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ERRATUM OF "THE SQUARES OF THE LAPLACIAN-DIRICHLET EIGENFUNCTIONS ARE GENERICALLY LINEARLY INDEPENDENT"

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Firstly, we would like to clarify that domains are, by definition, connected. This is not precisely stated in the definition of Σ_m in Section 2.1.

Secondly, in the proof of Theorem 2.4 we claim that "each $\Lambda_k(t)$ converges, as $t \to +\infty$, to an eigenvalue of the Laplacian-Dirichlet operator on $\hat{\Omega}$". In general, this is not true.

The result stated in Theorem 2.4, however, is true and can actually be strengthened as follows.

Theorem 2.4. *Let $(F_n)_{n\in\mathbb{N}}$, $(\mathcal{P}_n)_{n\in\mathbb{N}}$ and $(\mathcal{R}_n)_{n\in\mathbb{N}}$ be as in the statement of Theorem 2.3. Then, for every $m \in \mathbb{N} \cup \{+\infty\}$, a generic $\Omega \in \Sigma_m$ satisfies $\mathcal{P}_n$ for every $n \in \mathbb{N}$.*

A proof of this result can be based on the following strengthened version of Proposition 2.2.

Proposition 2.2 (Teytel)**.** *Let $m > 2$ and Ω_0, Ω_1 be two domains in Σ_m that are $\mathcal{C}^m$-differentiably isotopic. Then there exists an analytic curve $[0,1] \ni t \mapsto Q_t$ of C^m-diffeomorphisms such that Q_0 is equal to the identity, $Q_1(\Omega_0) = \Omega_1$ and the Laplacian-Dirichlet operator has simple spectrum on every domain $\Omega_t = Q_t(\Omega_0)$ for t in the open interval $(0,1)$.*

Teytel proves Proposition 2.2 in the case where Ω_0 and Ω_1 are $\mathcal{C}^m$-differentiably isotopic to the unit d-dimensional ball. His argument applies also, without modifications, to pairs of domains belonging to the same isotopy class.

The proof of Theorem 2.4, in its new formulation, works by replacing:

– "*Fix $m \in \mathbb{N} \cup \{+\infty\}$. [...] We are left to prove that $\hat{\mathcal{A}}_j$ is dense in Σ_m.*" with "Fix $m \in \mathbb{N} \cup \{+\infty\}$. Thanks to Theorem 2.3, a generic $\hat{\Omega} \in D_m$ satisfies $\mathcal{P}_n$ for every $n \in \mathbb{N}$. Fix one such $\hat{\Omega}$ and notice that, in particular, the spectrum $(\lambda_n^{\hat{\Omega}})_{n\in\mathbb{N}}$ is simple.

Define, for every $n \in \mathbb{N}$, the set

$$\hat{\mathcal{A}}_n = \{\Omega \in \Sigma_m \mid \Omega \text{ satisfies } \mathcal{P}_n\}.$$

The openness of $\hat{\mathcal{A}}_n$ in Σ_m can be proved following exactly the same argument used in the proof of Theorem 2.3 to show that each A_n is open in D_m. We are left to prove that $\hat{\mathcal{A}}_n$ is dense in Σ_m. Without loss of generality $m > 2$."

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– "*Moreover,* $t \mapsto \Omega_t$ *is an analytic path in* Σ_m*. [...] Since, for* t *small enough,* $\Lambda_k(t) = \lambda_{j_k}^{\Omega_t}$*, we deduce that* Ω *can be approximated arbitrarily well in* Σ_m *by an element of* $\hat{\mathcal{A}}_j$*.*" with "Moreover, each Ω_t is isotopic to Ω. It follows from Proposition 2.1 that we can fix t large enough in such a way that Ω_t verifies $\mathcal{P}_n$. Proposition 2.2 implies that there exists an analytic path of domains $s \mapsto \tilde{\Omega}_s$ such that $\tilde{\Omega}_0 = \Omega$, $\tilde{\Omega}_1 = \Omega_t$ and the spectrum of the Laplacian-Dirichlet operator on $\tilde{\Omega}_s$ is simple for every $s \in (0,1)$.

Hence, as in the proof of Theorem 2.3, we can deduce that $\tilde{\Omega}_s$ satisfies $\mathcal{P}_n$ for all but finitely many $s \in [0,1]$. In particular, Ω is in the closure of $\hat{\mathcal{A}}_n$."