C. R. Acad. Sci. Paris, Ser. I 356 (2018) 1075–1080

Contents lists available at ScienceDirect

C. R. Acad. Sci. Paris, Ser. I

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Number theory/Group theory

Explicit ring-theoretic presentation of Iwasawa algebras

Présentation explicite des algèbres d'Iwasawa en théorie des anneaux

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A R T I C L E I N F O

Article history:
Received 29 June 2018
Accepted 11 September 2018
Available online 30 October 2018

Presented by Jean-Loup Waldspurger

A B S T R A C T

Iwasawa algebras are completed group algebras of compact p-adic Lie groups. Ardakov and Venjakob have studied the structure theory and the ring-theoretic properties of such algebras. This article gives an explicit presentation by generators and relations of the Iwasawa algebras of uniform pro-p groups, i.e. the pro-p groups that admit a p-adic analytic manifold structure.

R É S U M É

Les algèbres d'Iwasawa sont des algèbres de groupes complétées des groupes de Lie p-adiques compacts. Ardakov et Venjakob ont étudié la structure et les propriétés d'anneaux de telles algèbres. Cette note donne une présentation explicite par générateurs et relations des algèbres d'Iwasawa des pro-p-groupes uniformes, c'est-à-dire des pro-p-groupes qui admettent une structure de variété analytique p-adique.

1. Introduction

Let p be a prime number. In this paper, we study the Iwasawa algebras of certain p-adic analytic pro-p groups. Recall that Lazard provides an extensive study of p-adic analytic Lie groups in his paper entiled *Groupes analytiques p-adiques* [5]. He defined the notion of p-saturated groups and characterized algebraically the notion of p-adic analytic groups as topological groups containing a topologically finitely generated open p-saturated pro-p group with integer valued filtration. Dixon, Sautoy, Mann and Segal in [3] use the notion of uniform pro-p groups to re-interpret Lazard's work from a group-theoretic perspective and deduced the same characterization of p-adic analytic groups; uniform pro-p groups play the same role as p-saturated pro-p groups with integer-valued filtration for $p > 2$ ([3, Notes, p. 81]).

The Iwasawa algebra (over $\mathbb{Z}_p$) of a uniform pro-p group G is the algebra of p-adic measures on G (sec 2.3). It is a non-commutative algebra, isomorphic as a $\mathbb{Z}_p$-module, to a commutative power series in several variables. Our goal is

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https://doi.org/10.1016/j.crma.2018.09.002
1631-073X/

to find relations between these variables and give an explicit ring theoretic presentation of the Iwasawa algebra using generators and relations.

Such an explicit presentation of Iwasawa algebras was originally given by Clozel in [2] for the first congruence kernel of $SL(2, \mathbb{Z}_p)$, and later generalized to the first congruence kernel of split, semi-simple, simply connected Chevalley groups over $\mathbb{Z}_p$ in [8]. Note that the first congruence kernel of Chevalley groups is a uniform pro-p group ([8, Sec. 4]). The result of this paper is an extension of [8]. Furthermore, in [7], we have deduced the explicit presentations of Iwasawa algebras for the pro-p Iwahori subgroup of $GL(n, \mathbb{Z}_p)$, which is not uniform pro-p but a p-saturated group in the terminology of Lazard. Thus, adapting arguments of [7], we hope that it might be possible to give explicit presentations of Iwasawa algebras for any p-saturated group. However, for this paper, we restrict to the case of uniform pro-p groups.

Such explicit presentations might be useful to make explicit computations in the Iwasawa algebra. For example, it is used by Clozel in the $SL(2)$ case (cf. [2]) to determine the center of the Iwasawa algebra. Apart from the above implication of our explicit presentation of the Iwasawa algebra, Dong Han and Feng Wei noted that our results about explicit presentation in [8] and [7] might shed some light on providing possible ways to answer the open question on the existence of non-trivial normal elements in $\Omega(G) := \Lambda(G) \otimes_{\mathbb{Z}_p} \mathbb{F}_p$ (cf. introduction and section 5 of [4]), where $\Lambda(G)$ is the Iwasawa algebra of G. An element $r \in \Omega(G)$ is normal if $r\Omega(G) = \Omega(G)r$. The question on the normal elements was originally posed in [1], later reformulated in [4] having dealt with the case for $SL(2, \mathbb{Z}_p)$ and $SL(3, \mathbb{Z}_p)$. As noted in [4], the normal elements help in constructing reflexive ideals in the Iwasawa algebra. The main question of Han and Wei is to find a mechanism for constructing ideals of completed group algebras without using central elements or closed normal subgroups which provide natural ways to construct ideals in the Iwasawa algebra (*loc. cit.*).

Our main result is the following.

Theorem *(see Theorem 4.4). For $p > 2$ the Iwasawa algebra of a uniform pro-p group is naturally isomorphic as a topological ring to $\mathcal{A}/\mathcal{R}$.*

Here $\mathcal{A}$ is the algebra of non-commutative power series over $\mathbb{Z}_p$ in several variables $X_1, \cdots, X_d$, with d equal to the dimension of the uniform pro-p group as an analytic manifold and $\mathcal{R}$ is a closed two sided ideal in $\mathcal{A}$ generated by a set of relations between these variables given by (3.1).

2. Brief review on analytic groups and Iwasawa algebras

In this section, we first briefly recall the notion of p-saturated groups and an ordered basis in the terminology of Lazard (section 2.1). Then, in section 2.2, we recall the definition of uniform pro-p groups following [3] and find relations between its ordered basis elements. Finally, in section 2.3 we recall the notion of the Iwasawa algebra and its topological filtration.

2.1. Lazard's p-saturated groups

Let G be a pro-p group. A *p-valuation* ω on G is a real valued function $\omega : G \setminus \{1\} \to (1/(p-1), \infty)$ satisfying

$$\begin{aligned} \omega(gh^{-1}) &\geq \min(\omega(g), \omega(h)), \\ \omega(g^{-1}h^{-1}gh) &\geq \omega(g) + \omega(h), \\ \omega(g^p) &= \omega(g) + 1, \end{aligned}$$

for $g, h \in G$ ([5, III.2.1.2]). By convention we set $\omega(1) = \infty$. The topology of G is defined by the filtration $G_\nu = \{g \in G \mid \omega(g) \geq \nu\}$ for all positive, real number ν.

An *ordered basis* of G is an ordered sequence of elements $(g_1, ..., g_d) \in G \setminus \{1\}$ such that the continuous map

$$\begin{aligned} \mathbb{Z}_p^d &\to G \\ (x_1, ..., x_d) &\mapsto g_1^{x_1} \cdots g_d^{x_d} \end{aligned}$$

is a homeomorphism and $\omega(g_1^{x_1}, \cdots g_d^{x_d}) = \min_{1 \leqslant i \leqslant d}(\omega(g_i) + \mathrm{val}_p(x_i))$ for $x_i \in \mathbb{Z}_p$, where val_p is the p-adic valuation of $\mathbb{Z}_p$.

Also by [5, III.2.1.6], G is defined to be *p-saturated* if any $g \in G$ satisfying $\omega(g) > \frac{p}{p-1}$ is a p-th power. Notice that, in [10], Schneider and Teitelbaum introduced a subclass of p-saturated groups satisfying the following hypothesis

(HYP) G is p-saturated and $\omega(g_i) + \omega(g_j) > p/(p-1)$ for $1 \leq i \neq j \leq d$.

According to the remark after lemma 4.3 of [10], the class of groups satisfying (HYP) is the same as the class of uniform pro-p groups.

2.2. Uniform pro-p-groups

The pro-p group G is said to be *powerful* if $[G,G] \le G^p$. Here $[G,G]$ and G^p denote the closures of the commutator subgroup and the subgroup generated by p-th powers, respectively.

The group G is said to be *uniform* in the sense of [3, Thm. 4.5] if and only if it is topologically finitely generated, powerful, and torsion-free.

If G is uniform, by the remark after lemma 4.3 of [10], we fix on G an integrally valued p-valuation ω and an ordered basis $(g_1, ..., g_d)$ of G such that $\omega(g_i) = 1$ for all $1 \le i \le d$ satisfying (HYP). Henceforth, throughout the text, we will assume that G is uniform and we will work with an ordered basis $(g_1, ..., g_d)$ and a p-valuation ω such that $\omega(g_i) = 1$. According to Proposition 4.3.2 of [3] and its proof, for $1 \le i < j \le d$, we have the following relation between the elements of the ordered basis

$$g_j^{-1} g_i^{-1} g_j g_i = \prod_{m=1}^{d} g_m^{\lambda_m(i,j)} \tag{2.1}$$

where $\lambda_m(i,j) \in p\mathbb{Z}_p$. These are the relations that we are going to use in order to find relations in the Iwasawa algebra of G.

2.3. Iwasawa algebra

We recall that the Iwasawa algebra $\Lambda(G)$ of G is a non-commutative algebra defined by taking the projective limit of group rings

$$\Lambda(G) := \varprojlim_{H \in \mathcal{H}} \mathbb{Z}_p[G/H]$$

where $\mathcal{H}$ is the set of open normal subgroups of G. As a $\mathbb{Z}_p$-module, the Iwasawa algebra is isomorphic to a commutative power series ([9, Chapter IV]). The isomorphism is given by

$$\delta : \mathbb{Z}_p[[X_1, ..., X_d]] \cong \Lambda(G) \tag{2.2}$$

$$1 + X_i \mapsto g_i. \tag{2.3}$$

Thus, any element $\lambda \in \Lambda(G)$ can be written in the form of power series

$$\lambda = \sum_{r \in \mathbb{N}^d} a_r X^r$$

where $a_r \in \mathbb{Z}_p, r = (r_1, ..., r_d) \in \mathbb{N}^d$, $X^r = X_1^{r_1} \cdots X_d^{r_d}$. The products of X_i's can also be thought as convolutions of the distributions inside the Iwasawa algebra identifying X_i with $g_i - 1$. The topology of the Iwasawa algebra is given by the powers of the maximal ideal $M_\Lambda := \{\lambda \in \Lambda(G) \mid \tilde{\omega}(\lambda) \ge 1\}$, where $\tilde{\omega}$ is the function

$$\tilde{\omega} : \Lambda(G) \backslash \{0\} \to [0, \infty)$$

given by

$$\tilde{\omega}\Big(\sum_r a_r X^r\Big) := \inf_r \Big(\mathrm{val}_p(a_r) + |r|\Big)$$

where $|r| := r_1 + \cdots + r_d$, with the convention that $\tilde{\omega}(0) := \infty$. Also note that, by [9, Chapter 6], $M_\Lambda^N := \{\lambda \in \Lambda(G) \mid \tilde{\omega}(\lambda) \ge N\}$ is indeed the N-th power of M_Λ.

Our objective is to find relations between the variables X_i's in (2.2) and find a *ring theoretic* description of the Iwasawa algebra in terms of generators and relations.

3. Generators and relations in the Iwasawa algebra

In this section, we are going to give the main result of this article, that is, Theorem 3.1, deducing relations in the Iwasawa algebra. We postpone its proof to section 4.

Let $\mathcal{A}$ be the non-commutative power series over $\mathbb{Z}_p$ in the variables $X_i, 1 \le i \le d$, ordered according to the ordered basis (g_i). The topology on $\mathcal{A}$ is given by powers of its maximal ideal $M_\mathcal{A} := (p, X_1, ..., X_d)$. We can define a valuation $w_\mathcal{A}$ given for:

$$f=\sum_r a_r X^r,$$

by $w_{\mathcal{A}}(f)=\inf_r(\mathrm{val}_p(a_r)+|r|)$.

We also define $M^N_{\mathcal{A}}$ and it is equal to the N-th power of $M_{\mathcal{A}}$.

By (2.1) and (2.2), we have the following relations in $\mathcal{A}$:

$$(1+X_j)(1+X_i)=(1+X_i)(1+X_j)\Big(\prod_{m=1}^{d}(1+X_m)^{\lambda_m(i,j)}\Big) \quad 1\le i<j\le d. \tag{3.1}$$

Let $\mathcal{R}$ be the closed two-sided ideal generated by these relations in $\mathcal{A}$. We are going to show the following theorem.

Theorem 3.1. *For $p>2$, the Iwasawa algebra of the uniform pro-p group G is isomorphic as a topological ring to the quotient $\mathcal{A}/\mathcal{R}$.*

4. Proof of Theorem 3.1

In this section, as in [2], the idea of the proof is to first prove Theorem 3.1 by reducing the Iwasawa algebra and $\mathcal{A}/\mathcal{R}$ modulo p and then lift the coefficients to $\mathbb{Z}_p$. We are going to prove a technical proposition (see Proposition 4.2) regarding a bound on the dimension of the n-th graded piece of $\mathcal{A}/\mathcal{R}$ modulo p, which is going to help us to prove Theorem 3.1 for the Iwasawa algebra modulo p.

By (2.2), we view the variable X_i as an element of the Iwasawa algebra and thus we have a natural map $\varphi:\mathcal{A}\to\Lambda(G)$ sending X_i to itself.

Lemma 4.1. *The map $\varphi:\mathcal{A}\to\Lambda(G)$ is surjective and continuous. Moreover, for $N\geqslant 0$, we have*

$$\varphi(\mathcal{M}^N_{\mathcal{A}})\subset\mathcal{M}^N_{\Lambda}.$$

Proof. Proof same as [2, Prop. 2.2]. □

This gives the continuous surjection $\varphi:\mathcal{B}:=\mathcal{A}/\mathcal{R}\to\Lambda(G)$.

Let

$$\Omega(G):=\Lambda(G)\otimes_{\mathbb{Z}_p}\mathbb{F}_p$$

be the Iwasawa algebra with finite coefficients ([11]), $\overline{\mathcal{A}}=\mathbb{F}_p\{\{X_1,\dots,X_d\}\}=\mathcal{A}\otimes\mathbb{F}_p$, $\overline{\mathcal{R}}$ be the image of $\mathcal{R}$ in $\overline{\mathcal{A}}$. Then, by [2, Lem 1.3], $\overline{\mathcal{R}}$ is the closed two-sided ideal generated in $\overline{\mathcal{A}}$ by the images of the relations in (3.1). This implies that we have the natural surjection

$$\overline{\varphi}:\overline{\mathcal{B}}:=\overline{\mathcal{A}}/\overline{\mathcal{R}}\to\Omega(G).$$

By abelian distribution theory, $\Omega(G)$ is isomorphic as a $\mathbb{F}_p$-module to the commutative power series ring in the variables X_i, $1\le i\le d$ over $\mathbb{F}_p$. Also, it is easy to show that

$$\mathcal{M}^N_{\Omega}=\{\lambda\in\Omega(G):v_{\Omega}(\lambda)\geqslant N\},$$

v_{Ω} being the usual valuation on power series, is the reduction of $\mathcal{M}^N_{\Lambda}$. It is also a (two-sided) ideal, $\mathcal{M}_{\Omega}$ being the maximal ideal.

Similarly, in $\mathcal{A}$ we have that the reduction mod p of $\mathcal{M}^N_{\mathcal{A}}$ is the ideal of series

$$\overline{f}=\sum_r a_r X^r \quad (a_r\in\mathbb{F}_p)$$

such that $|r|\geqslant N$. We obtain the maximal ideal $\mathcal{M}_{\overline{\mathcal{A}}}$ of $\overline{\mathcal{A}}$ by setting $N=1$. Moreover, $(\mathcal{M}_{\overline{\mathcal{A}}})^N=\mathcal{M}^N_{\overline{\mathcal{A}}}$.

Let $F^n\overline{\mathcal{A}}$ denote the natural filtration of $\overline{\mathcal{A}}$ by the powers of $\mathcal{M}_{\overline{\mathcal{A}}}$. We have $F^n\overline{\mathcal{A}}/F^{n+1}\overline{\mathcal{A}}=\mathrm{gr}^{\,n}\overline{\mathcal{A}}$. The filtration F^n induces a filtration on $\overline{\mathcal{B}}$:

$$F^n\overline{\mathcal{B}}=F^n\overline{\mathcal{A}}+\overline{\mathcal{R}}$$

and hence

$$\mathrm{gr}^n\overline{\mathcal{B}}=F^n\overline{\mathcal{A}}+\overline{\mathcal{R}}/F^{n+1}\overline{\mathcal{A}}+\overline{\mathcal{R}}.$$

Hence we have

$$\mathrm{gr}^n\overline{\mathcal{B}} = F^n\overline{\mathcal{A}}/F^{n+1}\overline{\mathcal{A}} + (F^n\overline{\mathcal{A}} \cap \overline{\mathcal{R}})$$

([2, Sec. 3]). Let S_n be the space of homogeneous commutative polynomials in the variables $\{X_i, 1 \le i \le d\}$ over $\mathbb{F}_p$ of degree n. Let Σ_n be the corresponding space of homogeneous non-commutative polynomials of degree n; hence $\Sigma_n \to F^n\overline{\mathcal{A}}/F^{n+1}\overline{\mathcal{A}}$, and therefore $\Sigma_n \to \mathrm{gr}^n\overline{\mathcal{B}}$, is surjective.

Proposition 4.2. *We have* $\dim \mathrm{gr}^n\overline{\mathcal{B}} \leqslant \dim S_n$.

Proof. The case $n=0$ and $n=1$ are obvious as $\mathrm{gr}^0\overline{\mathcal{B}}$ is generated by constant X_i^0 and $\mathrm{gr}^1\overline{\mathcal{B}}$ is generated by monomials $X_i, 1 \le i \le d$ of degree 1. For $n=2$, we will study relations in $\overline{\mathcal{R}}$ and use induction to complete the proof. From (3.1) and by definition of $\mathcal{R}$, we have for all $1 \le i < j \le d$,

$$(1+X_j)(1+X_i) - (1+X_i)(1+X_j)\Big(\prod_{m=1}^{d}(1+X_m)^{\lambda_m(i,j)}\Big) \in \mathcal{R}.$$

We expand

$$(1+X_m)^{\lambda_m(i,j)} = 1 + \lambda_m(i,j)X_m + \frac{\lambda_m(i,j)(\lambda_m(i,j)-1)}{2}X_m^2 + R_m(X_m)$$

with $R_m(X_m)$ of degree ≥ 3. As $\lambda_m(i,j) \in p\mathbb{Z}_p$ we have $\lambda_m(i,j)X_m = 0$ in $\mathrm{gr}^2\overline{\mathcal{B}}$. As $p>2$, 2 is invertible in $\mathbb{Z}_p$, and therefore

$$\frac{\lambda_m(i,j)(\lambda_m(i,j)-1)}{2}X_m^2 = 0 \qquad \text{in } \mathrm{gr}^2\overline{\mathcal{B}}$$

As in $\mathrm{gr}^2\overline{\mathcal{B}}$, polynomials with degree ≥ 3 equals 0, we obtain $R_m(X_m) = 0$ in $\mathrm{gr}^2\overline{\mathcal{B}}$. Altogether, we deduce that $(1+X_m)^{\lambda_m(i,j)} = 1$ in $\mathrm{gr}^2\overline{\mathcal{B}}$, which implies that

$$(1+X_j)(1+X_i) = (1+X_i)(1+X_j) \qquad \text{in } \mathrm{gr}^2\overline{\mathcal{B}}.$$

Hence

$$X_jX_i = X_iX_j \qquad \text{in } \mathrm{gr}^2\overline{\mathcal{B}}.$$

Therefore,

$$\dim \mathrm{gr}^2\overline{\mathcal{B}} \leqslant \dim S_2.$$

Now consider an arbitrary monomial of degree n,

$$X^k = X_{k_1} \dots X_{k_n}.$$

Following Lemma 3.2 of [2], we can change X^k into a well-ordered monomial ($b \to k_b$ increasing) by a sequence of transpositions. Consider a move $(b, b+1) \to (b+1, b)$ and assume $k_b > k_{b+1}$. We write

$$X^k = X^f X_{k_b} X_{k_{b+1}} X^e$$

where $\deg(f) = r$, $\deg(e) = s$ and $\deg(k) = n$. Then we have $X_{k_b}X_{k_{b+1}} = X_{k_{b+1}}X_{k_b}[F^3\overline{\mathcal{B}}]$. So we deduce that $X^f X_{k_{b+1}} X_{k_b} X^e \equiv X^k[F^{n+1}], n = r+s+2$. This reduces the number of inversions in X^k and proves Proposition 4.2. □

Theorem 4.3. *For $p>2$, the Iwasawa algebra mod p, $\Omega(G)$, is naturally isomorphic to $\overline{\mathcal{A}}/\overline{\mathcal{R}}$.*

Proof. (Cf. [2]). The natural map $\varphi : \mathcal{A} \to \Lambda(G)$ sends $\mathcal{M}_{\mathcal{A}}^n$ to $\mathcal{M}_{\Lambda}^n$. As $F^\bullet$ on $\overline{\mathcal{B}}$ is the filtration inherited from the natural filtration on $\overline{\mathcal{A}}$, we see that $\overline{\varphi}$ sends $F^n\overline{\mathcal{B}}$ to $\mathcal{M}_{\Omega}^n$. As $\overline{\varphi}$ is surjective, the natural map

$$\mathrm{gr}\overline{\varphi} : \mathrm{gr}^\bullet\overline{\mathcal{B}} \to \mathrm{gr}^\bullet\Omega(G)$$

is surjective. Moreover, it is an isomorphism because $\dim \mathrm{gr}\ ^n\overline{\mathcal{B}} \leqslant \dim S_n = \dim \mathrm{gr}\ ^n\Omega(G)$. (The last equality follows from Theorem 7.24 of [3]). Since the filtration on $\overline{\mathcal{B}}$ is complete, we deduce that $\overline{\varphi}$ is an isomorphism (cf. Theorem 4 (5), p. 31 of [6]). We have $\overline{\mathcal{B}}$ complete because $\overline{\mathcal{B}} = \overline{\mathcal{A}}/\overline{\mathcal{R}}$, where $\overline{\mathcal{R}}$ is closed, and therefore complete for the filtration induced from that of $\overline{\mathcal{A}}$. □

Theorem 4.4. *For $p>2$, the Iwasawa algebra $\Lambda(G)$ is naturally isomorphic to $\mathcal{A}/\mathcal{R}$.*

Proof. The reduction of φ is $\overline{\varphi}$. We recall that $\overline{\mathcal{R}}$ is the image of $\mathcal{R}$ in $\overline{\mathcal{A}}$. Let $f \in \mathcal{A}$ satisfies $\varphi(f) = 0$. We then have $\overline{f} \in \overline{\mathcal{R}}$ since $\overline{\mathcal{A}}/\overline{\mathcal{R}} \cong \Omega(G)$. So $f = r_1 + pf_1, r_1 \in \mathcal{R}$, $f_1 \in \mathcal{A}$. Then $\varphi(f_1) = 0$. Inductively, we obtain an expression $f = r_n + p^n f_n$ of the same type. Since $p^n f_n \to 0$ in $\mathcal{A}$ and $\mathcal{R}$ is closed, we deduce that $f \in \mathcal{R}$. $\square$

In conclusion, we have found an explicit presentation of the Iwasawa algebras by generators and relations for uniform pro-p groups which is a subclass of p-saturated groups. Since the pro-p Iwahori of $GL(n, \mathbb{Z}_p)$ is, in general, not uniform, we believe that adapting arguments of [7] it might be possible to give explicit presentations of the Iwasawa algebras for any p-saturated group.

Acknowledgements

I thank Konstantin Ardakov for posing me the question of the possibility of generalizing the explicit presentation of the Iwasawa algebra in [8] to more general groups.

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