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MODELLING AND OPTIMIZATION OF OUTPATIENT APPOINTMENT SCHEDULING

Mahmoud H. Alrefaei[1] and Ali Diabat[2]

Abstract. We consider the problem of appointment scheduling for outpatient departments in health care systems. The objective is to design an appointment system that minimizes the average waiting time per patient, while at the same time ensuring the effective use of resources, by maximizing doctor utilization and minimizing the average number of patients in the clinic. We model the appointment system problem as a multi-objective optimization problem with three objectives. Several new alternative appointment systems are considered, and the new systems are modelled and simulated using the software Arena. Subsequently, a new version of ranking and selection approaches is used to compare the alternative systems, by constructing a set of Pareto optimal solutions that consists of non-dominated systems with a predetermined level of confidence. Finally, we present the numerical results obtained by implementing the proposed procedure on an outpatient clinic, taking into account the no-show patients as well as the walk-in patients.

Keywords. Appointment system, ranking and selection, multi-objective optimization.

Mathematics Subject Classification. 90B50, 90B36, 81T80, 68M20.

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[1] Department of Mathematics and Statistics, Jordan University of Science and Technology, P.O. Box 3030, 22110 Irbid, Jordan. alrefaei@just.edu.jo

[2] Department of Engineering Systems & Management, Masdar Institute of Science & Technology, Abu Dhabi, United Arab Emirates. adiabat@masdar.ac.ae

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1. Introduction

The scheduling of outpatient departments is considered to be one of the major determinants of health care sector efficiency, aiming to provide excellent services for patient satisfaction and at the same time to effectively use the available resources. Among these services is the appointment system (AS), which is essentially a scheduling tool to determine the arrival of patients, in a way that improves the productivity of the outpatient department. Seeing as long waiting hours have remained a long term complaint from the patients' side, this improvement becomes a central concern for action.

Thus, in the current paper we study the appointment system and how it can be improved to better satisfy patients by reducing waiting times, as this has always been the most important determinant of service quality. The current situation in the clinic of interest is that patients arrive and register at the beginning of each session. Upon registration, they are assigned an order and must wait until their turn has come. In some cases, patients must wait very long hours to receive consultation from a specialist. Several appointment systems are modelled and simulated using Arena simulation software; then, these systems are evaluated based on three performance measures before the best systems are selected. These measures include the minimization of the average waiting time per patient, the maximization of doctor time utilization and the minimization of the average number of patients in the clinic. The consideration of three conflicting objectives leads to a multi-objective optimization problem, for which we propose a ranking and statistical selection method to construct a set that contains the best systems with a pre-specified level of confidence. Several researchers have assessed the operation of an outpatient department; see Jacobson and Swisher [12] and Cayirli and Veral [8] for overall health care and for outpatient scheduling.

In the literature, the popular appointment systems (AS's) range from single-block appointments, according to which patients arrive collectively at the beginning of a clinic session, to individual appointments, where they arrive independently. Most of the AS's can be considered as a modification and combination of these two types of systems. Any appointment system scenario consists of three parts; the appointment interval, the block size and the initial block of patients. The most used scenarios in hospitals are the single-block appointments which allocate a date rather than an exact appointment for patients, Babes and Sarma [3]. This scenario creates long waiting times for patients and at the same time maximizes the utilization of the doctors time. Many researchers including Klassen and Rohleder [14], Rohleder and Klassen [17] and Cayirli *et al.* [6, 7] have studied the individual-block/fixed-interval system; in this scenario, patients are scheduled individually in each time interval that equals to the mean consultation time of the doctor. Bailey [4] introduced the individual-block/fixed-interval with an initial block system which is similar to the individual-block/fixed-interval system but the number of patients assigned to the initial block is set to two patients rather than one patient. Brahimi and Worthington [5] have suggested three patients for the initial block instead of two. Soriano [18] has used the multiple block/fixed-interval rule that calls a fixed number of patients at the beginning of each block by introducing a scheduling system according to which two patients are scheduled at each time interval that is twice the consultation time. The variable-block/fixed-interval system assigns a number of patients that varies from one interval to another but with fixed appointment intervals, Liu and Liu [16]. The Individual-block/variable-interval rule assigns individual patients in unequal appointment intervals; Ho and Lau [11] have introduced this rule by setting smaller intervals at the beginning of the session but these intervals increase in time as the sessions proceed. Finally, multiple patient blocks with variable-intervals have been considered by Cayirli *et al.* [6]. Jerbi and Kamoun [13] have used a mixed goal programming approach to compare several approaches and select the best appointment system among the many alternatives. Nearly all the above studies have concentrated only on appointment rules, without taking into consideration factors such as patient characteristics, which can have an impact when designing the appointment systems. Alrefaei *et al.* [2] considered the problem as a multi-objective optimization and use ranking and selection to build a Pareto set on non-dominated solutions. For a more comprehensive review of appointment scheduling systems see Gupta and Denton [10] and Wijewickrama and Takakuwa [19].

In the current paper, we integrate modelling and simulation with an optimization technique for selecting an optimal set of systems for the appointment problem in an outpatient department. More specifically, we consider the case of a single doctor and we implement different appointment systems in Arena. Then, we apply a new version of ranking and statistical selection for the multi-objective optimization problem, in order to construct the Pareto set that will dominate all others. This paper is an extended version of the paper by Alrefaei *et al.* [2] presented in the International Conference on Operations Research and Statistics (ORS 2011) that was held in Penang, Malaysia.

This paper is structured as follows: Section 2 provides an overview of modelling assumptions and important aspects of the simulation approach. Section 3 presents the detailed construction of the Arena model, while Section 4 explains the ranking and selection procedure. Section 5 discusses the results, while Section 6 concludes the work with important findings.

2. Simulating appointment systems

In this section, we begin by discussing the flow of patients in the clinic. Then, we describe the method for collecting the required data before presenting the various appointment systems that are considered in this work. Finally, we conclude this section with the methodology for constructing the Arena simulation model.

Patients flow in the clinic: Patients start arriving at the clinic at 7:00 AM and they proceed to the registration desk to receive a number. They wait until this number is called in order to meet the doctor for consultation. Upon receiving consultation, a patient may either leave the clinic, or stay for laboratory

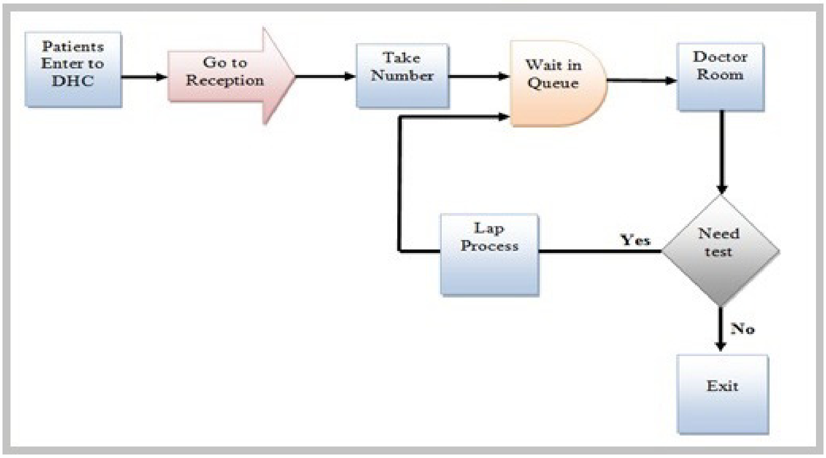

FIGURE 1. The flowchart of the patient in the clinic.

tests, depending on the doctor's suggestion. In the latter case, the patient is received in the lab and once tests are run, the patient must return to the queue for a second consultation with the doctor. Figure 1 illustrates the flowchart for a patient in the clinic.

Data Collection: There is continuous information exchange between the construction of the model and the collection of the required input data. Since data collection requires a significant portion of the total time for performing a simulation, it is collected in the early stages of the work. The data collection period lasted for a month and a half; it was collected under the same conditions, on the same day of each week for the same work shift and the same resources. The collected data included the arrival time for each patient, the type of patient (new or follow-up), the waiting time, the queue for the doctor's office, the service time inside the doctor's office, the number of patients sent to the lab and the time required for the patient to come back from the lab to the queue. This data was analyzed using the Arena input analyzer which enabled us to reach several important conclusions. Firstly, the consultation time is different according to the patient type: for new patients it is 8.86 min, for follow-up patients it is 6.3 min and for return patients from the lab it is 2.89 min. The lab time for return patients is 10.77 min. From the collected data, we have also noticed that 87.5% of all patients are follow-up patients, while 12.5% are new patients. Furthermore, we observed that only 9.5% of patients are sent for lab tests.

Modelling and simulating appointment systems: Any appointment system is viewed according to its preformed process. Here we discuss how to model an appointment system for the clinic at the outpatient department. First, we describe the current situation at the clinic and then we present the proposed

Table 1. Current situation of the clinic.

Clinic working hour	7 h
Doctor working hours	5 h
Number of resources (doctors)	1 doctor
Average served patients	33 patients
Average waiting time per patient	2.16 h
Average doctor's utilization	Almost 100

alternative appointment systems. Finally, we explain the structure of the Arena model.

The current appointment system consists of the following three main parts:

Waiting in the doctor queue: This clinic offers its services to three types of patients, namely new patients, follow-ups and return patients. The two first types require a number from the registration desk, while return patients are those who are advised to run lab tests for evaluation and thus they visit the doctor's office twice; this is why they are called return patients. When the patients are inserted into the system database and assigned numbers, they are requested to wait until it is their turn to see the doctor. The patients start arriving at 7:00 AM, while the doctor commences consultation at 9:00 AM, thus a queue is generated in these two hours and the first patient receives consultation at 9:00 AM.

Doctor diagnosing process: After entering the doctor's office for diagnosis, it is assumed that the patient will remain there for a service time which is dependent on the patient type; new, follow-up or return. New patient time is usually the longest due to the fact that the doctor must complete the patient's file and identify problems and conditions. Follow-up patient time is not as long, because the doctor already has a record for these patients and performs a mere check-up. Finally, return patients require the least time, as the doctor only examines the test results and provides the appropriate prescription based on these results.

Lab Process: As already mentioned, return patients are those who are sent for tests, the results of which are usually ready on the same day. On average, the lab process takes 10 min. Upon completion of the tests, return patients have a higher priority to see the doctor and thus they are taken in immediately without the need for a visit number.

Table 1 includes some statistics obtained from the collected data of the current situation.

At this point, it must be mentioned that appointment systems are affected by many factors, including the initial block of patients, in terms of the number of arrivals at the beginning of each session, the number of patients in the initial block and the length of the interval between the consecutive arriving blocks. Based on

TABLE 2. The description of the 9 scenarios considered in the project.

System	Class	Description
1	1	One patient each 8 min
2	2	One patient each 8 min but with initial arrival of 2 patients
3	3	One patient each 6 min for 30 min then one patient each 8 min for 2 h and a half then one patient each 10 min for the last 3 h
4	3	One patient each 8 min for 2 h then one patient each 10 min for the last 4 h
5	4	Two patients per arrival for each 20 min
6	4	Two patients per arrival for each 18 min
7	5	Three patients per arrival each 18 min for 1 hour then Two patients per arrival for each 18 min for the last 5 h
8	6	Two patients per arrival each 15 min for 1 hour then two patients per arrival each 18 min for the last 5 h
9	6	Two patients per arrival each 15 min for 30 min then two patients per arrival each 18 min for 3 h then two patients per arrival each 20 min for the last 2.5 h

these factors, the AS can be classified as follows:

Class 1: The individual-block/fixed-interval system. In this system, patients are assigned individual appointments with a fixed interval time between any two appointments that is equal to the mean consultation time between any two appointments.

Class 2: The individual-block/fixed-interval with an initial block system. This system is similar to the previous, except in this case the initial arrival block comprises of more than one patient.

Class 3: The individual-block/variable-interval system. In this system individual patients in each block are assigned with different appointment intervals between scheduled arrivals. Usually, these appointment intervals are increasing.

Class 4: The multiple-block/fixed-interval system. According to this system, a fixed number of patients (more than one) are assigned in each block but with fixed intervals, equal to the mean consultation time between two patients, multiplied by the number of patients in the block.

Class 5: The variable-block/fixed-interval system. In this system, a different number of patients in each appointment block are assigned with fixed intervals. Usually, the number of patients assigned at the beginning is higher and notes a gradual decrease thereafter.

Class 6: The multiple-block/variable-interval system. In this system, a fixed number of patients (more than one) with different appointment intervals (increasing appointment intervals) are assigned.

We have investigated 9 alternative systems based on this classification scheme, which are compared in order to identify the set that contains the systems exhibiting the best performance. Table 2 describes these 9 alternatives.

3. The Arena model

Arena uses what is known as CREATE module to control the arrival process to the clinic. To model this we used one CREATE module to generate the initial arrivals, one CREATE module for walk-in arrivals and another for each block and each times slot. For example, in order to model the arrival of 3 patients with 20 min intervals, for the first 2 h, then 2 patient arrivals with 15 min intervals for the next 2 h and finally one patient arrival each 10 min. For the last 2 h, we use 3 Create modules, each one with constant time between arrivals, with values equal to 20, 15 and 10, respectively, the number of entities per arrival is set as 3, 2 and 1, respectively; the maximum number of arrivals are set to 6, 8 and 12 and the first creation starts at 0.00, 120.00 and 240.00 min, respectively. The DECIDE module is used to decide whether the patient shows up or not and it is set to 90% in favor of showing up and 10% for no-show patients who are disposed immediately. Another DECIDE module is used to determine the arrival type; 12.5% for new patients and 87.5% for follow up patients. Then the ASSIGN module is used to assign attributes to the entity type; for new arrivals we assign doctor service times obtained from data collected as 4 + Expo(4.86); for follow-ups the doctor service time is assigned as Expo(6.32); finally, for return patients it is Expo(2.9). The priority for new, follow-up and return patients is set higher than that for walk-in patients, who have not booked an appointment. The PROCESS module is used to assign the process time required in the doctors office, which depends on the patient type. The DECIDE module is used to model whether the patient is sent for lab tests or not; from the collected data, the percentage of patients requested to go to lab test is 9.5%. The lab process time follows the distribution of 7.17 + Expo(3.65) min. In addition, the RECORD module is used to count the number of patients leaving the system. Finally, the DISPOSE module is used to let the entities leave the system.

Figure 2 provides a sketch of the Arena model for an appointment system.

4. The ranking and selection procedure

Ranking and Selection (R&S) procedures are statistical selection techniques designed to select the best, or a subset of systems containing the best, from a group of alternative systems whose behavior encounters some randomness. It is assumed that we can generate independent and normally distributed observations from each system. There are two types of R&S procedures, the indifference zone approach and the subset selection approach. In the indifference zone approach one needs to select the system with the largest mean (assuming we are maximizing a set of systems) with a predetermined level of confidence. If the difference between the selected and the actual best system is small, less than d^*, then the user is satisfied with the selection. Let X_{ik}, be the kth observation of the ith system and assume that $X_{i_1}, X_{i_2}, X_{i_{n_i}}$ are independent and normally distributed. Let $\mu_i = E(X_{i_k})$ and assume that $\mu_1 \leq \mu_2 \ldots \leq \mu_n$ be the ordered mean performance

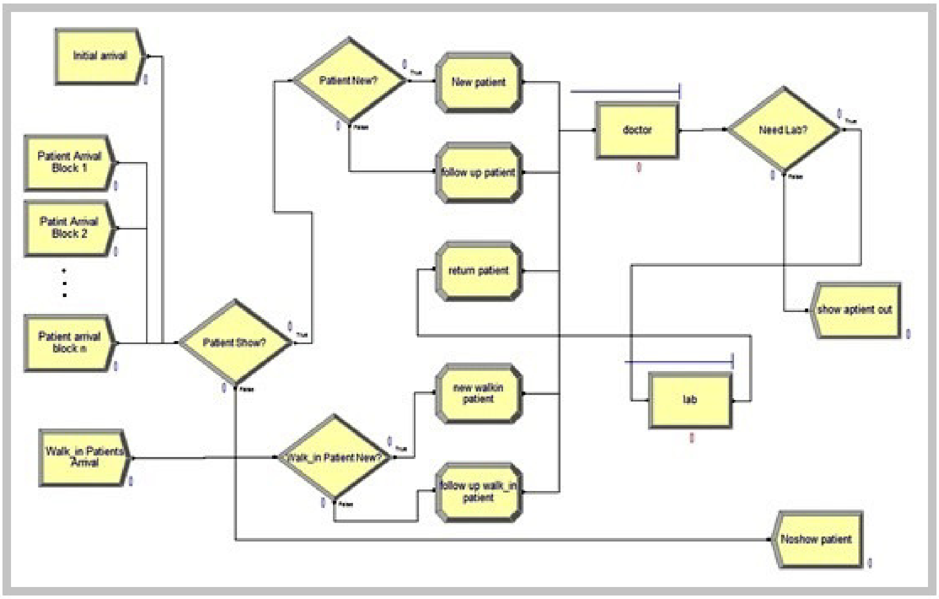

FIGURE 2. The Arena model for a general appointment system.

for the n systems, the practitioner then is interested in selecting the system n^*, where $\mu_n - \mu_{n^*} \leq d^*$, where d^* is predetermined by a practitioner, the region $[\mu_n - d^*, \mu_n]$ is called the indifference zone.

Let CS be the event of correct selection, *i.e.*, a system in the indifference zone is selected as the best, then it is sought to have $P(CS) \geq P^*$, where P^* is a predetermined level of confidence. R&S procedures usually consist of two phases. In the first phase, a sample of size n_0 is generated from each system, which is used to estimate the variances. Then, using the variances, the values of d^* and P^*, and the initial sample size, a second sample is calculated to guarantee the probability of correct selection. Most of the ranking and selection procedures have focused on a single objective function. However, in many applications, it is required to select a system based on multi-objective criteria; for example, in outpatient clinic departments managers are interested in minimizing the average waiting time per patient, maximizing the utilization of the doctor and minimizing the expected number of patients in the clinic.

Multi-objective optimization

Assume that f is an r-vector $f = (f_1, \ldots, f_r)$, where f_j is the expected performance of a complex stochastic system whose evaluation encounters some noise; $f_j(s) = E[h_j(s; Y_s)]$ where Y_s is a random variable that depends on the parameter s. The objective is to select a system that optimizes all the objective functions

$f_1, \ldots, f_r$. Note that, there may be no single optimal solution that solves all objective functions. One way of solving this problem is to give a weight for the various objective functions based on their importance and then they are aggregated into a single objective optimization problem

$$f(s) = \sum_{j=1}^{r} w_j f_j(s), \quad \text{where} \quad \sum_{j=1}^{r} w_j = 1, w_j \geq 0.$$

Then one can use any optimization problem to solve the aggregated problem, Alrefaei and Diabat [1] have used the simulated annealing algorithm for solving the multi-objective optimization (MOO) problem using the weighted sum method. However, selecting the weight is a major concern because it depends on the decision makers. Another way of solving the MOO is to construct a Pareto set, that consists of all non-dominated solutions. A solution A is said to dominate solution B if for all components of A; $f_j(A) \leq f_j(B), j = 1, \ldots, r$ and for at least one j, $f_j(A) < f_j(B)$. A solution A is said to be non-dominated if it is not dominated by any solution. Solution A belongs to the Pareto optimal set if it is not dominated by any other feasible solution, see Chen and Lee [9].

Note that in the stochastic model, the objective function values are known and have to be estimated by simulation, therefore the definition of the dominance is not valid in this situation; in this work, we propose a new definition for dominance based on comparison of alternative simulation systems.

Definition 4.1. A solution A is said to α-dominate solution B if the $(1-\alpha)100\%$ confidence interval of the difference $Z = f_j(A) - f_j(B)$ contained entirely in the closed interval $(-\infty, 0]$ and at least one confidence interval is contained in the open interval $(-\infty, 0)$. A solution A is said to be α-non-dominated solution if it is not α-dominated by any solution.

The following algorithm describes the proposed ranking and selection procedure for multi-objective optimization.

Algorithm 1 (RSMOO)**.**

Step 1: *For each system $i, i = 1, \ldots, n$, get n_0 samples for each objective $j, j = 1, \ldots, r$. For $k = 1, \ldots, n_0$, let X_{ijk} be the kth sample of system i for objective j.*

Step 2: *For each system i and objective $j; i = 1, \ldots, n,\ j = 1, \ldots, r$, calculate the initial estimates of the sample mean $\overline{X_{ij}}(1)$ and the sample variances S_{ij}^2 as follows:*

$$\overline{X_{ij}}(1) = \frac{1}{n_0} \sum_{k=1}^{n_0} X_{ijk}$$

$$S_{ij}^2 = \frac{1}{(n_0 - 1)} \sum_{k=1}^{n_0} (X_{ijk} - \overline{X_{ij}}(1))^2.$$

Step 3: *For $i = 1, \dots, n,\ j = 1, \dots, r$, compute the total sample size*

$$N_{ij} = \max\left\{ n_0 + 1, \left\lceil \frac{h_1^2 S_{ij}^2)}{d_j^*} \right\rceil \right\}$$

where d_j^* is the indifference value for objective j and h_1 is a constant that depends on confidence level P^* to gurantee the probability of correct selection and can be obtained from Table 10.11 of Law and Kelton [15], see the remark below to show how to evaluate the value of h_1.

Step 4: *For each system $i, i = 1, \dots, n$, and each objective $j, j = 1, \dots, r$, run the simulation for $N_{ij} - n_0$ more samples and compute the second sample mean*

$$\overline{X_{ij}}(2) = \frac{1}{N_{ij} - n_0} \sum_{k=1}^{N_{ij}-n_0} X_{ijk}.$$

Step 5: *For $i = 1, \dots, n$, and $j = 1, \dots, r$, define the weights*

$$W_{ij}(1) = \frac{n_0}{N_{ij}} \left[1 + \sqrt{(1 - \frac{N_{ij}}{n_0} \left(1 - \frac{(N_{ij} - n_0)(d_j^*)^2)}{h_1^2 S_{ij}^2} \right)} \right]$$

and $W_{ij}(2) = 1 - W_{ij}(1)$ and compute the overall weighted sample mean

$$\widehat{X_{ij}} = W_{ij}(1)\overline{X_{ij}}(1) + W_{ij}(2)\overline{X_{ij}}(2).$$

Step 6: *For each objective $j, j = 1, \dots, r$, rank the systems based on their weighted sample means $\widehat{X_{ij}}$.*

Step 7: *Construct an initial Pareto set P that consists of all systems that have the best performance in at least one objective function based on the ranks obtained in Step 6.*

Step 8: *For each system $s \in S$, let $\alpha = 1 - P^*$, construct the $100(1-\alpha)\%$ confidence intervals for the difference $f_j(s) - f_j(i)$, for all $i = 1, \dots, n, i \neq s$ and $j = 1, \dots, r$ as follows*

$$\left(\widehat{X_{sj}} - \widehat{X_{ij}}\right) \pm Z_{\left(1-\left(\frac{\alpha}{(n-1)}\right)\right)} \left[\sqrt{\frac{(S_{sj})^2}{N_{sj}} + \frac{(S_{ij})^2}{N_{ij}}} \right]$$

where $Z_{\left(1-\frac{\alpha}{n-1}\right)}$ is the $1 - \frac{\alpha}{n-1}$ upper critical points of the normal random variable. Note that since we have $n-1$ comparisons, to guarantee the $100(1-\alpha)\%$ confidence level, we build the $100(1-\frac{\alpha}{(n-1)})\%$ confidence interval for each comparison.

Step 9: *If there is a system s' that is α-non-dominated by all $s \in P$, then s' enters the Pareto set P.*

TABLE 3. The estimated mean and variance for the average waiting time per patient obtained by the initial 20 replication and the new sample sizes.

System	The average waiting time per patient	Variance	New sample sizes N_{ij}
1	10.90	36.22	85
2	14.84	53.45	125
3	12.74	102.11	238
4	9.72	53.27	124
5	9.00	60.54	141
6	10.06	22.82	54
7	11.30	44.97	105
8	14.61	89.70	209
9	12.22	43.47	102

Remark 4.2. Note that in Step 3, h_1 is a constant that can be evaluated by solving the integral equation:

$$\int_{\infty}^{\infty} (F(t+h_1))^{(n-1)} f(t)\mathrm{d}t = P^*$$

to guarantee the correct selection $P(CS) = P^*$, where $F(.)$ and $f(.)$ are the CDF and pdf for the standard normal distribution. h_1 can be obtained from Table 10.11 of Law and Kelton [15].

5. Results and discussion

In order to use ranking and selection method for multi objectives, we first ran the Arena simulation for each alternative an initial run of $n_0 = 20$ replications. Then we obtained the initial estimates of sample means and variances for the average waiting time per patient, doctor's utilization and the number of patients waiting in the queue using equations (1) and (2), respectively. The second sample sizes, N_{ij} for each system $i, i = 1, \ldots, n$ and objective $j, j = 1, \ldots, r$ are calculated using equation (3). We assume that we seek the probability of correct selection $P^* = 0.9$, the indifference zone d_j^* is selected as follows; for the average waiting time $d_1^* = 2$ min, for the utilization objective $d_2^* = 0.05$ and for the average number of patients in the queue $d_3^* = 0.5$. Tables 3, 4 and 5 include the summary statistics and the new sample sizes for the three objectives, respectively. Table 6 includes the maximum new sample sizes for all alternatives over the three objectives.

We ran Arena for each alternative based on the maximum number sample size for each system. New estimates of the sample means and variance for all systems and all objectives are calculated and the weighted sample means are calculated based on equations (4) and (5). The systems are ranked based on their weighted sample means for each objective. Tables 7–9 include these ranked systems including their sample means, variances and the number of total replications.

TABLE 4. The estimated mean and variance for the average number of patients in queue obtained by the initial 20 replication and the new sample sizes.

System	The Number of patients in Queue	Variance	New sample sizes (N_{ij})
1	1.85	1.65	62
2	2.83	3.17	119
3	1.90	2 87	108
4	1.3	1.49	56
5	1.20	1.10	42
6	1.42	0.57	22
7	1.66	1.62	61
8	2.37	2.33	87
9	1.72	0.85	32

TABLE 5. The estimated mean and variance for doctor utilization obtained by the initial 20 replication and the new sample sizes.

System	The estimated Doctor Utilization	Variance	New sample sizes (N_{ij})
1	88.14%	0.0054	21
2	89.49%	0.0104	39
3	83.52%	0.0117	44
4	80.92%	0.0047	18
5	74.22%	0.0071	27
6	80.49%	0.0111	42
7	76.80%	0.0167	63
8	85.23%	0.0076	29
9	81.88%	0.0126	47

TABLE 6. The new sample sizes for all systems.

System	N_{MAX}
1	85
2	125
3	238
4	124
5	141
6	54
7	105
8	209
9	102

The initial Pareto set is constructed by including the best system in at least one objective, so the initial set is $S = 1, 5$. Then each system in S is compared with all other alternatives not in S by constructing confidence intervals for the difference of the estimated means of the objective functions. So we construct the $1 - (\frac{0.1}{8})100\% = 98.75\%$ confidence intervals for $W_1 - W_i$ and $NIQ_1 - NIQ_i$

TABLE 7. The ranked systems based on the average waiting time per patients.

System	Average waiting time (min)	Variance	Number of replicates
5	8.11	31.244	141
4	8.77	40.797	124
6	10.55	33.633	54
8	11.65	53.903	209
1	11.68	52.679	85
2	11.72	51.362	125
3	12.14	99.015	238
9	13.8	81.265	102
7	14.19	128.47	105

TABLE 8. The ranked systems based on the average number of patients in queue.

System	Average number of patients in queue	Variance	Number of replicates
5	1.07	0.687	141
4	1.16	0.821	124
7	1.52	1.658	105
6	1.55	1.059	54
3	1.77	2.697	238
1	1.97	2.058	85
2	2.05	2.400	125
9	2.09	2.526	102
8	2.19	3.162	209

TABLE 9. The rank of systems based on the utilization of the doctor.

System	Average utilization	Variance	Number of replicates
1	86.9%	0.0063	85
2	86.6%	0.0078	125
8	86.1%	0.097	209
9	83.2%	0.0123	102
3	81.9%	0.0117	238
6	80.7%	0.0101	54
4	79.2%	0.0092	124
5	73.6%	0.0111	141
7	65.3%	0.0167	105

and $UT_5 - UT_i$ for each system $i = 1, 2, \ldots, 9, j \neq 1, 5$. The results are listed in Tables 10–12.

Since the confidence interval for $W_1 - W_4$ and $NIQ_1 - NIQ_4$ are contained entirely in $(0, \infty)$ (Note that we are interested in minimizing the waiting time and the average number of patients in system) and $U_5 - U_4$ is contained entirely in the interval $(-\infty, 0)$ (Note that we are interested in maximizing the doctor utilization), we conclude that System 4 is not dominated by any system, so it enters the Pareto

TABLE 10. The The lower and upper limits of the 90% confidence intervals for the difference in waiting time between alternative 1 and other alternatives.

System (j)	$W_1 - W_j$	W	LCI	UCI
2	−0.037	2.2751	−2.3121	2.2381
3	−0.4664	2.2807	−2.7472	1.8143
4	2.9058	2.1828	0.723	5.0886
6	1.1274	2.4981	−1.3706	3.6255
7	−2.5126	3.0425	−5.5551	0.5299
8	0.0257	2.0994	−2.0738	2.1251
9	−2.1207	2.6671	−4.7878	0.5465

TABLE 11. The lower and upper limits of the 90% confidence intervals for the difference in average number of patients in queue between alternative 1 and other alternatives.

System (j)	$NIQ_1 - NIQ_j$	HW	LCI	UCI
2	−0.0828	0.4669	−0.5497	0.3841
3	0.2028	0.4225	−0.2197	0.6253
4	0.8073	0.3935	0.4138	1.2008
6	0.4246	0.4691	−0.0445	0.8938
7	0.4528	0.4482	0.0046	0.901
8	−0.2206	0.4445	−0.6651	0.2239
9	−0.1232	0.4959	−0.6192	0.327

TABLE 12. The lower and upper limits of the 90% confidence intervals for the difference in doctor utilization between alternative 5 and other alternatives.

System (j)	$U_5 - U_j$	HW	LCI	UCI
2	−0.1292	0.0266	−0.1558	−0.1026
3	−0.0827	0.0254	−0.1081	−0.0573
4	−0.0556	0.0277	−0.0833	−0.0279
6	−0.0705	.0365	−0.107	−0.034
7	0.0832	0.0346	0.0486	0.1178
8	−0.1241	0.0251	−0.1492	−0.0991
9	−0.0955	0.0317	−0.1272	0.0638

set. Therefore, the final Pareto set is $S = \{1, 4, 5\}$. Table 13 includes the estimated average waiting time per patient, the estimated number of patients in queue and the doctor utilization for all members in the final Pareto set S.

6. CONCLUSION

In this paper we considered the problem of selecting an optimal appointment system to be implemented in an outpatient department clinic. We develop a

TABLE 13. The estimates of the performance of the members of the final Pareto set S.

System	Average waiting time per patient	Number of patients in queue	Doctor utilization
5	8.11	1.07	73.6%
4	8.77	1.16	79.2%
1	11.68	1.97	86.9%

multi-objective optimization problem to address this challenge, by considering the following important objectives: the minimization of the average waiting time in the clinic per patient, the maximization of doctor utilization and the minimization of the average number of patients waiting in the queue for consultation from the doctor. We propose a new version of the ranking and selection procedure to solve this problem. The procedure constructs a set containing the best systems with a predetermined level of confidence. The proposed alternative appointment systems are simulated using Arena software and then the R&S procedure is used to construct the Pareto set of appointment systems that are not dominated by any other systems. The final set contains three appointment systems and it is left to the decision maker to select which system must be adapted, that will best suit the clinic.

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