a nonlinear Neumann boundary condition, J. Partial Differ. Equ. 27 (2014) 217–228.
[7] Y. Li, Z.C. Zhang, L.P. Zhu, Classification of certain qualitative properties of solutions for the quasilinear parabolic equations, Sci. China Ser. A 61 (2018) 855–868.
[8] N. Ozalp, B. Selcuk, The quenching behavior of a nonlinear parabolic equation with a singular boundary condition, Hacet. J. Math. Stat. 44 (2015) 615–621.
[9] J.A. Pelesko, A.A. Bernstein, Modeling MEMS and NEMS, Chapman Hall and CRC Press, 2002.
[10] M.A. Rincon, J. Límaco, I. Liu, A nonlinear heat equation with temperature-dependent parameters, Math. Phys. Electron. J. 12 (2006).
[11] B. Selcuk, N. Ozalp, The quenching behavior of a semilinear heat equation with a singular boundary outflux, Q. Appl. Math. 72 (2014) 747–752.
[12] P. Souplet, Finite time blow-up for a non-linear parabolic equation with a gradient term and applications, Math. Methods Appl. Sci. 19 (1996) 1317–1333.
[13] Y. Yang, J.X. Yin, C.H. Jin, A quenching phenomenon for one-dimensional p-Laplacian with singular boundary flux, Appl. Math. Lett. 23 (2010) 955–959.
[14] Y. Yang, J.X. Yin, C.H. Jin, Quenching phenomenon of positive radial solutions for p-Laplacian with singular boundary flux, J. Dyn. Control Syst. 22 (2016) 653–660.
[15] Z.C. Zhang, Y. Li, Blowup and existence of global solutions to nonlinear parabolic equations with degenerate diffusion, Electron. J. Differ. Equ. 264 (2013).
[16] Z.C. Zhang, Y. Li, Classification of blowup solutions for a parabolic p-Laplacian equation with nonlinear gradient terms, J. Math. Anal. Appl. 436 (2016) 1266–1283.
[17] C.L. Zhao, Blow-up and Quenching for Solutions of Some Parabolic Equations, PhD Thesis, University of Louisiana, Lafayette, LA, USA, 2000.
[18] Y.H. Zhi, The boundary quenching behavior of a semilinear parabolic equation, Appl. Math. Comput. 218 (2011) 233–238.
[19] Y.H. Zhi, C.L. Mu, The quenching behavior of a nonlinear parabolic equation with nonlinear boundary outflux, Appl. Math. Comput. 184 (2007) 624–630.
[20] L.P. Zhu, Blowup time of solutions for a small diffusive parabolic problem with exponential source, Bound. Value Probl. 155 (2016) 1–15.
[21] L.P. Zhu, Z.C. Zhang, Rate of approach to the steady state for a diffusion-convection equation on annular domains, Electron. J. Qual. Theory Differ. Equ. 39 (2012).