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INVENTORY POLICY FOR DETERIORATING SEASONAL PRODUCTS WITH PRICE AND RAMP-TYPE TIME DEPENDENT DEMAND

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Abstract. In this paper, we investigate the problem of simultaneously determining the order quantity and optimal prices for deteriorating products with price and ramp-type time dependent demand. We assume that a retailer has the opportunity to adjust prices before the end of the sales season to increase demand, decrease deterioration, and improve revenues. A mathematical model is developed to jointly optimize the order quantity, time interval for any two successive price changes, and the corresponding prices. An algorithm is provided to find the optimal solution to the proposed model. Finally, we use a numerical example to verify the availability of this model.

Keywords. Deterioration, dynamic pricing, inventory, ramp-type time dependent demand.

Mathematics Subject Classification. 65k05.

1. Introduction

It is observed that demand of many perishable seasonal products (fruits, *e.g.*, mango, banana, vegetables, foodstuffs, *etc.*) over the entire sales season may vary with time and prices. In the meantime, such type of seasonal products suffers from depletion by direct spoilage. Thus deterioration of this kind of products is also a realistic phenomenon. It is well known that the value of perishable seasonal products decreases under deterioration during their normal storage period. As a

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result, firms face the problem of simultaneously deciding how much to order and how to price the purchased perishable product as well as when to adjust the price over the sales season in order to increase the demand and decrease the loss due to deterioration.

Existing related inventory models concentrating on deteriorating seasonal products can be classified into three types of models according to their demand characteristic: stock-dependent demand, time-dependent demand, and price-dependent demand. Since we mainly focus on last two demand patterns, we refer the readers to [21] for a review on inventory models with stock-dependent demand. In reality, demand for a product may change with time. Many researchers have considered the inventory model with time-dependent demand [2,3,7,9,10,12]. Among different time-dependent demand models, we are typically interested in the ramp-type time-dependent demand pattern in which demand increases up to a point of time then it becomes steady. In [11], a ramp-type time dependent demand pattern is firstly proposed by considering it as the combination of two different types of demand in two successive time periods. An EOQ model with ramp-type demand, Weibull distribution deterioration, and partial backlogging is considered in [25]. A production inventory model with a ramp-type demand pattern is developed in [16], where the finite production rate depends on the demand. In [4], an inventory model for deteriorating items using a ramp-type time-dependent demand rate with three time periods is developed. EOQ models for deteriorating items with time dependent quadratic demand are investigated in [13, 18]. A production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness is developed in [17].

Since demand is usually price sensitive, many researchers have developed inventory models for deteriorating items with price dependent demand. A deterministic production lot size inventory model for the items with price dependent demand is presented in [5]. A deterministic inventory model is addressed in [22], in which demand is dependent on price, time, and inventory level. They also extend the model to the case with a single price markdown. The newsvendor problem with multiple discount prices is analyzed in [14], in which the discount prices are set at equal intervals on a price domain. In [19], Shinn and Hwang investigate the problem of determining the order quantity in which demand is a convex function of price and the delay in payments is order-size dependent. In [20], Teng and Chang establish an economic production quantity model for deteriorating items when the demand is a function of price and on-display stock level. In [8], Guchhait et al develop an inventory model of a deteriorating item with stock and selling price dependent demand under two-level credit period. Recently, Some scholars perform the studies on the inventory decision problems for deteriorating items with price- and time-dependent demand. In [15], Maihami and Kamalabadi develop a joint pricing and inventory control model for non-instantaneous deteriorating items with price- and time-dependent demand, in which the demand is an increasing/decreasing exponential function of time and shortages are allowed. In [23], Wang and Huang present a production-inventory problem for a seasonal deteriorating product with price- and ramp-type time-dependent demand, in which the selling season for the deteriorating product is fixed. However, above mentioned papers consider the price decision with the assumption that the price is unchanged once it is determined. In [24], Wang *et al.* consider the pricing and lot-sizing decision in a supply chain with price-sensitive demand. In [1], Banerjee and Sharma present a deterministic inventory model for the product with a price and time dependent demand rate, in which the pricing strategies are changed and pricing decision is determined under the assumption that the times of changing price are pre-specified. In [26], You relaxes this assumption and investigates the problem of jointly determining the number of price settings and optimal prices for a perishable inventory system in which demand is time and price dependent. However, in [26], the optimal pricing strategy is obtained under the assumption that the time interval for any two successive price changes is equal. Our paper assumes that the time interval for any two successive price changes is variable and considers it as a decision variable. Another difference between this paper and [26] in that the later paper assumes the demand is a linear function of time whereas our paper assume as a non-linear function of time.

In this paper, we analyze a dynamic inventory model for deteriorating seasonal items with price and ramp-type time dependent demand. The demand pattern for such type products implies that the demand rate of the items increases with time up to certain time and then becomes steady when the price of the products remain unchanged. We assume that a retailer has the opportunity to adjust prices before the end of the sales season to improve revenue. The purpose of this paper aims to maximize the profit over the sales season by simultaneously determining (1) the optimal order quantity; (2) the time intervals for two successive price changes; and (3) the corresponding prices.

2. Parameters and assumptions

Consider an inventory system where a firm purchases a perishable item at the start of a sales season and sells it over the sales season. The following parameters and assumptions are used in developing the model:

2.1. Parameters

D_t: Demand rate for the item at time t, $D_t > 0$.
h: Inventory holding cost per unit per unit of time.
d: Deterioration cost per unit.
L: Time horizon over the sales season.
θ: Constant deterioration rate for product $0 < \theta < 1$.
p_i: ith price over the sales season.$1 \le i \le n$ (decision variable)
T_i: Time interval for two successive price changes over the sales season, $1 \le i \le n$ (decision variable).
L_i: Total time elapsed up to and including the time interval of adopting the ith price P_i, $L_i = \sum_{k=1}^{i} T_k$. Define $L_0 = 0$.

$I_i(t)$: Inventory level at time $[L_{i-1}+t]$ during the period of adopting ith price $p_i(0 \le t \le T_i), I_i(t) \ge 0$.

S_i: Sales amount from the start of the sales season up to the end of time interval T_i.

Q: Initial order quantity at the start of the sales season (decision variable).

2.2. Assumptions

The demand rate for the item is assumed to be a function of time and price and is of the form

$$D_t = D(t) - cp_i \tag{2.1}$$

where c is a known constant.

For perishable products, the demand rate usually increases with time up to certain time and then becomes steady when the price of the products remain unchanged. Therefore, we assume $D(t)$ is a time dependent ramp-type function defined as follows:

$$D(t) = A\mathrm{e}^{b(t-(t-\mu)H(t,\mu))}, \mu > 0 \tag{2.2}$$

where $A > 0$ is the initial demand rate and $b > 0$ is the rate with which the demand rate increases. $H(t,\mu)$ is the well-known Heaviside's function [6,16] defined as

$$H(t,\mu) = \begin{cases} 1 \text{ if } t \ge \mu \\ 0 \text{ if } t < \mu. \end{cases} \tag{2.3}$$

We assume the item deteriorates at a constant rate. The retailer adopts a dynamic pricing strategy during the sales season. Under this strategy, as time goes on, the retailer changes price to increase product sale quantity. The time interval for any two successive price changes is not necessarily equal. It is observed that determining the time interval for any two successive price changes is equivalent to determining when to change price. The purpose of this paper aims to maximize the retailer's profit over the finite time horizon L by simultaneously determining (1) the optimal order quantity Q, (2) the time intervals for two successive price changes $T_i, 1 \le i \le n$, and (3) the corresponding prices $p_i, 1 \le n \le n$.

3. Model formulation

Let S_i represent the sales amount from the start of the sales season up to the end of time interval T_i.Then, we have

$$S_i = \sum_{m=1}^{i} \int_0^{T_m} D_t \mathrm{d}t = S_{i-1} + \int_0^{T_i} D_t \mathrm{d}t. \tag{3.1}$$

Using ramp type function $D(t)$ defined by equation (2.2), equation (3.1) can be written as

$$S_i = S_{i-1} + \int_0^{T_i} (A\mathrm{e}^{b(L_{i-1}+t} - cp_i)\mathrm{d}t = S_{i-1} + \frac{A\mathrm{e}^{bLi-1}(\mathrm{e}^{bT_i} - 1)}{b} - cp_i T_i, \text{ if } 0 \le t \le \mu \tag{3.2}$$

$$S_i = S_{i-1} + \int_0^{T_i} (A\mathrm{e}^{b(L_{i-1}+\mu} - cp_i)\mathrm{d}t = S_{i-1} + (A\mathrm{e}^{b(L_{i-1}+\mu)} - cp_i)T_i, \text{ if } \mu \le t \le T_i \tag{3.3}$$

Let $I_i(t)$ be the inventory level at time $L_{i-1} + t$ during the period of adopting the ith price $p_i(0 \le t \le T_i), i = 1, 2, 3, \ldots, n$, where L_{i-1} is the total time elapsed up to and including the time interval of adopting the $(i-1)$th price p_{i-1}.The differential equation governing the instantaneous states of $I_i(t)$ is given by

$$\frac{\mathrm{d}I_i(t)}{\mathrm{d}t} + \theta I_i(t) = -D_{L_{i-1}+t}. \tag{3.4}$$

Using ramp type function $D(t)$, equation (3.4) becomes respectively

$$\frac{\mathrm{d}I_i(t)}{\mathrm{d}t} + \theta I_i(t) = -A\mathrm{e}^{b(L_{i-1}+t)} + cp_i, 0 \le t \le \mu \tag{3.5}$$

with the condition $I_i(L) = Q - S_{i-1}$

$$\frac{\mathrm{d}I_i(t)}{\mathrm{d}t} + \theta I_i(t) = -A\mathrm{e}^{b(L_{i-1}+\mu)} + cp_i \quad \mu_i \tag{3.6}$$

with the condition $I_i(L_{i-1}) = Q - S_{i-1}$.

Solutions to equations (3.5) and (3.6) are as follows:

$$I_i(t) = \frac{A\mathrm{e}^{bL_{i-1}-\theta t}}{b+\theta}(1 - \mathrm{e}^{(b+\theta)t}) + \frac{c(1-\mathrm{e}^{-\theta t})p_i}{\theta} + (Q - S_{i-1})\mathrm{e}^{-\theta t} \qquad 0 \le t \le \mu \tag{3.7}$$

$$I_i(t) = \frac{-A\mathrm{e}^{b\mu} + cp_i}{\theta} + \frac{\mathrm{e}^{(L_{i-1}-t)}\theta(A\mathrm{e}^{b\mu} - cp_i + Q\theta - S_{i-1}\theta)}{\theta} \qquad \mu \le t \le T_i. \tag{3.8}$$

Our object is to determine the optimal order quantity, optimal time interval for any two successive price changes and the optimal price to maximize the profit over the sales season. Since the demand has two components in two successive time periods, the relationship among demand transfer points and the time interval for two successive price changes can be completely characterized by illustrating following three cases:

Case 1: The time interval for two successive price changes starts and ends within the time interval $[0, \mu], T_i < \mu$.

Case 2: The time interval for two successive price changes starts in the time interval $[0, \mu]$ but ends in $[\mu, L].T_i$ is not necessarily greater than μ but $T_i < L$.

Case 3: The time interval for two successive price changes starts and ends within the time interval $[\mu, L], T_i < L - \mu$.

We first derive profit functions for each of the cases, then combine those using an algorithm to determine the optimal order quantity and optimal pricing strategy for the season to maximize total profit.

3.1. Case 1

In this case, the time interval for two successive price changes starts and ends within the time interval $[0, \mu]$. Obviously, retailer's sales revenue is defined as

$$\begin{aligned} R_i &= p_i \int_0^{T_i} D_t \mathrm{d}t = p_i \int_0^{T_i} (A\mathrm{e}^{b(L_{i-1}+t)} - cp_i)\mathrm{d}t \\ &= \frac{A\mathrm{e}^{bL_{i-1}}(\mathrm{e}^{bT_i} - 1)p_i}{b} - cT_i p_i^2. \end{aligned} \tag{3.9}$$

Based on (3.7), retailer's holding cost and deteriorating cost during the time interval of adopting price p_i can be expressed as, respectively

$$\begin{aligned} HC_i = h \int_0^{T_i} I_i(t)\mathrm{d}t &= \frac{hA\mathrm{e}^{bL_{i-1}}}{b+\theta} \left(\frac{1-\mathrm{e}^{-\theta T_i}}{\theta} + \frac{1-\mathrm{e}^{bT_i}}{b} \right) \\ &+ \frac{hc(T_i + \frac{\mathrm{e}^{-\theta T_i}}{\theta} - 1)p_i}{\theta} - \frac{h(Q - S_{i-1})(\mathrm{e}^{-\theta T_i} - 1)}{\theta} \end{aligned} \tag{3.10}$$

$$\begin{aligned} DC_i = d\theta \int_0^{T_i} I_i(t)\mathrm{d}t &= \frac{d\theta A\mathrm{e}^{bL_{i-1}}}{b+\theta} \left(\frac{1-\mathrm{e}^{-\theta T_i}}{\theta} + \frac{1-\mathrm{e}^{bT_i}}{b} \right) \\ &+ \frac{d\theta c(T_i + \frac{\mathrm{e}^{-\theta T_i}}{\theta} - 1)p_i}{\theta} - \frac{d\theta(Q - S_{i-1})(\mathrm{e}^{-\theta T_i} - 1)}{\theta}. \end{aligned} \tag{3.11}$$

Therefore, the unit time profit function during the time interval of adopting price can be obtained as

$$\begin{aligned} \mathit{\Pi}(T_i, p_i, Q) &= \frac{R_i - HC_i - DC_i}{T_i} \\ &= \frac{A\mathrm{e}^{bL_{i-1}}(\mathrm{e}^{bT_i} - 1)p_i}{bT_i} - cp_i^2 - \frac{(h+d\theta)A\mathrm{e}^{bL_{i-1}}}{(b+\theta)T_i} \left(\frac{1-\mathrm{e}^{-\theta T_i}}{\theta} + \frac{1-\mathrm{e}^{bT_i}}{b} \right) \\ &- \frac{(h+d\theta)c(T_i + \frac{\mathrm{e}^{-\theta T_i}}{\theta} - 1)p_i}{\theta T_i} + \frac{(h+d\theta)(Q - S_{i-1})(\mathrm{e}^{-\theta T_i} - 1)}{\theta T_i}. \end{aligned} \tag{3.12}$$

The above function is a composite function of Q, T_i and p_i. Here our object is to find the values of Q, T_i and p_i optimally to maximize the unit time profit. Our problem can be denoted as the following constrained problem (P1)

$$
\begin{aligned}
(\mathrm{P1}) \qquad \max \Pi(T_i, p_i, Q) \qquad (3.13)\\
\text{subject to } p_i < \frac{A\mathrm{e}^{b(L_{i-1}+T_i)}}{c} \qquad (3.14)\\
0 < T_i < \mu \qquad (3.15)\\
S_i < Q \qquad (3.16)\\
I_i(L_i) > 0 \qquad (3.17)\\
L_i \leq L. \qquad (3.18)
\end{aligned}
$$

3.2. Case 2

The maximum profit of the retailer is determined in this case if the time interval for two successive price changes starts in the time interval $[0, \mu]$ but ends in the time interval $[\mu, L]$. In this case, retailer's sales revenue is expressed as

$$
\begin{aligned}
R_i &= p_i \int_{L_{i-1}}^{L_i} D_t \mathrm{d}t = p_i \int_{L_{i-1}}^{\mu} (A\mathrm{e}^{bt} - cp_i)\mathrm{d}t + p_i \int_{\mu}^{L_i} (A\mathrm{e}^{b\mu} - cp_i)\mathrm{d}t \\
&= \frac{A(\mathrm{e}^{b\mu} - \mathrm{e}^{bL_{i-1}})p_i}{b} - cT_i p_i^2. \qquad (3.19)
\end{aligned}
$$

Based on (3.7) and (3.8), retailer's holding cost and deteriorating cost can be expressed as, respectively

$$
\begin{aligned}
HC_i &= h \int_0^{T_i} I_i(t)\mathrm{d}t = h \int_0^{\mu} I_i(t)\mathrm{d}t + h \int_{\mu}^{T_i} I_i(t)\mathrm{d}t \\
&= \frac{hA\mathrm{e}^{bL_{i-1}}(b(1-\mathrm{e}^{-\theta\mu}) + \theta(1-\mathrm{e}^{b\mu}))}{b\theta(b+\theta)} + \frac{hcp_i(\mu\theta + \mathrm{e}^{-\theta\mu} - 1)}{\theta^2} \\
&\quad + h(Q - S_{i-1})\frac{1-\mathrm{e}^{-\theta\mu}}{\theta} + \frac{h}{\theta^2}(A\mathrm{e}^{b(L_{i-1}+\mu)} - cp_i) \\
&\quad \times (\mathrm{e}^{\theta(L_{i-1}-\mu)} - \mathrm{e}^{\theta(L_{i-1}-T_i)} + \theta(\mu - T_i)) \\
&\quad + \frac{h}{\theta^2}(\mathrm{e}^{\theta(L_{i-1}-T_i)} - \mathrm{e}^{\theta(L_{i-1}-\mu)}(Q - S_{i-1})) \qquad (3.20)
\end{aligned}
$$

$$
\begin{aligned}
DC_i &= d\theta \int_0^{T_i} I_i(t)\mathrm{d}t = d\theta \int_0^{\mu} I_i(t)\mathrm{d}t + d\theta \int_{\mu}^{T_i} I_i(t)\mathrm{d}t \\
&= \frac{dA\mathrm{e}^{bL_{i-1}}(b(1-\mathrm{e}^{\theta\mu}) + \theta(1-\mathrm{e}^{b\mu}))}{b(b+\theta)} + \frac{dcp_i(\mu\theta + \mathrm{e}^{-\theta\mu} - 1)}{\theta} \\
&\quad + d(Q - S_{i-1})(1-\mathrm{e}^{-\theta\mu}) + \frac{d}{\theta}(A\mathrm{e}^{b(L_{i-1}+\mu)} - cp_i) \\
&\quad \times (\mathrm{e}^{\theta(L_{i-1}-\mu)} - \mathrm{e}^{\theta(L_{i-1}-T_i)} + \theta(\mu - T_i)) \\
&\quad + \frac{d}{\theta}(\mathrm{e}^{\theta(L_{i-1}-T_i)} - \mathrm{e}^{\theta(L_{i-1}-\mu)}(Q - S_{i-1})). \qquad (3.21)
\end{aligned}
$$

Thus, retailer's unit time profit function during the time interval of adopting price p_i can be obtained as

$$\begin{aligned}\Pi(T_i, p_i, Q) = {} & \frac{A(\mathrm{e}^{b\mu} - \mathrm{e}^{bL_{i-1}})p_i}{bT_i} - cp_i^2 \\ & - \frac{(h+d\theta)A\mathrm{e}^{bL_{i-1}}(b(1-\mathrm{e}^{\theta\mu})+\theta(1-\mathrm{e}^{b\mu}))}{b\theta(b+\theta)T_i} - \frac{(h+d\theta)cp_i(\mu\theta+\mathrm{e}^{-\theta\mu}-1)}{\theta^2 T_i} \\ & - (h+d\theta)(Q-S_{i-1})\frac{1-\mathrm{e}^{-\theta\mu}}{\theta T_i} \\ & - \frac{(h+d\theta)(A\mathrm{e}^{b(L_{i-1}+\mu)}-cp_i)(\mathrm{e}^{\theta(L_{i-1}-\mu)}-\mathrm{e}^{\theta(L_{i-1}-T_i)}+\theta(\mu-T_i))}{\theta^2 T_i} \\ & - (h+d\theta)\frac{\mathrm{e}^{\theta(L_{i-1}-T_i)} - \mathrm{e}^{\theta(L_{i-1}-\mu)}(Q-S_{i-1})}{\theta^2 T_i}. \end{aligned} \tag{3.22}$$

Similar to Case 1, our problem can be denoted as the following constrained problem (P2):

$$\text{(P2)} \qquad \max \Pi(T_i, p_i, Q) \tag{3.23}$$

$$\text{subject to } p_i < \frac{A\mathrm{e}^{b\mu}}{c} \tag{3.24}$$

$$S_i < Q \tag{3.25}$$

$$I_i(L_i) > 0 \tag{3.26}$$

$$L_i \leq L \tag{3.27}$$

$$T_i > 0. \tag{3.28}$$

3.3. Case 3

In this case, the time interval of price change starts and ends within the time interval $[\mu, L]$. Retailer's sales revenue is expressed as

$$R_i = p_i \int_0^{T_i} D_t \mathrm{d}t = p_i \int_0^{T_i} (A\mathrm{e}^{b\mu} - cp_i)\mathrm{d}t = A\mathrm{e}^{b\mu}T_i p_i - cT_i p_i^2. \tag{3.29}$$

Based on (3.8), the retailer's holding cost and deteriorating cost can be obtained as, respectively

$$\begin{aligned} HC_i = h\int_0^{T_i} I_i(t)\mathrm{d}t = {} & \frac{-h(A\mathrm{e}^{b(L_{i-1}+\mu)} - cp_i)(\mathrm{e}^{(L_{i-1}-T_i)\theta} - \mathrm{e}^{L_{i-1}\theta} + T_i\theta)}{\theta^2} \\ & + \frac{h(\mathrm{e}^{L_{i-1}\theta} - \mathrm{e}^{(L_{i-1}-T_i)\theta})(Q-S_{i-1})}{\theta} \end{aligned} \tag{3.30}$$

$$\begin{aligned} DC_i = d\theta\int_0^{T_i} I_i(t)\mathrm{d}t = {} & \frac{-d(A\mathrm{e}^{b(L_{i-1}+\mu)} - cp_i)(\mathrm{e}^{(L_{i-1}-T_i)\theta} - \mathrm{e}^{L_{i-1}\theta} + T_i\theta)}{\theta} \\ & + d(\mathrm{e}^{L_{i-1}\theta} - \mathrm{e}^{(L_{i-1}-T_i)\theta})(Q-S_{i-1}). \end{aligned} \tag{3.31}$$

Thus, retailer's unit time profit function during the time interval of adopting price can be obtained as

$$\Pi(T_i, p_i, Q) = Ae^{b\mu}p_i - cp_i^2 - \frac{(h+d\theta)(Ae^{b(L_{i-1}+\mu)} - cp_i)(e^{(L_{i-1}-T_i)\theta} - e^{L_{i-1}\theta} + T_i\theta)}{\theta^2 T_i}$$
$$+ \frac{(h+d\theta)(e^{(L_{i-1}\theta} - e^{(L_{i-1}-T_i)\theta)})(Q - S_{i-1})}{\theta T_i}. \tag{3.32}$$

Similar to Case 1, our problem can be denoted as the following constrained problem (P3):

$$\text{(P3)} \quad \max\ \Pi(T_i, p_i, Q) \tag{3.33}$$
$$\text{subject to}\ \ p_i < \frac{Ae^{b\mu}}{c} \tag{3.34}$$
$$S_i < Q \tag{3.35}$$
$$I_i(L_i) > 0 \tag{3.36}$$
$$T_i > 0 \tag{3.37}$$
$$L_i \le L. \tag{3.38}$$

4. An algorithm for model solution

In this section, we develop an algorithm to determine the retailer's maximum profit, optimal order quantity and optimal pricing strategy over the sales season. Since the demand pattern is partitioned two time periods over the entire sales season, the three cases mentioned above may arise in different combinations. To deal with different demand patterns in different successive time periods, we use the following algorithm to determine the maximum retailer's total profit, optimal pricing strategy for the entire sales season and the initial order quantity at the start of the sales season.

Algorithm

Step 1: Let $L_i = 0$, $S_i = 0$, $n = 0$, and determine the initial value of Q (*i.e.* Q_0).

Step 2: Solve problem (P1), $L_i = L_i + T_i, S_i + S_i + \int_0^{T_i} D_t\mathrm{d}t, n = n + 1$.

Step 3: If $\mu < L_i < L$, then $L_i = L_i - T_i, S_i = S_i - \int_0^{T_i} D_t\mathrm{d}t, n = n - 1$ go to Step 5.

Step 4: If $L_i = L$, perform Procedure 1.

Step 5: Solve problem (P2).

Step 6: $L_i = L_i + T_i, S_i = S_i + \int_0^{T_i} D_t\mathrm{d}t, n = n+1$, if $L_i = L$, perform Procedure 1.

Step 7: Solve problem (P3) and go to Step 6.

Procedure 1

If $S_i \neq Q_0$,let $Q_0 = S_i, L_i = 0, S_i = 0, n = 0$. Perform Steps 2–7.

If $S_i = Q_0$,then optimal order quantity $Q^* = Q_0$,optimal pricing interval $T_i^* = T_i$, optimal price $p_i^* = p_i$, calculate profit $\Pi(T_i^*, p_i^*, Q^*)$ and total profit $\Pi_T(T_i^*, p_i^*, Q^*) = \sum_i T_i \times \Pi(T_i^*, p_i^*, Q^*), i = 1, 2, 3, \ldots$ Stop.

In order to perform above Algorithm, we firstly have to determine the initial value of Q (*i.e.* Q_0). We can consider initial Q_0 as the optimal order quantity Q_S^* obtained from the following model (P4) with static pricing strategy.

$$\text{(P4)} \quad \max \mathit{\Pi}(p,Q) = \frac{p}{L}\left(\int_0^{\mu} D_t \mathrm{d}t + \int_{\mu}^{L} D_t \mathrm{d}t\right)$$

$$- \frac{(h+d\theta)}{L}\left(\int_0^{\mu}\left(\frac{A\mathrm{e}^{-\theta t}}{b+\theta}(1-\mathrm{e}^{(b+\theta)t}) + \frac{c(1-\mathrm{e}^{-\theta t})p}{\theta} + Q\mathrm{e}^{-\theta t}\right)\right)\mathrm{d}t$$

$$+ \frac{(h+d\theta)}{L}\int_{\mu}^{L}\left(\frac{1}{\theta}(A\mathrm{e}^{b\mu} - cp)(\mathrm{e}^{-\theta t} - 1) + Q\mathrm{e}^{-\theta t}\right)\mathrm{d}t \tag{4.1}$$

$$p > 0 \tag{4.2}$$

$$Q > 0. \tag{4.3}$$

5. Numerical example

To illustrate the practicality of the proposed model we take the parameter values as $h = 0.60$, $\theta = 0.02$, $A = 50$, $b = 0.02$, $d = 5$, $\mu = 1.0$, $c = 1$. The entire season lengths are assumed as 21 and 15, respectively.

By solving model (P4), we can obtain the optimal price p_S^* and order quantity Q_S^* with static pricing strategy (shown in Tab. 1).

Table 1. Optimal price and order quantity with static pricing strategy.

L	p_S	Q_S	$\mathit{\Pi}_T = \mathit{\Pi}(p,Q) \times L$
20	28.185	455.989	9399.199
15	27.643	350.003	7168.979

Let the initial $Q_0 = Q_S^*$,by applying the above mentioned algorithm, the results can be obtained as in Tables 2 and 3.

We see from Table 2 that the final optimal value of n is 3, and corresponding values of Q is 508.943, $T_1 = 1.102$, $T_2 = 16.054$, $T_3 = 2.844$, $p_1 = 25.078$, $p_2 = 25.658$, $p_3 = 25.017$. The corresponding maximum total profit is 9994.250. In addition, $n = 3$ implies that prices are set three times at time periods 0.0, 1.102–17.156, respectively. The optimal prices 25.078, 25.658, 25.017 are set during time intervals [0,1.102], [1.102, 17.156], [17.15620.000], respectively.

Similarly, we can observe from Table 3 that the final optimal value of n is 3, and corresponding values of Q is 381.856, $T_1 = 0.908$, $T_2 = 12.454$, $T_3 = 1.638$, $p_1 = 25.078$, $p_2 = 25.594$, $p_3 = 25.222$ The corresponding maximum profit is 7986.257. In addition, $n = 3$ implies that prices are set three times at time periods 0.0, 0.90813.362, respectively. The optimal prices 25.078, 25.594, 25.222 are set during time intervals [0,0.908], [0.908, 13.362], [13.362,15.000], respectively.

TABLE 2. Optimal order quantity, time interval for any two successive price changes, corresponding prices and profit ($L = 20$).

Iteration time	i	T_i	L_i	$p_i(T_i)$	S_i	$\Pi(T_i, p_i)$
1	1	0.871	0.871	25.078	25.425	330.377
$Q_0 = 455.989$	2	14.736	15.607	25.630	396.155	507.121
	3	4.393	20.000	24.758	511.483	649.061
2	1	1.124	1.124	25.078	25.425	291.917
$Q_0 = 511.483$	2	16.106	17.230	25.659	436.859	489.550
	3	2.847	20.000	25.016	508.940	625.402
3	1	0.963	0.963	25.078	25.425	293.750
$Q_0 = 508.837$	2	16.190	17.153	25.658	434.947	490.344
	3	2.847	20.000	25.016	508.940	625.402
4	1	0.896	0.896	25.078	25.425	293.679
$Q_0 = 508.940$	2	16.260	17.156	25.657	435.023	490.351
	3	2.844	20.000	25.017	508.947	625.360
5	1	1.054	1.054	25.078	25.426	293.674
$Q_0 = 508.947$	2	16.102	17.156	25.658	435.027	490.349
	3	2.844	20.000	25.017	508.942	625.355
6	1	0.897	0.897	25.078	25.426	293.678
$Q_0 = 508.942$	2	16.259	17.156	25.658	435.023	490.351
	3	2.844	20.000	25.017	508.943	625.358
7	1	1.102	1.102	25.078	25.425	293.678
$Q_0 = 508.943$	2	16.054	17.156	25.658	435.024	490.350
	3	2.844	20.000	25.017	508.943	625.357

TABLE 3. Optimal order quantity, time interval for any two successive price changes, corresponding prices and profit ($L = 15$).

Iteration time	i	T_i	L_i	$p_i(T_i)$	S_i	$\Pi(T_i, p_i)$
1	1	1.013	1.013	25.078	25.426	403.830
$Q_0 = 350.003$	2	11.355	12.368	25.581	314.503	541.200
	3	2.632	15.000	25.052	382.829	649.595
2	1	0.895	0.895	25.078	25.425	381.080
$Q_0 = 382.829$	2	12.497	13.392	25.595	346.368	530.569
	3	1.608	15.000	25.227	381.831	635.537
3	1	0.887	0.887	25.078	25.425	381.772
$Q_0 = 381.831$	2	12.474	13.361	25.594	339.589	530.891
	3	1.639	15.000	25.221	381.857	635.970
4	1	1.112	1.112	25.078	25.426	381.754
$Q_0 = 381.857$	2	12.250	13.362	25.594	339.609	530.883
	3	1.638	15.000	25.221	381.856	635.958
5	1	0.908	0.908	25.078	25.426	381.754
$Q_0 = 381.856$	2	12.454	13.362	25.594	339.609	530.883
	3	1.638	15.000	25.222	381.856	635.959

We can see from Tables 2 and 3 that the price should be set low initially, raised slightly and then decreased again in the three successive intervals. Similarly, time interval for any two successive price changes and the sales revenue during these time intervals also follow a similar change pattern. This implies that, for deteriorating seasonal products with price and ramp-type time dependent demand, at the beginning of sale season, the demand is usually relatively low, in order to increase demand and sale revenue, the sale price should be kept low. As the time goes up, the demand increases accordingly, the sale price can be increased gradually. However, at the end of sale season, the demand will decrease, in order to decrease the loss due to deterioration and stimulate the demand, the sale price should be decreased.

We also obtain that the optimal profits over the entire season length $L = 20$ and $L = 15$ with static pricing strategy ($n = 1$) are 9399.199 and 7168.979, respectively. Comparing this value with that of the dynamic pricing strategy, we find that the dynamic pricing strategy outperforms the static pricing strategy. This implies that, under dynamic pricing strategy, product inventory and demand change as time goes on, retailer's price adjustment can influence the demand and decrease the loss due to deterioration.

6. Conclusions

This paper deals with a dynamic inventory problem for the deteriorating seasonal products with price and ramp-type time dependent demand. The contribution of this paper can be summarized as follows: firstly, this paper relaxes the assumption in [26] that the time interval for any two successive price changes is equal, and considers the time interval for any two successive price changes as a decision variable. Secondly, this paper is an extension of previous paper, where the inventory model for the deteriorating seasonal products with ramp-type time dependent demand has been discussed. Whereas both price and ramp-type time dependent demand is considered in this paper. It is shown that the inventory model with dynamic price proposed in this paper outperforms the inventory model with static price.

The proposed model in this paper can be extended in several ways. For example, we might extend the proposed demand function to the three time periods classified time dependent ramp-type function. We could also consider the deteriorate rate for the season products as a time varying function. The demand could also be generalized as a function of the price, time, stock level, advertising and product quality. Furthermore, we could perform some analysis to investigate if there is some condition on the parameters for joint optimization. Dynamic programming would possibly be used to handle such a problem.

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