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COORDINATING A SUPPLY CHAIN WITH NEGATIVE EFFECT OF RETAILER'S LOCAL PROMOTION ON GOODWILL AND REFERENCE PRICE

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Abstract. This paper investigates a distribution channel consisting of a manufacturer and a retailer under a cooperative program, where the manufacturer determines the national advertising and quality improving effort, while the retailer decides the local promotion effort and may undertake parts of the costs of national advertising and quality improving of the manufacturer. It is assumed that the manufacturer's national advertising and quality improving efforts positively affect the brand goodwill and reference price, whereas the retailer's local promotion effort damages them. Three scenarios of the non-cooperative and cooperative scenarios in the decentralized supply chain, and the centralized supply chain scenario, are analyzed. The corresponding equilibrium strategies and profits are obtained and compared, which shows that the cooperative program can achieve payoff-Pareto-improving, but cannot coordinate completely the supply chain. Furthermore, a revenue sharing contract combined with two-subsidy policy is designed to coordinate the decentralized supply chain. Numerical simulation and sensitivity analysis of the coordinating results on the key system parameters are provided to verify the effectiveness of the contract, and some managerial insights are provided.

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1. Introduction

Supply chain management (SCM) is to apply a total systems approach to managing the entire flow of information, materials, and services in fulfilling a customer demand (see [8]). Since supply chain members are often separate and independent economic entities, a completely integrated solution may result in optimal system performance, this solution is not always in the best interest of every individual member in the system. As a result, independent supply chain members are usually more keen in optimizing their individual objectives rather than that of the entire system. A key issue in SCM is to design mechanisms that can align their individual objectives and coordinate their activities so as to optimize system performance (see [33]). The coordination mechanisms are divided into the operation coordination mechanisms (*e.g.*, contract) (see [5–7, 10]) and the marketing coordination mechanisms (*e.g.*, support programs) (see [21, 26, 29]).

Keywords. Supply chain coordination, cooperative program, revenue sharing contract, reference price, local promotion.

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Supply chain contracts are regarded as useful tools to coordinate the decentralized supply chain as an integrated firm. Revenue sharing contract as an important coordination scheme has attracted a lot of attention of the researchers and practitioner. As for the operation coordination mechanisms, Cachon [6] carefully reviewed the frequently used contracts including buy-back contract, revenue-sharing contract, and wholesale price contract in SC coordination. Pan *et al.* [37] considered a supply chain channel with two manufacturers and one retailer, where manufacturers could choose either a wholesale price contract or a revenue-sharing contract with the retailer to coordinate the supply chain. Feng *et al.* [16] studied a revenue-sharing with reliability contract in an N-stage supply chain. According to the research, we can obtain the strengths and limitations of the RSC contract. One of its main strengths is the mitigation of the double marginalization effect when the demand depends on price because the wholesale and, consequently, retail price turns out to be lower than that in a scenario without a RSC. Recent research in supply chain management has shown that a RSC works perfectly to coordinate a one-supplier, one-retailer chain in applications on the video-rental industry [7]. Some limitations of the RSC also exist. For instance, a RSC does not apply when supply chain competition occurs, when the retailer's effort affects the consumer demand, or when the implementation of information and auditing systems is too complex.

Cooperative program is a cost sharing and promotional mechanism in vertical supply chain, where a partner undertakes parts of the cost incurred by the other partner's promotion efforts, which can increase the rate of consumer demand and improve the whole channel's profit. As a significant market tool to affect consumers purchase behavior and enhance supply chain cooperative operation management, cooperative advertising has been extensively researched. There exists much literature investigating cooperative advertising, which can be divided into two types: static and dynamic. Bergen and John [2] proposed two formal models by considering wholesale price and retail price to analyze effects of advertising "spillovers", differentiation across competing retailers and differentiation across competing manufacturer on the participation rate. Huang and Li [23] and Huang *et al.* [24] developed models to reflect different power structure and corresponding advertising effort. It was shown that the cooperative advertising increased the channel's total profit, but was insufficient for channel coordination. Yue *et al.* [46] extended the model of Huang *et al.* [24] to research cooperative advertising with price elasticity. Xie and Wei [43] and Yang [44] solved a bargaining problem where the players could determine how to divide the extra profit, and determined cooperative advertising, wholesale price and retail price simultaneously based on Nash bargaining model. Chen [9] dealt with a news-vendor problem to the case of a two-level supply chain consisting of one manufacturer and one retailer, and investigated the combined effects of the cooperative advertising mechanism, the return policy and the channel coordination. Wang *et al.* [41] considered cooperative advertising issues of a monopolistic manufacturer with competing duopolistic retailers. Yang *et al.* [45] found a model to address cooperative advertising by incorporating the effect of retailer's fairness concerns.

Chintagunta and Jain [11] formulated a dynamic model to determine the equilibrium marketing effort levels for a manufacturer and a retailer of distribution channel. Jørgensen *et al.* [27] put forward a dynamic model with cooperative advertising based on Nerlove–Arrow framework, by assuming that both the manufacturer and retailer could make long term and short term advertising efforts to enhance the brand goodwill and demand rate. It was concluded that the manufacturer ought to support both types of retailer's advertising rather than only one type. In addition, supporting one type was still more profitable than supporting none. Jørgensen *et al.* [28] improved the model of Jørgensen *et al.* [27] with assumption of the decreasing marginal returns to goodwill. It was indicated that whether the goodwill stock had a decreasing marginal effect on demand rate or not, the cooperative advertising program was a coordinating mechanism. Jørgensen *et al.* [29] and Jørgensen and Zaccour [26] explored cooperative advertising even the retailer's local promotion might erode the brand goodwill. Bass *et al.* [1] gave research on cooperative advertising and pricing simultaneously by allowing for market expansion and market share effects. Jørgensen *et al.* [30] characterized how a manufacturer had to construct an incentive to stimulate the retailer to advertise at the same level of the vertical integration scenario. He *et al.* [20] focused on the cooperative advertising and pricing simultaneously by using the stochastic version of Sethi model (see [39]). He *et al.* [21] investigated cooperative advertising channel composed of a manufacturer and two independent competing retailers. The result illustrated that the manufacturer preferred to support for its retailer more in competition environment. Then He *et al.* [22] amended the above model by considering a consumer goods manufacturer selling through a retailer in competition with outside retailers. Chutani and Sethi[12] designed cooperative advertising and pricing strategies in a dynamic durable goods duopoly with a manufacturer and two independent competing retailers.

The aforementioned papers only incorporated marketing tool such as advertising, pricing to investigate cooperative programme, without the consideration of product quality which is an important operations tool affecting consumer purchase decision. The combination of the marketing and operations management research has attracted considerable attention from scholars. Dalalah [13] developed a novel supply chain to investigate the optimal pricing and manufacturing rate. Vörös [40] constructed a dynamic model to investigate pricing, quality and productivity improvement decisions. Nair and Narasimhan [35] formulated a dynamic game model to study dynamic of competing with quality- and advertising-based goodwill. Giovanni [15] examined the case where both advertising and quality improvement facilitated the build-up of brand goodwill by assuming that the manufacturer improved the product quality and undertook parts of the cost of retailer's advertisement. The result shown that the retailer was always better off with a cooperative program.

In addition, the above papers explored the problem of making firms' operations strategies by just considering the internal factors such as sales price, quality and brand goodwill, without taking the consumer reference price, a significant external factor affecting consumer purchase decisions, into account in the cooperative promotion program model. Reference price is a price in consumers' mind to be compared with the shelf price of a specific product. If current price is lower than reference price, it affects positively, otherwise negatively, on the demand rate [31,42]). Mazumdar *et al.* [34] reviewed the reference price comprehensively, and indicated that the reference price could be affected by many factors, such as prior purchase price, price sensitivity, advertising, quality, brand loyalty and so forth. Considering the effect of asymmetric reference price, Kopalle *et al.* [32] obtained the dynamic pricing policy by using dynamic programming. Fibich *et al.* [17] generated a dynamic pricing strategy under symmetric and asymmetric effects of reference price on demand function. An explicit solution to the optimization problem was obtained by applying optimal control theory. Popescu and Wu [38] handled a dynamic pricing problem of a monopolist firm in a market with repeated interactions, where demand was sensitive to the firm's pricing history. Their results indicated the firm's optimal pricing strategies converged over the long run to a constant price, and managers who ignored long term implications of their pricing strategy would consistently price too low to lose revenue systematically. Some interesting results about dynamic pricing with reference price effect were proposed in Fibich *et al.* [18], Geng *et al.* [19], Nasiry and Popescu [36]. The above mentioned papers mainly focused on dynamic pricing strategies, whereas they didn't consider the firm's other measures, such as advertising and improving production quality which may affect the reference price. Zhang *et al.* [47] firstly established a differential game model to study cooperative advertising by taking reference price into account and a new supply chain coordination mechanism was introduced. Since reference price has a significant effect on consumer purchase decision, we take it into account in our model. Zhang *et al.* [48] developed an advertising model where goodwill affected by advertising effort has a positive effect on reference price and market demand and obtained the optimal advertising strategies in finite and infinite horizons.

According to the above analysis and enlightening from the model formulated in Jørgensen *et al.* [29] and Zhang *et al.* [47], we characterize a distribution channel consisting of a manufacturer and a retailer, where the manufacturer's national advertising and quality improving efforts have positive effects on the brand goodwill and reference price, whereas the retailer's local promotion has negative effect on both of them. Additionally, it is assumed that manufacturer's national advertising effort, retailer's local promotion effort, the brand goodwill and reference price positively affect the demand rate. Three scenarios including the non-cooperative and cooperative scenarios of the decentralized supply chain, and the centralized supply chain scenario, are analyzed, and the corresponding equilibrium strategies and profits are obtained. Motivated by the observing power shift from manufacturer to retailer in Huang and Li [23] and Huang *et al.* [24], in the decentralized supply chain system, we study the case that the retailer is the leader and the manufacturer is the follower. The equilibrium strategies, steady-state variables and profits are compared, which indicate the cooperative program can achieve payoff-Pareto-improving, but cannot coordinate completely the supply chain. Furthermore, a revenue sharing contract combined two-subsidy policy is introduced to coordinate the decentralized supply chain. Numerical example and sensitivity analysis of the coordinating results on the key system parameters are given to verify the effectiveness of the presented contract, meanwhile some managerial insights are obtained.

The rest of this paper is organized as follows. A dynamic game model is formulated in Section 2. The equilibria of the three scenarios are obtained, and the comparisons of strategies, steady-state and profits are provided in Section 3. A contract is introduced to coordinate the decentralized supply chain in Section 4. Numerical analysis is presented in Section 5. The paper is concluded in Section 6.

2. The model

Consider a distribution channel consisting of a manufacturer and a retailer. The retailer purchases items from the manufacturer, and then sells them to consumers. To increase demand, the manufacturer invests in national advertising and improving quality, and the retailer invests in local promotion.

Although, in most marketing literature, it has been assumed that retailer's local promotion has a positive effect on the brand's goodwill, some scholars have supposed an opposite assumption that the negative effect of retailer's promotion on the brand's goodwill may also occur (Davis *et al.* [14], Jamal *et al.* [25], Jørgensen and Zaccour [26]). This hypothesis is in line with a recommendation of advertising executives, that frequent promotions will destroy the brand's image (see Blattberg and Neslin [3]). One reason can be that consumers come to believe that frequent promotions are used as a "cover up" for insufficient quality, holding the view that high quality products need little or no promotion (see Jørgensen *et al.* [29]). According to the above empirical and analytical results, we suppose that the local promotion effort of the retailer will damage both the brand's goodwill and reference price.

Let $a(t) \geq 0$, $q(t) \geq 0$ and $b(t) \geq 0$ represent the manufacturer's national advertising effort, quality improving effort and the retailer's local promotion effort at time t, respectively. The manufacturer's national advertising and quality improving efforts have positive impacts on the brand goodwill, which is denoted by $G(t)$, whereas the retailer's local promotion effort negatively affects the brand goodwill. Thus, the dynamics of the brand goodwill $G(t)$ can be described by the following differential equation

$$\dot{G}(t) = \theta_1 a(t) + \theta_2 q(t) - \theta_3 b(t) - \delta G(t), \quad G(0) = G_0, \tag{2.1}$$

where $G_0 > 0$ is initial brand goodwill level, θ_1, θ_2 and θ_3 are positive constants, which represent the effects of manufacturer's national advertising effort, quality improving effort and the retailer's local promotion effort, respectively, on the brand goodwill, and $\delta > 0$ is depreciation coefficient of the brand goodwill.

Reference price is viewed as a predictive price expectation and formed by consumers' shopping experience and current purchase environment [4, 31]. When consumers decide whether to buy a product or not, current price and reference price would have significant effects on their purchase decisions. According to Mazumdar *et al.* [34], reference price can be affected by advertising, product quality, previous price and so on. Differentiating from the assumption that the local promotion positively affects the reference price, we assume that the manufacturer's national advertising and quality improving efforts have positive effects on the reference price, yet the retailer's local promotion effort damages the reference price. Let $r(t)$ denote the reference price at time t, and p represent sales price. Thus, the dynamics of reference price $r(t)$ can be described by the following differential equation

$$\dot{r}(t) = \alpha(p - r(t)) + \nu_1 a(t) + \nu_2 q(t) - \nu_3 b(t), r(0) = r_0, \tag{2.2}$$

where $r_0 > 0$ is initial reference price, and ν_1, ν_2 and ν_3 are all positive constants, which represent the effects of the manufacturer's national advertising effort, quality improving effort and the retailer's local promotion effort, respectively, on the reference price. $\alpha > 0$ is interpreted as "memory parameter", which reflects the memory impact on the reference price.

This paper focuses on investigating the manufacturer's optimal national advertising and quality improving efforts which affect positively on the brand goodwill and reference price as well as the retailer's optimal local promotion effort, which damages the brand goodwill and reference price. Hence, we view the sales price p as an exogenous variable to avert the price decision. Additionally, the dynamic sales price may be considered in our model, but frequent change of sales price would lead to negative effects on the brand goodwill and the consumer purchasing decision. Thus the sales price p is treated as a given constant in the sales period.

Both the manufacturer's national advertising and the retailer's local promotion efforts have direct and positive impacts on consumer demand rate, while the quality improving effort has indirect effect on consumer demand rate. In addition, the brand goodwill and reference price have direct effects on consumer demand rate. Consequently, the demand rate $D(t)$ is given by:

$$D(t) = \mu_1 a(t) + \mu_2 b(t) + \mu_3 G(t) + \mu_4 (r(t) - p), \tag{2.3}$$

where μ_1, μ_2, μ_3 and μ_4 are all positive constants, which represent the impacts of the manufacturer's national advertising effort, the retailer's local promotion effort, the brand goodwill and reference price effect on the demand rate, respectively. In equation (2.3), the items $\mu_1 a(t)$ and $\mu_2 b(t)$ reflect that the manufacturer's national advertising and retailer's local promotion efforts have instant effects on the consumer demand. The item $\mu_3 G(t)$ represents that the brand goodwill have positive effect on the current sales. The item $\mu_4(r(t) - p)$ represents the reference price effect on the consumer demand. The reference price effect has positive impact on the consumer when $r > p$ and *vice versa*. The demand function specification in equation (2.3) reflects the retailer's tradeoff: promoting strongly will boost the current demand, but it will damage the goodwill and reference price which have positive effects on the current demand. The demand which is linear in the supply chain efforts, the goodwill and the price gap, has been extensively used in numerous literature by scholars, such as Jørgensen *et al.* [29], Zhang *et al.* [47] and Zhang *et al.* [48].

Similar to previous literature such as He *et al.* [21], Jørgensen *et al.* [27] and Zhang *et al.* [47], the manufacturer's national advertising, quality improving and the retailer's local promotion costs, respectively, are convex increasing and, and take quadratic form for simplicity, *i.e.*,

$$C_a(a) = \frac{1}{2} c_a a^2, \quad C_q(q) = \frac{1}{2} c_q q^2, \quad C_b(b) = \frac{1}{2} c_b b^2,$$

where c_a, c_q and c_b are positive constants.

Let $\pi_M \geq 0$ and $\pi_R \geq 0$ represent the marginal profits of the manufacturer and the retailer, respectively. To stimulate the manufacturer to invest in national advertising and improving quality efforts, the retailer bears parts of the costs of national advertising and quality improving efforts. Let $\phi_a(t)$ and $\phi_q(t)$, $0 \leq \phi_i(t) \leq 1$, $i \in \{a, q\}$, denote the participation rates that the retailer shares the costs of national advertising and quality improving efforts, respectively.

When the manufacturer and the retailer play the game over an infinite horizon, the objective function of the manufacturer is

$$\begin{aligned} \max_{a(\cdot),q(\cdot)} J_M = \int_0^\infty \mathrm{e}^{-\rho t} \Bigg[&\pi_M(\mu_1 a(t) + \mu_2 b(t) + \mu_3 G(t) + \mu_4(r(t) - p(t))) \\ &- \frac{1}{2}(1 - \phi_q(t)) c_q q^2(t) - \frac{1}{2}(1 - \phi_a(t)) c_a a^2(t) \Bigg] \mathrm{d}t, \end{aligned} \tag{2.4}$$

and that of the retailer is

$$\begin{aligned} \max_{b(\cdot),\phi_i(\cdot)} J_R = \int_0^\infty \mathrm{e}^{-\rho t} \Bigg[&\pi_R(\mu_1 a(t) + \mu_2 b(t) + \mu_3 G(t) + \mu_4(r(t) - p(t))) \\ &- \frac{1}{2}\phi_q(t) c_q q^2(t) - \frac{1}{2}\phi_a(t) c_a a^2(t) - \frac{1}{2} c_b b^2(t) \Bigg] \mathrm{d}t, \end{aligned} \tag{2.5}$$

where $\rho > 0$ denotes discount rate.

When the retailer and manufacturer are integrated as a whole firm, the whole firm's objective function is

$$\max_{a(\cdot),q(\cdot),b(\cdot)} J_F = \int_0^\infty \mathrm{e}^{-\rho t}\Bigg[(\pi_M+\pi_R)(\mu_1 a(t)+\mu_2 b(t)+\mu_3 G(t)+\mu_4(r(t)-p(t))) - \frac{1}{2}c_q q^2(t) - \frac{1}{2}c_a a^2(t) - \frac{1}{2}c_b b^2(t)\Bigg]\mathrm{d}t. \tag{2.6}$$

According to (2.1), (2.2), (2.4) and (2.5), we form a differential game with two players, five control variables $a(t)$, $q(t)$, $b(t)$, $\phi_a(t)$, $\phi_q(t)$ and two state variables $G(t)$ and $r(t)$. The control variables are constrained by

$$a(t) \geq 0, \quad q(t) \geq 0, \quad b(t) \geq 0, \quad \phi_i \in [0,1], i \in \{a,q\}.$$

Later in this paper, the time argument is omitted when there is no confusion.

3. Equilibria and comparisons

According to the supply chain construction and whether cooperation exists in the supply chain members, we describe three scenarios as follows.

Non-cooperative scenario of the decentralized supply chain. In this scenario the manufacturer and retailer will adopt a non-cooperative program, namely, the retailer as the leader will not undertake the costs of the manufacturer's national advertising and quality improving efforts. The subscript "N" is used to represent "non-cooperative scenario of the decentralized setting".

Cooperative scenario of the decentralized supply chain. In this scenario the manufacturer and retailer will adopt a cooperative program, that is, the retailer as the leader bears parts of the costs of the manufacturer's national advertising and quality improving efforts. We use the subscript "C" for referring to "cooperative scenario of the decentralized setting".

Centralized supply chain. In this scenario the manufacturer and retailer are integrated as a whole firm, and make the optimal strategies to maximize the whole firm's profit. We use subscript "I" to signify "the centralized supply chain".

We derive the equilibria of the two scenarios of the decentralized supply chain, and the optimal strategies of the centralized supply chain, and further compare the corresponding equilibrium strategies and profits in the following subsections.

3.1. Equilibria in the non-cooperative scenario

This scenario is played in a non-cooperative Stackelberg game with the retailer as the leader. The sequence of the events is as follows: first, the retailer announces the non-cooperative program, namely $\phi_a = \phi_q = 0$, and the local promotion strategy. Second, the manufacturer makes the national advertising and quality improving strategies according to the decisions announced by the retailer. Since in this scenario the retailer's participation rates are $\phi_a = \phi_q = 0$, which don't affect the manufacturer's quality improving and national advertising strategies, the equilibrium strategies obtained are the same as those obtained in a Nash game scenario.

Proposition 3.1 characterizes the equilibrium strategies in the non-cooperative scenario. For the smoothness of the paper, the proofs for this proposition and all subsequent propositions and corollaries are presented in the appendix.

Proposition 3.1. *The equilibrium advertising and quality improving strategies of the manufacturer are*

$$a^{\mathrm{N}} = \frac{\pi_M}{c_a}A, \tag{3.1}$$

$$q^{\mathrm{N}} = \frac{\pi_M}{c_q}Q, \tag{3.2}$$

and the local promotion strategy of the retailer is

$$b^{\mathrm{N}} = \begin{cases} \dfrac{\pi_R}{c_b} B, & \mu_2 > \dfrac{\theta_3\mu_3}{\rho+\delta} + \dfrac{\nu_3\mu_4}{\rho+\alpha} \\ 0, & \text{otherwise,} \end{cases} \tag{3.3}$$

where

$$A = \mu_1 + \frac{\theta_1\mu_3}{\rho+\delta} + \frac{\nu_1\mu_4}{\rho+\alpha}, \tag{3.4}$$

$$Q = \frac{\theta_2\mu_3}{\rho+\delta} + \frac{\nu_2\mu_4}{\rho+\alpha}, \tag{3.5}$$

$$B = \mu_2 - \frac{\theta_3\mu_3}{\rho+\delta} - \frac{\nu_3\mu_4}{\rho+\alpha}. \tag{3.6}$$

The manufacturer's and retailer's value functions are given by

$$V_M^{\mathrm{N}}\left(G^{\mathrm{N}}, r^{\mathrm{N}}\right) = \frac{\pi_M\mu_3}{\rho+\delta} G^{\mathrm{N}} + \frac{\pi_M\mu_4}{\rho+\alpha} r^{\mathrm{N}} + l, \tag{3.7}$$

$$V_R^{\mathrm{N}}\left(G^{\mathrm{N}}, r^{\mathrm{N}}\right) = \frac{\pi_R\mu_3}{\rho+\delta} G^{\mathrm{N}} + \frac{\pi_R\mu_4}{\rho+\alpha} r^{\mathrm{N}} + m, \tag{3.8}$$

where

$$l = \frac{\pi_M^2}{2\rho}\left(\frac{A^2}{c_a} + \frac{Q^2}{c_q}\right) + \frac{\pi_M\pi_R}{c_b\rho} B^2 - \frac{\pi_M\mu_4 p}{\rho+\alpha},$$

$$m = \frac{\pi_M\pi_R}{\rho}\left(\frac{A^2}{c_a} + \frac{Q^2}{c_q}\right) + \frac{\pi_R^2}{2c_b\rho} B^2 - \frac{\pi_R\mu_4 p}{\rho+\alpha}.$$

It is shown from Proposition 3.1 that the equilibrium strategies are constants, due to the fact that the present game is a linear state game. Such a property may be less satisfactory in a differential game with an infinite horizon. To enable the equilibrium strategies and profits comparisons that we present in subsection 3.4, the property of the equilibrium seems to be the price that must be paid (Jørgensen *et al.* [29]). On another hand, the constant strategies are easy to carry out from a managerial perspective.

From Proposition 3.1, we can easily obtain the following results.

(i) It is shown from (3.1) that a^N consists of $\pi_M\mu_1/c_a$, $\pi_M\theta_1\mu_3/(c_a(\rho+\delta))$ and $\pi_M\nu_1\mu_4/(c_a(\rho+\alpha))$. The same result has been obtained in Zhang *et al.* [47]. The first part is used to get direct effect on demand rate. The second and third parts are utilized to obtain the effect of national advertising effort on the brand goodwill and reference price, which can affect demand rate directly. Comparing with the manufacturer's national advertising effort obtained in Jørgensen *et al.* [29], the result obtained in (3.1) shows that there are two new terms in a^N, *i.e.*, the first and third parts. The first part comes from the direct impact of manufacturer's national advertising effort on demand rate. Considering the reference price effect on demand rate results in the third part.

(ii) As represented in (3.2), the equilibrium quality improving strategy consists of two parts, which are $\pi_M\theta_2\mu_3/(c_q(\rho+\delta))$ and $\pi_M\nu_2\mu_4/(c_q(\rho+\alpha))$, resulting from that the manufacturer's quality improving effort have positive effect on the brand goodwill and reference price, which affect the demand rate directly and positively. In our model, the manufacturer's quality improving effort doesn't affect the demand rate directly, hence, (3.2) misses the first part as in (3.1).

(iii) It can be found from (3.3) that only if μ_2 is sufficiently large would the retailer invest in local promotion effort. Otherwise, the retailer would abstain from the local promotion effort. If $\mu_2 > \theta_3\mu_3/(\rho+\delta) + \nu_3\mu_4/(\rho+\alpha)$, it is shown from (3.3) that the retailer's local promotion effort b^N consists of $\pi_R\mu_2/c_b$,

$-\pi_R\theta_3\mu_3/(c_b(\rho+\delta))$ and $-\pi_R\nu_3\mu_4/(c_b(\rho+\alpha))$. In this paper, we assume that the retailer's local promotion effort damages the brand goodwill and reference price, which is indicated from the second and third parts. Comparing with the retailer's local promotion effort obtained in Jørgensen *et al.* [29], (3.3) shows that there is a new term in b^N, *i.e.*, the third part, due to considering the reference price. The retailer will decrease the local promotion effort when the retailer takes the negative effect of the promotion effort on the reference price into account.

By substituting (3.1), (3.2) and (3.3) into (2.1) and (2.2), we obtain the brand goodwill and reference price as

$$G^{\mathrm{N}}(t) = (G_0 - G^{\mathrm{N}}_{\infty})\mathrm{e}^{-\delta t} + G^{\mathrm{N}}_{\infty},$$
$$r^{\mathrm{N}}(t) = (r_0 - r^{\mathrm{N}}_{\infty})\mathrm{e}^{-\alpha t} + r^{\mathrm{N}}_{\infty},$$

where $G^{\mathrm{N}}_{\infty} = (\theta_1 a^{\mathrm{N}} + \theta_2 q^{\mathrm{N}} - \theta_3 b^{\mathrm{N}})/\delta$ and $r^{\mathrm{N}}_{\infty} = p + (\nu_1 a^{\mathrm{N}} + \nu_2 q^{\mathrm{N}} - \nu_3 b^{\mathrm{N}})/\alpha$.

To keep nonnegative steady-state for the brand goodwill and reference price, we impose the following conditions

$$\theta_1 a^{\mathrm{N}} + \theta_2 q^{\mathrm{N}} > \theta_3 b^{\mathrm{N}},\ \nu_1 a^{\mathrm{N}} + \nu_2 q^{\mathrm{N}} > \nu_3 b^{\mathrm{N}}.$$

From Proposition 3.1, it is easy to obtain Corollary 3.2.

Corollary 3.2. *The equilibrium strategies a^{N}, q^{N} and b^{N} presented in (3.1)–(3.3)) satisfy*

(1) *a^{N} is increasing in π_M, μ_1, μ_3, μ_4, θ_1, ν_1, and decreasing in c_a, ρ, δ and α;*
(2) *q^{N} is increasing in π_M, μ_3, μ_4, θ_2, ν_2, and decreasing in c_q, ρ, δ and α;*
(3) *b^{N} in the first expression of (3.3) is increasing in π_R, μ_2, ρ, δ, α, and decreasing in c_b, μ_3, μ_4, θ_3 and ν_3.*

The following results directly hold from Corollary 3.2.

(i) It is reasonable for both the manufacturer and retailer to increase national advertising, quality improving and local promotional efforts when their marginal profits increase, but reduce their efforts when cost coefficients increase.
(ii) To obtain more profits, the manufacturer increases national advertising effort when μ_1 increases, and the retailer increases local promotion effort when μ_2 increases to stimulate the demand.
(iii) The manufacturer increases the national advertising effort when either of θ_1 and ν_1 increases, and increases quality improving effort when either θ_2 or ν_2 increases. The manufacturer ought to increase the national advertising and quality improving efforts, whereas the retailer ought to cut down on the local promotion effort, as either of μ_3 and μ_4 increases. It is rational that the rise of the brand goodwill and reference price, derived from the increase of the national advertising and quality improving efforts, leads to the growth of the demand rate and profit. In this way, the retailer prefers to decrease the local promotion effort to free-ride on the manufacturer's efforts. Additionally, it is advisable for the retailer to decrease the local promotion effort to reduce its negative effect on the brand goodwill and reference price, when either θ_2 or ν_2 increases.
(iv) The manufacturer reduces the national advertising and quality improving efforts, yet the retailer increases the local promotion effort, when any of δ, ρ and α increases. The increase in δ restrains the accumulation of the brand goodwill, and thereby lessens the effect of the national advertising and improving efforts on the demand, which makes the manufacturer decrease these efforts. The greater ρ is, the less the manufacturer is likely to invest for the long-term. On account of that the local promotion effort boosts sales temporarily, it is valid for the retailer to increase the local promotion effort. The larger α is, the less consumer loyalty is. When α increases, the manufacturer has less interest to establish the goodwill and reference price, whereas the retailer will increase the local promotion effort to stimulate the consumers to buy the items to obtain short-term payoff, even if the retailer's local promotion effort damages the brand goodwill and reference price.

3.2. Equilibria in the cooperative scenario

In this scenario, the retailer supports the manufacturer's national advertising and quality improving efforts. The game structure is a Stackelberg game and the sequence of the events is as follows: The retailer first announces the cooperative programme and the local promotion strategy. Then the manufacturer decides the national advertising and quality improving efforts according to the decisions announced by the retailer.

Proposition 3.3 characterizes the equilibrium strategies in the cooperative scenario.

Proposition 3.3. *The equilibrium advertising and quality improving strategies of the manufacturer are*

$$a^{\mathrm{C}} = \frac{\pi_M}{c_a(1-\phi_a)} A, \tag{3.9}$$

$$q^{\mathrm{C}} = \frac{\pi_M}{c_q(1-\phi_q)} Q, \tag{3.10}$$

and the retailer's local promotion strategy is

$$b^{\mathrm{C}} = \begin{cases} \dfrac{\pi_R}{c_b} B, & \text{if } \mu_2 > \dfrac{\theta_3 \mu_3}{\rho+\delta} + \dfrac{\nu_3 \mu_4}{\rho+\alpha} \\ 0, & \text{otherwise.} \end{cases} \tag{3.11}$$

The retailer's equilibrium participation rates ϕ_a and ϕ_q are

$$\phi_a = \phi_q = \begin{cases} \dfrac{2\pi_R - \pi_M}{2\pi_R + \pi_M}, & \text{if } \; 2\pi_R > \pi_M \\ 0, & \text{otherwise.} \end{cases} \tag{3.12}$$

The manufacturer's and retailer's value functions, respectively, are

$$V_M^{\mathrm{C}}\left(G^{\mathrm{C}}, r^{\mathrm{C}}\right) = \frac{\pi_M \mu_3}{\rho+\delta} G^{\mathrm{C}} + \frac{\pi_M \mu_4}{\rho+\alpha} r^{\mathrm{C}} + s, \tag{3.13}$$

$$V_R^{\mathrm{C}}\left(G^{\mathrm{C}}, r^{\mathrm{C}}\right) = \frac{\pi_R \mu_3}{\rho+\delta} G^{\mathrm{C}} + \frac{\pi_R \mu_4}{\rho+\alpha} r^{\mathrm{C}} + h, \tag{3.14}$$

where

$$s = \frac{\pi_M(2\pi_R + \pi_M)}{4\rho}\left(\frac{A^2}{c_a} + \frac{Q^2}{c_q}\right) + \frac{\pi_M \pi_R}{c_b \rho} B^2 - \frac{\pi_M \mu_4 p}{\rho+\alpha},$$

$$h = \frac{(2\pi_R + \pi_M)^2}{8\rho}\left(\frac{A^2}{c_a} + \frac{Q^2}{c_q}\right) + \frac{\pi_R^2}{2c_b \rho} B^2 - \frac{\pi_R \mu_4 p}{\rho+\alpha}.$$

From Proposition 3.3, we have the following results.

(i) The retailer's local promotion effort is the same as that in Proposition 3.1. If the participation rates ϕ_a and ϕ_q are zero, the manufacturer advertising and quality improving efforts are the same as those in Proposition 3.1. If the participation rates $0 < \phi_i < 1$, $i \in \{a, q\}$, differentiating (3.9) and (3.10) with respect to ϕ_a and ϕ_q, respectively, we obtain

$$\frac{\partial a^{\mathrm{C}}}{\partial \phi_a} = \frac{\pi_M}{c_a(1-\phi_a)^2} A > 0, \tag{3.15}$$

$$\frac{\partial q^{\mathrm{C}}}{\partial \phi_q} = \frac{\pi_M}{c_q(1-\phi_q)^2} Q > 0, \tag{3.16}$$

which show that the more the retailer undertakes the costs of the manufacturer's national advertising and quality improving efforts, the more the manufacturer will spend on the national advertising and quality improving efforts.

If the participation rates $0 < \phi_i < 1$, $i \in \{a, q\}$, substituting the first expression in (3.12) into (3.9) and (3.10), respectively, we have

$$a^{\mathrm{C}} = \frac{\pi_M + 2\pi_R}{2c_a} A, \tag{3.17}$$

$$q^{\mathrm{C}} = \frac{\pi_M + 2\pi_R}{2c_q} Q. \tag{3.18}$$

Differentiating (3.17) and (3.18) with respect to π_R, respectively, yields

$$\frac{\partial a^{\mathrm{C}}}{\partial \pi_R} = \frac{1}{c_a} A > 0, \tag{3.19}$$

$$\frac{\partial q^{\mathrm{C}}}{\partial \pi_R} = \frac{1}{c_q} Q > 0. \tag{3.20}$$

The results obtained from (3.19) and (3.20) are different from those obtained as (1) and (2) in Corollary 4.1, which show that the retailer's marginal profit has no effect on the manufacturer's national advertising and quality improving efforts, whereas (3.19) and (3.20) show that the manufacturer's national advertising and quality improving efforts increase in the retailer's marginal profit π_R.

(ii) It is shown from (3.12) that the retailer's participation rates ϕ_a and ϕ_q depend on both manufacturer's and retailer's marginal profits π_M and π_R. If $2\pi_R > \pi_M$, the retailer will support the manufacturer's national advertising and quality improving efforts. Specifically, if $\pi_M = 0$, the retailer will undertake all the costs of the national advertising and quality improving efforts, otherwise the retailer will support nothing.

By substituting (3.9), (3.10) and (3.11) into (2.1) and (2.2), we obtain the brand goodwill and reference price as

$$G^{\mathrm{C}}(t) = (G_0 - G_\infty^{\mathrm{S}})\mathrm{e}^{-\delta t} + G_\infty^{\mathrm{C}},$$
$$r^{\mathrm{C}}(t) = (r_0 - r_\infty^{\mathrm{S}})\mathrm{e}^{-\alpha t} + r_\infty^{\mathrm{C}},$$

where $G_\infty^{\mathrm{C}} = (\theta_1 a^{\mathrm{C}} + \theta_2 q^{\mathrm{C}} - \theta_3 b^{\mathrm{C}})/\delta$ and $r_\infty^{\mathrm{C}} = p + (\nu_1 a^{\mathrm{C}} + \nu_2 q^{\mathrm{C}} - \nu_3 b^{\mathrm{C}})/\alpha$.

The following result comes directly from Proposition 3.3.

Corollary 3.4. *For $2\pi_R > \pi_M$, ϕ_a and ϕ_q are increasing in π_R and decreasing in π_M, respectively.*

From Corollary 3.4, we obtain that the retailer's participation rates ϕ_a and ϕ_q increase if the retailer's marginal profit increases or the manufacturer's marginal profit decreases. If the retailer's marginal profit increases, according to (3) in Corollary 3.2 and Corollary 3.4, the retailer will increase the local promotion effort and the participation rates ϕ_a and ϕ_q, which stimulates the manufacturer to establish higher brand goodwill and reference price, by increasing its national advertising and quality improving efforts. On the other hand, if the manufacturer's marginal profit increases, the manufacturer will increase the national advertising and quality improving efforts to build up higher brand goodwill and reference price, whereas the retailer will decrease the participation rates ϕ_a and ϕ_q to reduce the operations costs and free-ride on the manufacturer's national advertising and quality improving efforts.

3.3. Integrated supply chain

In this subsection, decision-making is centralized and the optimal strategies are obtained to maximize the whole firm's profit.

Proposition 3.5 characterizes the firm's optimal strategies.

Proposition 3.5. *The optimal advertising, quality improving and local promotion strategies in the integrated supply chain are both constants*, i.e.,

$$a^{\mathrm{I}} = \frac{\pi_M + \pi_R}{c_a} A, \tag{3.21}$$

$$q^{\mathrm{I}} = \frac{\pi_M + \pi_R}{c_q} Q, \tag{3.22}$$

$$b^{\mathrm{I}} = \begin{cases} \dfrac{\pi_M + \pi_R}{c_b} B, & \mu_2 > \dfrac{\theta_3 \mu_3}{\rho + \delta} + \dfrac{\nu_3 \mu_4}{\rho + \alpha} \\ 0, & \textit{otherwise.} \end{cases} \tag{3.23}$$

The firm's value function is given by

$$V_F^{\mathrm{I}}\left(G^{\mathrm{I}}, r^{\mathrm{I}}\right) = \frac{(\pi_M + \pi_R)\mu_3}{\rho + \delta} G^{\mathrm{I}} + \frac{(\pi_M + \pi_R)\mu_4}{\rho + \alpha} r^{\mathrm{I}} + k, \tag{3.24}$$

where

$$k = \frac{(\pi_M + \pi_R)^2}{2\rho} \left[\frac{A^2}{c_a} + \frac{Q^2}{c_q} + \frac{B^2}{c_b} \right] - \frac{(\pi_M + \pi_R)\mu_4 p}{\rho + \alpha}.$$

By substituting (3.21), (3.22) and (3.23) into (2.1) and (2.2), we obtain the brand goodwill and reference price as

$$G^{\mathrm{I}}(t) = (G_0 - G_\infty^{\mathrm{I}})\mathrm{e}^{-\delta t} + G_\infty^{\mathrm{I}},$$
$$r^{\mathrm{I}}(t) = (r_0 - r_\infty^{\mathrm{I}})\mathrm{e}^{-\alpha t} + r_\infty^{\mathrm{I}},$$

where $G_\infty^{\mathrm{I}} = (\theta_1 a^{\mathrm{I}} + \theta_2 q^{\mathrm{I}} - \theta_3 b^{\mathrm{I}})/\delta$ and $r_\infty^{\mathrm{I}} = p + (\nu_1 a^{\mathrm{I}} + \nu_2 q^{\mathrm{I}} - \nu_3 b^{\mathrm{I}})/\alpha$.

To keep nonnegative steady-state for the brand goodwill and reference price, we impose the following conditions

$$\theta_1 a^{\mathrm{I}} + \theta_2 q^{\mathrm{I}} > \theta_3 b^{\mathrm{I}}, \ \nu_1 a^{\mathrm{I}} + \nu_2 q^{\mathrm{I}} > \nu_3 b^{\mathrm{I}}.$$

3.4. Comparison of results

In this subsection, we analyze the differences in strategies, steady-state variables and players' outcomes when shifting from a centralized supply chain to the non-cooperative and cooperative scenarios of the decentralized supply chain. In the following proposition, we present the comparisons of the equilibrium strategies.

Proposition 3.6. *Comparing the equilibrium strategies obtained in different decision scenarios, we can obtain the following results*

(i) *The manufacturer's national advertising effort relationships among the three scenarios are* $a^{\mathrm{I}} > a^{\mathrm{C}} > a^{\mathrm{N}}$, *if* $2\pi_R > \pi_M$, *otherwise* $a^{\mathrm{I}} > a^{\mathrm{C}} = a^{\mathrm{N}}$;
(ii) *The manufacturer's improving quality effort relationships among the three scenarios are* $q^{\mathrm{I}} > q^{\mathrm{C}} > q^{\mathrm{N}}$, *if* $2\pi_R > \pi_M$, *otherwise* $q^{\mathrm{I}} > q^{\mathrm{C}} = q^{\mathrm{N}}$;
(iii) *The retailer's local promotional effort relationships among the three scenarios are* $b^{\mathrm{I}} > b^{\mathrm{C}} = b^{\mathrm{N}}$.

From Proposition 3.6, we can obtain that the manufacturer will increase the national advertising and quality improving efforts when the retailer's participation rates ϕ_a and ϕ_q are nonzero. Hence, for stimulating the manufacturer to increase the national advertising and quality improving efforts, when $2\pi_R > \pi_M$, the retailer will support the manufacturer's efforts.

Note that

$$a^{\mathrm{C}} - a^{\mathrm{N}} = \phi_a a^{\mathrm{C}} \Longleftrightarrow \frac{a^{\mathrm{C}} - a^{\mathrm{N}}}{a^{\mathrm{C}}} = \phi_a,$$
$$q^{\mathrm{C}} - q^{\mathrm{N}} = \phi_q q^{\mathrm{C}} \Longleftrightarrow \frac{q^{\mathrm{C}} - q^{\mathrm{N}}}{q^{\mathrm{C}}} = \phi_q,$$

indicate that the manufacturer increases the national advertising and quality improving efforts, relative to the non-cooperative scenario, by the same percentage as the participation rate offered. The results are in accord with that obtained in Jørgensen *et al.* [29]. Furthermore, Proposition 3.6 indicates that the firm has higher national advertising, quality improving and local promotion efforts in the centralized supply chain situation.

Proposition 3.7. *For any participation rate $\phi_i \in (0,1)$, $i \in \{a, q\}$, the equilibrium brand goodwill and reference price have the following properties*

$$\text{(i) } G_\infty^{\mathrm{C}} > G_\infty^{\mathrm{N}},$$
$$\text{(ii) } r_\infty^{\mathrm{C}} > r_\infty^{\mathrm{N}}.$$

It is shown from Proposition 3.7 that the equilibrium brand goodwill and reference price in the cooperative scenario are respectively higher than those in the non-cooperative scenario. This is because, in the cooperative scenario, the retailer bears parts of costs of the manufacturer's national advertising and quality improving efforts, which contributes to establishing higher brand goodwill and reference price. The relationships between G_∞^{I} and G_∞^{C}, G_∞^{I} and G_∞^{N}, r_∞^{I} and r_∞^{C}, r_∞^{I} and r_∞^{N} are uncertain.

Proposition 3.8. *For any participation rate $\phi_i \in (0,1)$, $i \in \{a, q\}$, the profits among the three scenarios satisfy*

$$\text{(i) } V_M^{\mathrm{C}}(G_0, r_0) > V_M^{\mathrm{N}}(G_0, r_0), \ \forall G_0 > 0, r_0 > 0,$$
$$\text{(ii) } V_R^{\mathrm{C}}(G_0, r_0) > V_R^{\mathrm{N}}(G_0, r_0), \ \forall G_0 > 0, r_0 > 0,$$
$$\text{(iii) } V_F^{\mathrm{I}}(G_0, r_0) > V^{\mathrm{C}}(G_0, r_0) > V^{\mathrm{N}}(G_0, r_0), \forall G_0 > 0, r_0 > 0,$$

where $V^{\mathrm{C}}(G_0, r_0) = V_M^{\mathrm{C}}(G_0, r_0) + V_R^{\mathrm{C}}(G_0, r_0)$ and $V^{\mathrm{N}}(G_0, r_0) = V_M^{\mathrm{N}}(G_0, r_0) + V_R^{\mathrm{N}}(G_0, r_0)$.

In the cooperative scenario, the retailer undertakes parts of the costs of the manufacturer's national advertising and quality improving efforts, and the manufacturer will increase the national advertising and quality improving efforts, hence the manufacturer and the retailer will have higher demand rate and obtain more profit in the cooperative scenario. It is shown from (iii) of the Proposition 3.8 that the profit of the centralized supply chain is higher than that of the decentralized supply chain, which concludes that the decentralized supply chain construction damages the profit of the supply chain system.

From (3.7), (3.8), (3.13) and (3.14), we have

$$V_R^{\mathrm{C}}(G_0, r_0) - V_R^{\mathrm{N}}(G_0, r_0) - (V_M^{\mathrm{C}}(G_0, r_0) - V_M^{\mathrm{N}}(G_0, r_0)) = \left(\frac{1}{c_a}A^2 + \frac{1}{c_q}Q^2\right) \frac{(2\pi_R - \pi_M)(2\pi_R - 3\pi_M)}{8\rho} \cdot \quad (3.25)$$

It is shown from (3.25) that the improvement of the retailer's profit is less than that of the manufacturer if $\pi_M/2 < \pi_R < 3\pi_M/2$, and the improvement of the retailer's profit is more than that of the manufacturer if $\pi_R > 3\pi_M/2$. That is to say the higher marginal profit of the retailer will stimulate the retailer to take part in the cooperative program.

From the above analysis, we obtain some results as follows. First, the decentralized supply chain construction damages the profits of the supply chain system. Second, the cooperative program always leads to a payoff-Pareto-improving situation. Third, just using the cooperative program cannot coordination the supply chain completely. This raises a question that whether there exists a contract, in which the decentralized supply chain can be coordinated completely and the players can achieve a win-win situation. We will try to solve the problem in the following section.

4. Coordination of the decentralized supply chain

From the comparative results of Section 3.4, we obtain the following results: when the retailer and the manufacturer integrate as a whole firm, the firm's optimal national advertising, quality improving and local promotion efforts are lager than that in the decentralized supply chain. Meanwhile the centralized supply chain system has higher profits than the decentralized supply chain system. Consequently, it is necessary to design a coordination contract which helps the decentralized supply chain have the same performance as an integrated one and the players obtain the win-win situation, even if they make their strategy independently for their own objective function. In this section, we design a revenue sharing and two-subsidy contract to achieve this objective. It should be mentioned that the two-subsidy policy was first presented in Zhang *et al.* [47]. The subscript "D" is used to represent "a revenue sharing and two-subsidy contract situation".

Applying the revenue sharing contract and two-subsidy policy, the retailer and the manufacturer first set four parameters: participant rates ϕ_a^{D}, ϕ_q^{D} and ϕ_b^{D}, $0 \le \phi_i^{\mathrm{D}} \le 1,\ i \in \{a, b, q\}$, and revenue sharing fraction ψ, $0 \le \psi \le 1$. The contract works as follows: the retailer undertakes parts of the costs of the manufacturer's national advertising and quality improving efforts with participant rates ϕ_a^{D} and ϕ_q^{D}. Meanwhile the manufacturer shares part of the cost of the retailer's local promotion with participant rates ϕ_b^{D} and shares ψ of the revenue that the manufacturer generates. Under this mechanism, the objective function of the manufacturer is

$$\max_{a(\cdot),q(\cdot)} J_M = \int_0^\infty \mathrm{e}^{-\rho t}\Bigg[\pi_M(1-\psi)(\mu_1 a(t) + \mu_2 b(t) + \mu_3 G(t) + \mu_4(r(t) - p(t))) \tag{4.1}$$

$$-\frac{c_q}{2}(1-\phi_q^{\mathrm{D}}(t))q^2(t) - \frac{c_a}{2}\left(1-\phi_a^{\mathrm{D}}(t)\right)a^2(t) - \frac{c_b}{2}\phi_b^{\mathrm{D}}(t)b^2(t)\Bigg]\mathrm{d}t \tag{4.2}$$

and that of the retailer is

$$\max_{b(\cdot)} J_R = \int_0^\infty \mathrm{e}^{-\rho t}\Bigg[(\pi_R + \psi\pi_M)(\mu_1 a(t) + \mu_2 b(t) + \mu_3 G(t) + \mu_4(r(t) - p(t))) \tag{4.3}$$

$$-\frac{c_q}{2}\phi_q^{\mathrm{D}}(t)q^2(t) - \frac{c_a}{2}\phi_a^{\mathrm{D}}(t)a^2(t) - \frac{c_b}{2}\left(1-\phi_b^{\mathrm{D}}(t)\right)b^2(t)\Bigg]\mathrm{d}t. \tag{4.4}$$

When the parameters ϕ_a^{D}, ϕ_q^{D}, ϕ_b^{D} and ψ are fixed, the equilibrium national advertising and quality improving efforts of the manufacturer and the local promotion effort of the retailer, and the corresponding value functions are given by the following proposition.

Proposition 4.1. *When the players adopt the a revenue sharing and two-subsidy contract, and the parameters ϕ_a^{D}, ϕ_q^{D}, ϕ_b^{D} and ψ are fixed, the equilibrium advertising and quality improving strategies of the manufacturer are*

$$a^{\mathrm{D}} = \frac{\pi_M(1-\psi)}{c_a(1-\phi_a^{\mathrm{D}})}A, \tag{4.5}$$

$$q^{\mathrm{D}} = \frac{\pi_M(1-\psi)}{c_q(1-\phi_q^{\mathrm{D}})}Q, \tag{4.6}$$

and the equilibrium local promotion strategy of the retailer is

$$b^{\mathrm{D}} = \begin{cases} \dfrac{(\pi_R + \psi\pi_M)}{c_b(1-\phi_b^{\mathrm{D}})}B, & \mu_2 > \dfrac{\theta_3\mu_3}{\rho+\delta} + \dfrac{\nu_3\mu_4}{\rho+\alpha} \\ 0, & \textit{otherwise.} \end{cases} \tag{4.7}$$

The manufacturer's and retailer's value functions, respectively, are

$$V_M^{\mathrm{D}}\left(G^{\mathrm{D}}, r^{\mathrm{D}}\right) = \frac{(1-\psi)\pi_M\mu_3}{\rho+\delta}G^{\mathrm{D}} + \frac{(1-\psi)\pi_M\mu_4}{\rho+\alpha}r^{\mathrm{D}} + w, \tag{4.8}$$

$$V_R^{\mathrm{D}}\left(G^{\mathrm{D}}, r^{\mathrm{D}}\right) = \frac{(\pi_R+\psi\pi_M)\mu_3}{\rho+\delta}G^{\mathrm{D}} + \frac{(\pi_R+\psi\pi_M)\mu_4}{\rho+\alpha}r^{\mathrm{D}} + u, \tag{4.9}$$

where

$$\begin{aligned} w &= \frac{(\pi_R+\psi\pi_M)B^2}{c_b\rho(1-\phi_b^D)}\left(\pi_M(1-\psi) - \frac{\phi_b^D(\pi_R+\psi\pi_M)}{2(1-\phi_b^D)}\right) - \frac{(1-\psi)\pi_M\mu_4 p}{\rho+\alpha} \\ &\quad + \frac{\pi_M^2(1-\psi)^2}{2c_a\rho(1-\phi_a^D)}A^2 + \frac{\pi_M^2(1-\psi)^2}{2c_q\rho(1-\phi_q^D)}Q^2 \\ u &= \frac{(1-\psi)\pi_M A^2}{\rho c_a(1-\phi_a^D)}\left(\pi_R+\psi\pi_M - \frac{\phi_a^D(1-\psi)\pi_M}{2(1-\phi_a^D)}\right) - \frac{(\pi_R+\psi\pi_M)\mu_4 p}{\rho+\alpha} \\ &\quad + \frac{(1-\psi)\pi_M Q^2}{\rho c_q(1-\phi_q^D)}\left(\pi_R+\psi\pi_M - \frac{\phi_q^D(1-\psi)\pi_M}{2(1-\phi_q^D)}\right) + \frac{(\pi_R+\psi\pi_M)^2}{2\rho c_b(1-\phi_b^D)}B^2. \end{aligned}$$

It is obvious that only if $V_M^{\mathrm{D}} \geq V_M^{\mathrm{C}}$, $V_R^{\mathrm{D}} \geq V_R^{\mathrm{C}}$, $a^{\mathrm{D}} = a^{\mathrm{I}}$, $q^{\mathrm{D}} = q^{\mathrm{I}}$ and $b^{\mathrm{D}} = b^{\mathrm{I}}$, the decentralized supply chain can be coordinated and the contract can be accepted by the players. Consequently, we can get the following proposition.

Proposition 4.2. *If $c_b/B^2\left(A^2/c_a + Q^2/c_q\right) \geq 1$, when the participation rates ϕ_a^{D}, ϕ_q^{D} and ϕ_b^D, and the profit sharing fraction ψ satisfy*

$$\phi_a^{\mathrm{D}} = \phi_q^{\mathrm{D}} = \frac{\psi\pi_M+\pi_R}{\pi_M+\pi_R}, \tag{4.10}$$

$$\phi_b^{\mathrm{D}} = \frac{(1-\psi)\pi_M}{\pi_M+\pi_R}, \tag{4.11}$$

$$\psi_{\min} \leq \psi \leq \psi_{\max}, \tag{4.12}$$

where

$$\begin{aligned} \psi_{\min} &= \max\left\{0, \frac{L_1}{L_3}\right\}, \ \psi_{\max} = \frac{L_2}{L_3}, \\ L_1 &= \frac{\pi_R(\pi_M-\pi_R)}{2\rho}\left(\frac{A^2}{c_a}+\frac{Q^2}{c_q}\right) - \frac{\pi_M\pi_R}{2c_b\rho}B^2, \\ L_2 &= \frac{\pi_M\pi_R}{2\rho}\left(\frac{A^2}{c_a}+\frac{Q^2}{c_q}\right) + \frac{\pi_M(\pi_M-\pi_R)}{2c_b\rho}B^2, \\ L_3 &= V_M^{\mathrm{N}} + \frac{\pi_M\pi_R}{2\rho}\left(\frac{A^2}{c_a}+\frac{Q^2}{c_q}\right) + \frac{\pi_M(\pi_M-\pi_R)}{2c_b\rho}B^2, \end{aligned}$$

the decentralized supply chain system can be coordinated completely.

If $c_b/B^2\left(A^2/c_a + Q^2/c_q\right) < 1$, when $\pi_M/\pi_R \geq 1 - c_b/B^2\left(A^2/c_a + Q^2/c_q\right)$, and the participation rates ϕ_a^{D}, ϕ_q^{D} and ϕ_b^D, and the profit sharing fraction ψ satisfy (4.10), (4.11) and (4.12), the decentralized supply chain system can be coordinated completely. When $\pi_M/\pi_R < 1 - c_b/B^2\left(A^2/c_a + Q^2/c_q\right)$, the decentralized supply chain system cannot be coordinated completely.

It is shown from Proposition 4.2 that if $c_b/B^2\left(A^2/c_a+Q^2/c_q\right) \geq 1$, the decentralized supply chain system can be coordinated completely by using the revenue sharing contract and two-subsidy policy. If $c_b/B^2\left(A^2/c_a+Q^2/c_q\right) < 1$, whether the decentralized supply chain system can be coordinated completely or not, depends on the relationship between π_M/π_R and $1-c_b/B^2\left(A^2/c_a+Q^2/c_q\right)$. If the manufacturer's marginal profit π_M is higher than $\pi_R\left(1-c_b/B^2\ \left(A^2/c_a+Q^2/c_q\right)\right)$, the players will accept this mechanism and the decentralized supply chain can be coordinated completely. From the above analysis, we conclude that if the players' marginal profit ratio is higher than a threshold, the players can achieve the win-win situation. If the manufacturer's marginal profit π_M is lower than $\pi_R\left(1-c_b/B^2\left(A^2/c_a+Q^2/c_q\right)\right)$, the retailer obtains a lower profit than that in the decentralized supply chain system, thus the retailer will not accept this mechanism. Consequently, the decentralized supply chain cannot be coordinated completely and the players cannot achieve the win-win situation. In this condition, we can use a lump sum transfer contract to allocate additional profits resulting from the two-subsidy policy. The lump sum fee is the benefit player pays to the loss player to ensure that the loss player is willing to sign the two-subsidy contract. The range of lump sum fee is $P\in[P_1,P_2]$, where $P_1 = \min\left\{|V_M^{\mathrm{D}}-V_M^{\mathrm{N}}|,|V_R^{\mathrm{D}}-V_R^{\mathrm{N}}|\right\}$ and $P_2 = \max\left\{|V_M^{\mathrm{D}}-V_M^{\mathrm{N}}|,|V_R^{\mathrm{D}}-V_R^{\mathrm{N}}|\right\}$, and V_M^{N}, V_R^{N}, V_M^{D} and V_R^{D} are presented in (3.7), (3.8), (4.8) and (4.9). The value of the lump fee P is determined by the players' negotiation ability. In this paper, we will not discuss the value of P in detail. Because of this, the decentralized supply chain can be coordinated completely and the players can achieve the win-win situation with combining the two-subsidy policy and the a lump sum transfer contract.

From (4.10) and (4.11), we can find that ϕ_a^{D} and ϕ_q^{D} increase in ψ, and ϕ_b^{D} decreases in ψ. Namely, when the manufacturer increases the revenue sharing fraction ψ, she should decrease the participation rate ϕ_b^{D}, whereas the retailer increases the participation rates ϕ_a^{D} and ϕ_q^{D}. Comparing ϕ_a^{D} and ϕ_q^{D} expressed in (4.10) with ϕ_a and ϕ_q presented in (3.12), we get $\phi_a^{\mathrm{D}}=\phi_q^{\mathrm{D}}>\phi_a=\phi_q$, that is, the retailer will increases the participation rates in the revenue sharing and two-subsidy contract condition. Furthermore, we find that $\phi_a^{\mathrm{D}}-\phi_a$ and $\phi_q^{\mathrm{D}}-\phi_q$ increase in ψ. Additionally, we always have $\phi_b^{\mathrm{D}}>0$ unless $\psi=1$. The goodwill and reference price are the same as those in the centralized supply chain, due to the same national adverting, quality improving and local promotion efforts.

Remark 4.3. In the finite sales period model, we also can obtain the corresponding equilibrium (optimal) strategies and coordination results. Our results of the finite sales period model coincide with those of the infinite sales period model, where the cooperative program can reach the payoff-Pareto-improving situation but cannot coordinate the supply chain completely, and a revenue sharing contract combined with two-subsidy policy can coordinate the decentralized supply chain completely.

5. Numerical analysis

In this section, we present numerical analysis to illustrate the obtained equilibrium (optimal) strategies under different decision scenarios and the coordination results. Through the numerical analysis, we obtain some managerial insights.

5.1. Numerical example

Consider the following system parameters: $\theta_1=0.5$, $\theta_2=0.5$, $\theta_3=0.5$, $\nu_1=0.3$, $\nu_2=0.3$, $\nu_3=0.3$, $\mu_1=1$, $\mu_2=1$, $\mu_3=0.5$, $\mu_4=0.5$, $\alpha=0.3$, $\delta=0.3$, $p=15$, $r_0=13$, $G_0=5$, $\rho=0.1$, $\pi_M=5$, $\pi_R=4$, $c_a=1$, $c_q=1$, $c_b=0.5$.

According to Proposition 3.1, we obtain the equilibrium strategies in the non-cooperative scenario as $a^N=10$, $q^N=5$ and $b^N=2$, and the corresponding profits $V_M^N=668.75$ and $V_R^N=1025$.

Basing on Proposition 3.3, we get the equilibrium strategies in the cooperative scenario as $a^N=13$, $q^N=6.5$ and $b^N=2$, and the retailer's optimal participation rates $\phi_a=\phi_q=0.2308$, and the corresponding profits $V_M^C=853.25$ and $V_R^C=1081.3$.

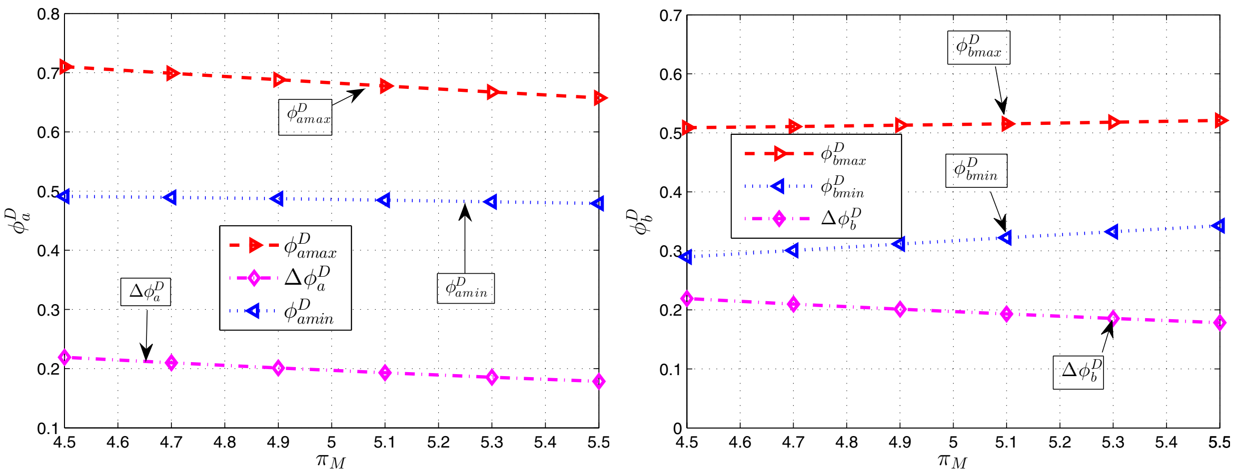

FIGURE 1. The participation rates $\phi_a^D(\phi_q^D)$ and ϕ_b^D with different π_M.

According to Proposition 3.5, we generate the integrated firm's optimal strategies in the integrated supply chain scenario as $a^I = 18$, $q^I = 9$ and $b^I = 4$, and the corresponding profits of integrate supply chain $V_F^I = 2109.4$.

Comparing the equilibrium strategies obtained in the non-cooperative and cooperative scenarios, we find that the advertising level and improving quality level in the cooperative scenario are higher than their counterparts in the non-cooperative scenario due to the retailer's cost subsidy. With the higher advertising level and improving quality level, the manufacturer and retailer will obtain higher reference price and goodwill, which ultimately result in higher consumer demand and profits. Moreover, we find $V_M^C > V_M^N$, $V_R^C > V_R^N$ and $V_F^I > V_M^C + V_R^C$, which mean that the cooperative program can reach the payoff-Pareto-improving situation but cannot coordinate the supply chain completely.

According to Proposition 4.2, we obtain that the supply chain can be coordinated completely for $\psi \in [0.0747, 0.4293]$, $\phi_a(\phi_q) \in [0.4859, 0.6830]$ and $\phi_b \in [0.3170, 0.5141]$. Under such situations, the profit range of manufacturer is $[668.75, 1084.4]$ and that of retailer is $[1025, 1440.65]$.

5.2. Sensitivity analysis on coordinating results

The sensitivity analysis on coordination results with respect to the system parameters π_M, π_R, α and δ are presented in Figures 1–6. Among them, in Figures 2 and 4–6, the revenue sharing fraction plane is divided into three regions by two curves $\psi_{\max}$ and $\psi_{\min}$, which is respectively generated by $V_M^D = V_M^N$ and $V_R^D = V_R^N$ respectively. Also, $\Delta\psi$ denotes $\psi_{\max} - \psi_{\min}$ representing the biggest variation range for the given system parameter. Consequently, the decentralized supply chain can be coordinated in the region, where below the curve $\psi_{\max}$ ensures $V_M^D > V_M^N$ and above the curve $\psi_{\min}$ ensures $V_R^D > V_R^N$. According to the given π_M, π_R and ψ, we can obtain the variation range of $\phi_a^D(\phi_q^D)$ and ϕ_b^D, which are depicted in Figures 1 and 3; $\Delta\phi_a$ and $\Delta\phi_b$ representing $\phi_{a\max}^D - \phi_{a\min}^D$ and $\phi_{b\max}^D - \phi_{b\min}^D$ respectively. Note that when α or δ changes, ϕ_a and ϕ_q is positively related with ψ respectively, whereas ϕ_b is negatively related with ψ. Consequently, when α or δ changes, we omit the corresponding analysis about $\phi_a(\phi_q)$ and ϕ_b.

(1) Sensitivity analysis of the marginal profit of manufacturer π_M.
Notice from Figure 1 that with the increase of π_M, the curves $\phi_{a\max}^D$ and $\phi_{a\min}^D$ decrease. Because the curve $\phi_{a\max}^D$ decreases faster than the curve $\phi_{a\min}^D$, the curve $\Delta\phi_a^D$ also decreases in π_M. At the same time, we find that the curves $\phi_{b\max}^D$ and $\phi_{b\min}^D$ increase when π_M increases. Moreover, the curve $\phi_{b\min}^D$ increases

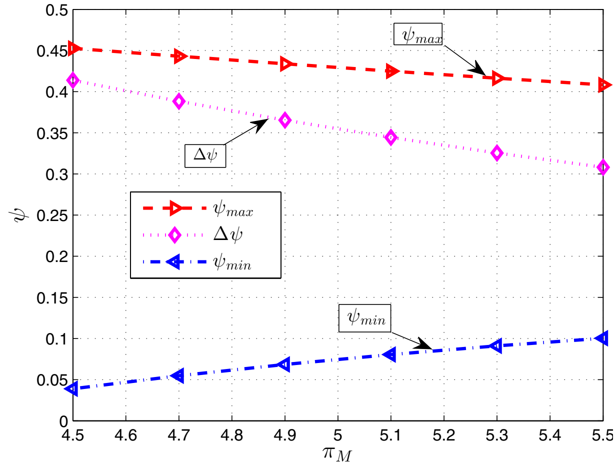

FIGURE 2. Coordinating scenarios with different π_M.

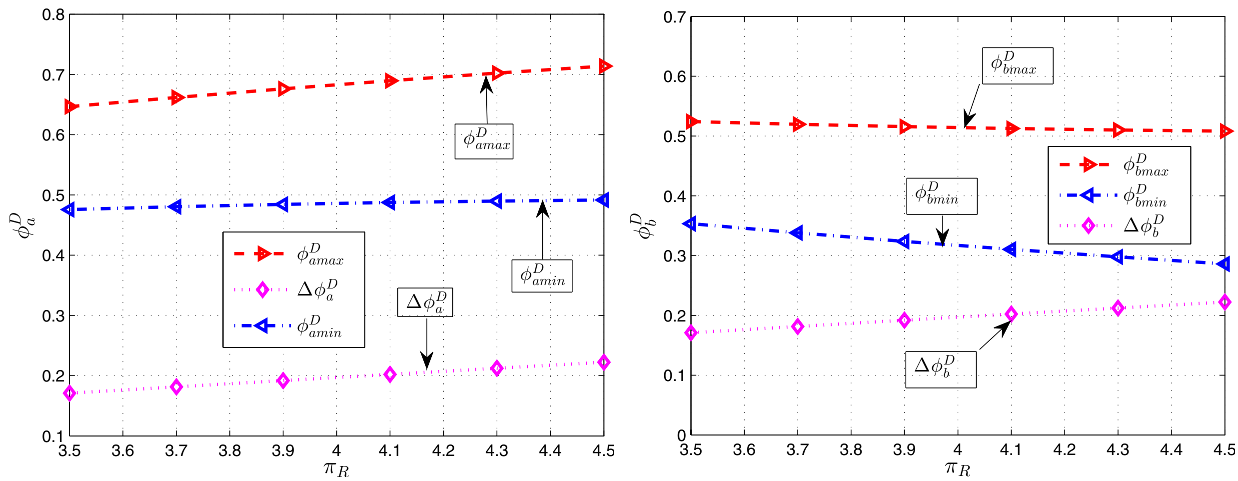

FIGURE 3. The participation rates $\phi^D_a(\phi^D_q)$ and ϕ^D_b with different π_R.

faster than the curve $\phi^D_{b\max}$. Consequently, the curve $\Delta\phi^D_b$ decreases in π_M. Those results indicate that, with the increase of π_M the manufacturer can gain more profits, the retailer will decrease the subsidy for the manufacturer's national advertising and improving quality costs, whereas the manufacturer will increase the subsidy for the retailer's local promotion cost. It is shown from Figure 2 that the curve $\psi_{\max}$ decreases in π_M, whereas $\psi_{\min}$ increases in π_M, which ultimately results in the decreasing of $\Delta\psi$. This result shows that a higher marginal profit of manufacturer gives both channel members smaller room to negotiate to obtain the supply chain coordination.

(2) Sensitivity analysis of the marginal profit of retailer π_R.
It is interesting to find from Figure 3 that with the increase of π_R, the curves $\phi^D_{a\max}$ and $\phi^D_{a\min}$ increase, whereas the curves $\phi^D_{b\max}$ and $\phi^D_{b\min}$ decrease. Because the curve $\phi^D_{a\max}$ increases faster than the curve $\phi^D_{a\min}$, and the curve $\phi^D_{b\min}$ decreases faster than the curve $\phi^D_{b\max}$, the curves $\Delta\phi_a$ and $\Delta\phi_b$ increase in π_R. The results indicate that when the retailer's marginal profit increases, the manufacturer will obtain more subsidy from the retailer and give less subsidy for the retailer's local promotion costs. Consequently, the

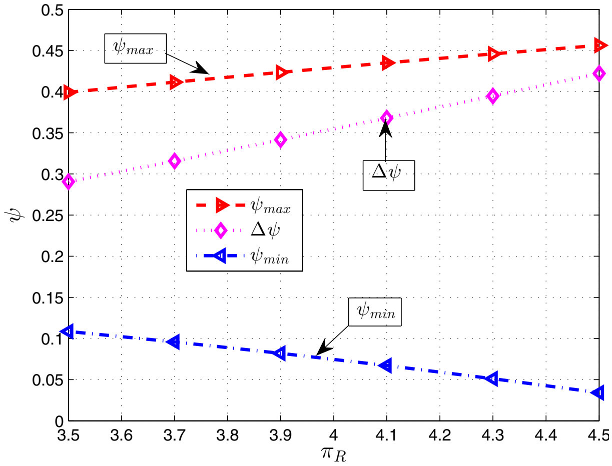

FIGURE 4. Coordinating scenarios with different π_R.

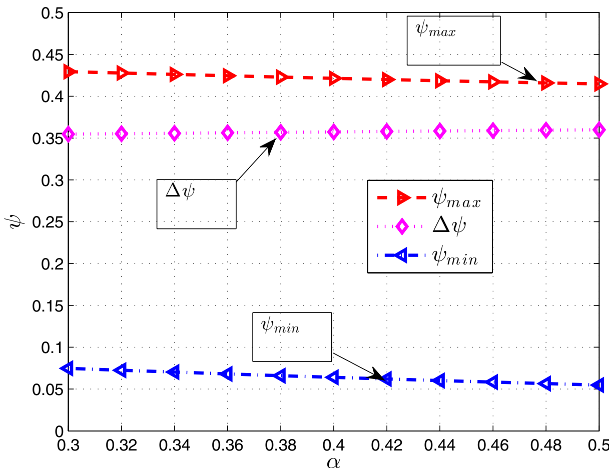

FIGURE 5. Coordinating scenarios with different α.

manufacturer can accept a higher revenue sharing fraction and the retailer a lower revenue sharing fraction. This reveals that a higher retailer's margin profit will endow the retailer a greater degree of flexibility to coordinate the whole supply chain.

(3) Sensitivity analysis of the memory parameter α and depreciation coefficient δ.
It is shown from Figures 5 and 6 that when α and δ increase, the curves $\psi_{\max}$ and $\psi_{\min}$ decrease. Since the curve $\psi_{\min}$ decreases faster than the curve $\psi_{\max}$, the curve $\Delta\psi$ increases. Higher values of α and δ reduce the manufacturer's and retailer's investments in the national advertising, improving quality and local promotion, and then decrease the effect of the two-subsidy policy on the supply chain coordination. This reveals that the channel members will obtain a larger room to coordinate the whole supply chain and reach the win-win situation for large α and δ.

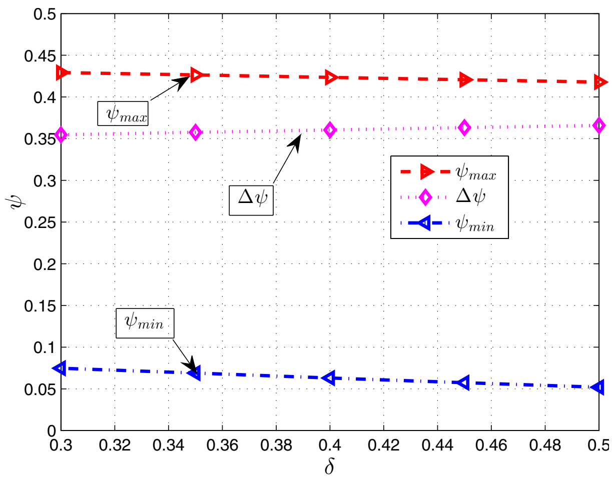

FIGURE 6. Coordinating scenarios with different δ.

6. Conclusions

A distribution channel with a manufacturer and a retailer is considered, where the manufacturer spends on national advertising and quality improving, while the retailer undertakes the expenditure of local promotion. Since reference price occupies a significant position in consumer making decisions whether to buy a product or not, we take the reference price into consideration. In our model, the brand goodwill and reference price would be positively affected by the manufacturer's national advertising and quality improving efforts, yet negatively affected by the retailer's local promotion. Three scenarios including the non-cooperative and cooperative scenarios of the decentralized supply chain, and the centralized supply chain scenario, are analyzed and the corresponding equilibrium strategies and profits are obtained. The equilibrium strategies, steady-state variables and profits are compared. From the comparisons, we obtain some results. Firstly, the optimal strategies and profits of the centralized supply chain are higher than that in the decentralized supply chain. Meanwhile, in the decentralized supply chain if the retailer adopts a cooperative program, the manufacturer has a higher national advertising and quality improving efforts. Secondly, in the decentralized supply chain system, the cooperative program can achieve a profit-Pareto-improving condition, but cannot coordinate the supply chain completely. Furthermore, a revenue sharing contract combined with two-subsidy policy can coordinate the decentralized supply chain. The effectiveness of the presented contract is verified through numerical example and sensitivity analysis of the coordinating results on the key system parameters, and some managerial insights are provided.

Extensions of our model for future research involve the following. Firstly, this paper revolves a channel of distribution with a manufacturer and a retailer in a dynamic model where there is no competing manufacturers or retailers. A research on multiple manufacturer or multiple retailer situations where adding competitive factors such as sales price, product quality, brand goodwill and so on, will make our model more practical. Besides, the marginal profits of the manufacturer and retailer are assumed to be constants, which means that the manufacturer and retailer are unable to adjust the wholesale price and retailer price, additionally the quality improving effort has no effect on the cost of production. As presented in Giovanni [15] that quality improving effort would increase the cost of production, our model can be extended by considering that the quality improving effort increases the cost of production, and the manufacturer and retailer can adjust the wholesale price and retailer price.

Appendices

Proof of Proposition 3.1

Proof. Equilibrium strategies are obtained by solving the following Hamilton–Jacobi–Bellman (HJB) equations

$$\rho V_M^{\mathrm{N}}\left(G^{\mathrm{N}}, r^{\mathrm{N}}\right) = \max_{a,q}\left\{\pi_M\left(\mu_1 a + \mu_2 b + \mu_3 G^{\mathrm{N}} + \mu_4\left(r^{\mathrm{N}} - p\right)\right) - \frac{1}{2}c_q q^2 - \frac{1}{2}c_a a^2 + \frac{\partial V_M^{\mathrm{N}}}{\partial G^{\mathrm{N}}}\left(\theta_1 a + \theta_2 q - \theta_3 b - \delta G^{\mathrm{N}}\right)\right.$$
$$\left. + \frac{\partial V_M^{\mathrm{N}}}{\partial r^{\mathrm{N}}}\left(\alpha\left(p - r^{\mathrm{N}}\right) + \nu_1 a + \nu_2 q - \nu_3 b\right)\right\}, \tag{A.1}$$
$$\rho V_R^{\mathrm{N}}\left(G^{\mathrm{N}}, r^{\mathrm{N}}\right) = \max_{b}\left\{\pi_R\left(\mu_1 a + \mu_2 b + \mu_3 G^{\mathrm{N}} + \mu_4\left(r^{\mathrm{N}} - p\right)\right) - \frac{1}{2}c_b b^2 + \frac{\partial V_R^{\mathrm{N}}}{\partial G^{\mathrm{N}}}\left(\theta_1 a + \theta_2 q - \theta_3 b - \delta G^{\mathrm{N}}\right)\right.$$
$$\left. + \frac{\partial V_R^{\mathrm{N}}}{\partial r^{\mathrm{N}}}\left(\alpha\left(p - r^{\mathrm{N}}\right) + \nu_1 a + \nu_2 q - \nu_3 b\right)\right\}. \tag{A.2}$$

For notational convenience, let

$$K_1 = \theta_1\frac{\partial V_M^{\mathrm{N}}}{\partial G^{\mathrm{N}}} + \nu_1\frac{\partial V_M^{\mathrm{N}}}{\partial r^{\mathrm{N}}},\ K_2 = \theta_2\frac{\partial V_M^{\mathrm{N}}}{\partial G^{\mathrm{N}}} + \nu_2\frac{\partial V_M^{\mathrm{N}}}{\partial r^{\mathrm{N}}},\ K_3 = \theta_3\frac{\partial V_M^{\mathrm{N}}}{\partial G^{\mathrm{N}}} + \nu_3\frac{\partial V_M^{\mathrm{N}}}{\partial r^{\mathrm{N}}},$$
$$K_4 = \theta_1\frac{\partial V_R^{\mathrm{N}}}{\partial G^{\mathrm{C}}} + \nu_1\frac{\partial V_R^{\mathrm{N}}}{\partial r^{\mathrm{N}}},\ K_5 = \theta_2\frac{\partial V_R^{\mathrm{N}}}{\partial G^{\mathrm{N}}} + \nu_2\frac{\partial V_R^{\mathrm{N}}}{\partial r^{\mathrm{N}}},\ \ K_6 = \theta_3\frac{\partial V_R^{\mathrm{N}}}{\partial G^{\mathrm{N}}} + \nu_3\frac{\partial V_R^{\mathrm{N}}}{\partial r^{\mathrm{S}}}.$$

The first-order conditions for the maximization of the right-hand sides of (A.1) and (A.2) provide

$$a^{\mathrm{N}} = \frac{1}{c_a}\left(\pi_M\mu_1 + K_1\right), \tag{A.3}$$
$$q^{\mathrm{N}} = \frac{K_2}{c_q}, \tag{A.4}$$
$$b^{\mathrm{N}} = \frac{1}{c_b}\left(\pi_R\mu_2 - K_3\right). \tag{A.5}$$

Substituting (A.3), (A.4) and (A.5) into (A.1) and (A.3), respectively, yields

$$\rho V_M^{\mathrm{N}}(G^{\mathrm{N}}, r^{\mathrm{N}}) = \left(\pi_M\mu_3 - \delta\frac{\partial V_M^{\mathrm{N}}}{\partial G^{\mathrm{N}}}\right)G^{\mathrm{N}} + \left(\pi_M\mu_4 - \alpha\frac{\partial V_M^{\mathrm{N}}}{\partial r^{\mathrm{N}}}\right)\left(r^{\mathrm{N}} - p\right)$$
$$+ \frac{1}{2c_a}\left(\pi_M\mu_1 + K_1\right)^2 + \frac{1}{2c_q}K_2^2 + \frac{1}{c_b}\left(\pi_M\mu_2 - K_3\right)\left(\pi_R\mu_2 - K_6\right), \tag{A.6}$$
$$\rho V_R^{\mathrm{N}}(G^{\mathrm{N}}, r^{\mathrm{N}}) = \left(\pi_R\mu_3 - \delta\frac{\partial V_R^{\mathrm{N}}}{\partial G^{\mathrm{N}}}\right)G^{\mathrm{N}} + \left(\pi_R\mu_4 - \alpha\frac{\partial V_R^{\mathrm{N}}}{\partial r^{\mathrm{N}}}\right)\left(r^{\mathrm{N}} - p\right)$$
$$+ \frac{1}{c_a}\left(\pi_M\mu_1 + K_1\right)\left(\pi_R\mu_1 + K_4\right) + \frac{K_2K_5}{c_q} + \frac{1}{2c_b}\left(\pi_R\mu_2 - K_6\right)^2. \tag{A.7}$$

We shall show that linear value functions satisfy (A.6) and (A.7). Consequently, we define

$$V_M^{\mathrm{N}}\left(G^{\mathrm{N}}, r^{\mathrm{N}}\right) = l_1 G^{\mathrm{N}} + l_2 r^{\mathrm{N}} + l, \tag{A.8}$$
$$V_R^{\mathrm{N}}\left(G^{\mathrm{N}}, r^{\mathrm{N}}\right) = m_1 G^{\mathrm{N}} + m_2 r^{\mathrm{N}} + m, \tag{A.9}$$

where l_1, l_2, l, m_1, m_2 and m are constant parameters to be identified.

Substituting (A.8), (A.9) and the derivatives of the value functions into (A.6) and (A.7), respectively, yields

$$\begin{aligned}\rho l_1 G^{\mathrm{N}} + \rho l_2 r^{\mathrm{N}} + \rho l = {} & \left(\pi_M \mu_3 - \delta l_1\right) G^{\mathrm{N}} + \left(\pi_M \mu_4 - \alpha l_2\right) r^{\mathrm{N}} \\ & + \frac{1}{2c_q}(\theta_2 l_1 + \nu_2 l_2)^2 + \frac{1}{2c_a}\left(\pi_M \mu_1 + \theta_1 l_1 + \nu_1 l_2\right)^2 \\ & + \frac{1}{c_b}(\pi_M \mu_2 - \theta_3 l_1 - \nu_3 l_2)(\pi_R \mu_2 - \theta_3 m_1 - \nu_3 m_2), \\ & + (\alpha l_2 - \pi_M \mu_4) p \end{aligned} \tag{A.10}$$

$$\begin{aligned}\rho m_1 G^{\mathrm{N}} + \rho m_2 r^{\mathrm{N}} + \rho m = {} & \left(\pi_R \mu_3 - \delta m_1\right) G^{\mathrm{N}} + \left(\pi_R \mu_4 - \alpha m_2\right) r^{\mathrm{N}} \\ & + \frac{1}{c_a}\left(\pi_R \mu_1 + \theta_1 m_1 + \nu_1 m_2\right)\left(\pi_M \mu_1 + \theta_1 l_1 + \nu_1 l_2\right) \\ & + \frac{1}{2c_b}\left(\pi_R \mu_2 - \theta_3 m_1 - \nu_3 m_2\right)^2 + (\alpha m_2 - \pi_R \mu_4) p \\ & + \frac{1}{c_q}(\theta_2 m_1 + \nu_2 m_2)(\theta_2 l_1 + \nu_2 l_2). \end{aligned} \tag{A.11}$$

It can be verified that the following constant parameters satisfy (A.10) and (A.11)

$$\begin{aligned} l_1 &= \frac{\pi_M \mu_3}{\rho + \delta},\ l_2 = \frac{\pi_M \mu_4}{\rho + \alpha}, \\ l &= \frac{\pi_M^2}{2c_a \rho} A^2 + \frac{\pi_M^2}{2c_q \rho} Q^2 + \frac{\pi_M \pi_R}{c_b \rho} B^2 - \frac{\pi_M \mu_4 p}{\rho + \alpha}, \\ m_1 &= \frac{\pi_R \mu_3}{\rho + \delta}, m_2 = \frac{\pi_R \mu_4}{\rho + \alpha}, \\ m &= \frac{\pi_M \pi_R}{c_a \rho} A^2 + \frac{\pi_M \pi_R}{c_q \rho} Q^2 + \frac{\pi_R^2}{2c_b \rho} B^2 - \frac{\pi_R \mu_4 p}{\rho + \alpha}. \end{aligned}$$

□

Proof of Proposition 3.3

Proof. To obtain Stackelberg equilibrium strategies, the manufacturer's national advertising effort a and quality improving effort q are viewed as a function of the retailer's promotion effort b, ϕ_a and ϕ_q, respectively. The HJB equations of the manufacturer and retailer respectively are

$$\begin{aligned}\rho V_M^{\mathrm{C}}\left(G^{\mathrm{C}}, r^{\mathrm{C}}\right) = \max_{a,q} \Bigg\{ & \pi_M\left(\mu_1 a + \mu_2 b + \mu_3 G^{\mathrm{C}} + \mu_4\left(r^{\mathrm{S}} - p\right)\right) - \frac{1}{2} c_q (1 - \phi_q) q^2 \\ & - \frac{1}{2} c_a (1 - \phi_a) a^2 + \frac{\partial V_M^{\mathrm{C}}}{\partial G^{\mathrm{C}}}\left(\theta_1 a + \theta_2 q - \theta_3 b - \delta G^{\mathrm{C}}\right) \\ & + \frac{\partial V_M}{\partial r^{\mathrm{C}}}\left(\alpha\left(p - r^{\mathrm{C}}\right) + \nu_1 a + \nu_2 q - \nu_3 b\right) \Bigg\}, \end{aligned} \tag{A.12}$$

$$\begin{aligned}\rho V_R^{\mathrm{C}}\left(G^{\mathrm{C}}, r^{\mathrm{C}}\right) = \max_{b, \phi_a, \phi_q} \Bigg\{ & \pi_R\left(\mu_1 a + \mu_2 b + \mu_3 G^{\mathrm{C}} + \mu_4\left(r^{\mathrm{C}} - p\right)\right) - \frac{1}{2} c_q \phi_q q^2 \\ & - \frac{1}{2} c_a \phi_a a^2 + \frac{\partial V_R^{\mathrm{C}}}{\partial G^{\mathrm{C}}}\left(\theta_1 a + \theta_2 q - \theta_3 b - \delta G^{\mathrm{C}}\right) \\ & - \frac{1}{2} c_b b^2 + \frac{\partial V_R^{\mathrm{C}}}{\partial r^{\mathrm{C}}}\left(\alpha\left(p - r^{\mathrm{C}}\right) + \nu_1 a + \nu_2 q - \nu_3 b\right) \Bigg\}. \end{aligned} \tag{A.13}$$

For notational convenience, let

$$H_1=\theta_1\frac{\partial V_M^{\mathrm{C}}}{\partial G^{\mathrm{S}}}+\nu_1\frac{\partial V_M^{\mathrm{C}}}{\partial r^{\mathrm{C}}},\ H_2=\theta_2\frac{\partial V_M^{\mathrm{C}}}{\partial G^{\mathrm{C}}}+\nu_2\frac{\partial V_M^{\mathrm{C}}}{\partial r^{\mathrm{C}}},\ H_3=\theta_3\frac{\partial V_M^{\mathrm{C}}}{\partial G^{\mathrm{C}}}+\nu_3\frac{\partial V_M^{\mathrm{C}}}{\partial r^{\mathrm{C}}},$$
$$H_4=\theta_1\frac{\partial V_R^{\mathrm{C}}}{\partial G^{\mathrm{C}}}+\nu_1\frac{\partial V_R^{\mathrm{C}}}{\partial r^{\mathrm{C}}},\ H_5=\theta_2\frac{\partial V_R^{\mathrm{C}}}{\partial G^{\mathrm{C}}}+\nu_2\frac{\partial V_R^{\mathrm{C}}}{\partial r^{\mathrm{C}}},\quad H_6=\theta_3\frac{\partial V_R^{\mathrm{C}}}{\partial G^{\mathrm{C}}}+\nu_3\frac{\partial V_R^{\mathrm{C}}}{\partial r^{\mathrm{S}}}\cdot$$

The first-order conditions for the maximization of the right-hand side of (A.12) provide the national advertising and quality improving strategies

$$a^{\mathrm{C}}=\frac{\pi_M\mu_1+H_1}{c_a(1-\phi_a)},\tag{A.14}$$

$$q^{\mathrm{C}}=\frac{H_2}{c_q(1-\phi_q)}\cdot\tag{A.15}$$

Substituting (A.14) and (A.15) into (A.13) yields

$$\begin{aligned}\rho V_R^{\mathrm{C}}\left(G^{\mathrm{C}},r^{\mathrm{C}}\right)=\max_{b,\phi_a,\phi_q}\Bigg\{&\left(\pi_R\mu_3-\delta\frac{\partial V_R^{\mathrm{C}}}{\partial G^{\mathrm{S}}}\right)G^{\mathrm{C}}+\left(\pi_R\mu_4-\alpha\frac{\partial V_R^{\mathrm{C}}}{\partial r^{\mathrm{S}}}\right)\left(r^{\mathrm{S}}-p\right)\\&-\frac{\phi_a(\pi_M\mu_1+H_1)^2}{2c_a(1-\phi_a)^2}+\frac{(\pi_R\mu_1+H_4)\left(\pi_M\mu_1+H_1\right)}{c_a(1-\phi_a)}\\&+\frac{H_5H_2}{c_q(1-\phi_q)}-\frac{\phi_qH_2^2}{2c_q(1-\phi_q)^2}+(\pi_R\mu_2-H_6)b-\frac{c_b}{2}b^2\Bigg\}.\end{aligned}\tag{A.16}$$

Performing the maximization of the right-hand side of (A.16), we obtain

$$b^{\mathrm{C}}=\frac{(\pi_R\mu_2-H_6)}{c_b},\tag{A.17}$$
$$\phi_a=\frac{2\left(\pi_R\mu_1+H_4\right)-\left(\pi_M\mu_1+H_1\right)}{2\left(\pi_R\mu_1+H_4\right)+\left(\pi_M\mu_1+H_1\right)},\tag{A.18}$$
$$\phi_q=\frac{2H_5-H_2}{2H_5+H_2}\cdot\tag{A.19}$$

Substituting (A.14), (A.15), (A.17), (A.18) and (A.19) into (A.12) and (A.16), respectively, yields

$$\begin{aligned}\rho V_M^{\mathrm{C}}\left(G^{\mathrm{C}},r^{\mathrm{C}}\right)=&\left(\pi_M\mu_3-\delta\frac{\partial V_M^{\mathrm{C}}}{\partial G^{\mathrm{C}}}\right)G^{\mathrm{C}}+\left(\pi_M\mu_4-\alpha\frac{\partial V_M^{\mathrm{C}}}{\partial r^{\mathrm{C}}}\right)\left(r^{\mathrm{C}}-p\right)\\&+\frac{1}{4c_q}H_2(2H_5+H_2)+\frac{1}{c_b}\left(\pi_M\mu_2-H_3\right)\left(\pi_R\mu_2-H_6\right)\\&+\frac{1}{4c_a}\left(\pi_M\mu_1+H_1\right)\left(2\left(\pi_R\mu_1+H_4\right)+\left(\pi_M\mu_1+H_1\right)\right),\end{aligned}\tag{A.20}$$

$$\begin{aligned}\rho V_R^{\mathrm{C}}\left(G^{\mathrm{C}},r^{\mathrm{C}}\right)=&\left(\pi_R\mu_3-\delta\frac{\partial V_R^{\mathrm{C}}}{\partial G^{\mathrm{C}}}\right)G^{\mathrm{C}}+\left(\pi_R\mu_4-\alpha\frac{\partial V_R^{\mathrm{C}}}{\partial r^{\mathrm{C}}}\right)\left(r^{\mathrm{C}}-p\right)\\&+\frac{1}{2c_q}H_5(2H_5+H_2)-\frac{1}{8c_q}(4H_5^2-H_2^2)+\frac{1}{2c_b}(\pi_R\mu_2-H_6)^2\\&+\frac{1}{2c_a}\left(\pi_M\mu_1+H_1\right)\left(2\left(\pi_R\mu_1+H_4\right)+\left(\pi_M\mu_1+H_1\right)\right)\\&-\frac{1}{8c_a}(4(\pi_R\mu_1+H_4)^2-(\pi_M\mu_1+H_1)^2).\end{aligned}\tag{A.21}$$

We shall show that linear value functions satisfy (A.20) and (A.21). Hence, we define

$$V_M^{\mathrm{C}}\left(G^{\mathrm{C}}, r^{\mathrm{C}}\right) = s_1 G^{\mathrm{C}} + s_2 r^{\mathrm{C}} + s, \tag{A.22}$$
$$V_R^{\mathrm{C}}\left(G^{\mathrm{C}}, r^{\mathrm{C}}\right) = h_1 G^{\mathrm{C}} + h_2 r^{\mathrm{C}} + h, \tag{A.23}$$

where s_1, s_2, s, h_1, h_2 and h are constant parameters to be identified.

Substituting (A.22), (A.23) and their derivations into (A.20) and (A.21), we obtain

$$\begin{aligned}\rho s_1 G^{\mathrm{C}} + \rho s_2 r^{\mathrm{C}} + \rho s = {} & \left(\pi_M \mu_3 - \delta s_1\right) G^{\mathrm{C}} + \left(\pi_M \mu_4 - \alpha s_2\right) r^{\mathrm{C}} + \left(\alpha s_2 - \pi_M \mu_4\right) p \\ & + \frac{1}{4c_a}\left(\pi_M \mu_1 + \theta_1 s_1 + \nu_1 s_2\right)\left(2\left(\pi_R \mu_1 + \theta_1 h_1 + \nu_1 h_2\right) + \left(\pi_M \mu_1 + \theta_1 s_1 + \nu_1 s_2\right)\right) \\ & + \frac{1}{4c_q}(\theta_2 s_1 + \nu_2 s_2)(2(\theta_2 h_1 + \nu_2 h_2) + \theta_2 s_1 + \nu_2 s_2) \\ & + \frac{1}{c_b}\left(\pi_M \mu_2 - \theta_3 s_1 - \nu_3 s_2\right)\left(\pi_R \mu_2 - \theta_3 h_1 - \nu_3 h_2\right),\end{aligned} \tag{A.24}$$

$$\begin{aligned}\rho h_1 G^{\mathrm{C}} + \rho h_2 r^{\mathrm{C}} + \rho h = {} & \left(\pi_R \mu_3 - \delta h_1\right) G^{\mathrm{C}} + \left(\pi_R \mu_4 - \alpha h_2\right) r^{\mathrm{C}} + \left(\alpha h_2 - \pi_R \mu_4\right) p \\ & + \frac{1}{2c_a}(\pi_M \mu_1 + \theta_1 s_1 + \nu_1 s_2)(2(\pi_R \mu_1 + \theta_1 h_1 + \nu_1 h_1) + (\pi_M \mu_1 + \theta_1 s_1 + \nu_1 s_2)) \\ & + \frac{1}{2c_q}(\theta_2 h_1 + \nu_2 h_2)(4(\theta_2 h_1 + \nu_2 h_2)^2 - (\theta_2 s_1 + \nu_2 s_2)^2) \\ & - \frac{1}{8c_a}(4(\pi_R \mu_1 + \theta_1 h_1 + \nu_1 h_2)^2 - (\pi_M \mu_1 + \theta_1 s_1 + \nu_1 s_2)^2) \\ & + \frac{1}{2c_b}(\pi_R \mu_2 - \theta_3 h_3 - \nu_3 h_2)^2.\end{aligned} \tag{A.25}$$

It is easy to verify that the following constants are content with (A.24) and (A.25)

$$\begin{aligned} s_1 &= \frac{\pi_M \mu_3}{\rho + \delta}, \;\; s_2 = \frac{\pi_M \mu_4}{\rho + \alpha}, \\ s &= \frac{\pi_M (2\pi_R + \pi_M)}{4 c_a \rho} A^2 + \frac{\pi_M (2\pi_R + \pi_M)}{4 c_q \rho} Q^2 + \frac{\pi_M \pi_R}{c_b \rho} B^2 - \frac{\pi_M \mu_4 p}{\rho + \alpha}, \\ h_1 &= \frac{\pi_R \mu_3}{\rho + \delta}, \;\; h_2 = \frac{\pi_R \mu_4}{\rho + \alpha}, \\ h &= \frac{(2\pi_R + \pi_M)^2}{8 c_a \rho} A^2 + \frac{(2\pi_R + \pi_M)^2}{8 c_q \rho} Q^2 + \frac{\pi_R^2}{2 c_b \rho} B^2 - \frac{\pi_R \mu_4 p}{\rho + \alpha}. \end{aligned}$$

□

Proof of Corollary 3.4

Proof. Differentiating (3.12) with respect to π_M and π_R respectively yields

$$\frac{\partial \phi_a}{\partial \pi_M} = \frac{-4\pi_R}{(2\pi_R + \pi_M)^2} < 0, \quad \frac{\partial \phi_a}{\partial \pi_R} = \frac{4\pi_M}{(2\pi_R + \pi_M)^2} > 0, \tag{A.26}$$
$$\frac{\partial \phi_q}{\partial \pi_M} = \frac{-4\pi_R}{(2\pi_R + \pi_M)^2} < 0, \quad \frac{\partial \phi_q}{\partial \pi_R} = \frac{4\pi_M}{(2\pi_R + \pi_M)^2} > 0. \tag{A.27}$$

□

Proof of Proposition 3.5

Proof. The proof is similar to that of Propositions 3.1 and 3.3, hence it is omitted here. □

Proof of Proposition 3.6

Proof. Using (3.1), (3.2), (3.3), (3.9), (3.10), (3.11), (3.21), (3.22) and (3.23), we can easily get Proposition 3.6. □

Proof of Proposition 3.7

Proof.

(i) Computing the difference $G_\infty^{\mathrm{C}} - G_\infty^{\mathrm{N}}$, one obtains

$$G_\infty^{\mathrm{C}} - G_\infty^{\mathrm{N}} = \frac{\theta_1 \pi_M \phi_a}{c_a \delta(1-\phi_a)} A + \frac{\theta_2 \pi_M \phi_q}{c_q \delta(1-\phi_q)} Q > 0.$$

(ii) Computing the difference $r_\infty^{\mathrm{C}} - r_\infty^{\mathrm{N}}$, we get

$$r_\infty^{\mathrm{C}} - r_\infty^{\mathrm{N}} = \frac{\nu_1 \pi_M \phi_a}{c_a \delta(1-\phi_a)} A + \frac{\nu_2 \pi_M \phi_q}{c_q \delta(1-\phi_q)} Q > 0.$$ □

Proof of Proposition 3.8

Proof.

(i) Using (3.7) and (3.13), we obtain

$$V_M^{\mathrm{C}}(G_0, r_0) - V_M^{\mathrm{N}}(G_0, r_0) = \frac{\pi_M(2\pi_R - \pi_M)}{4\rho} \left(\frac{1}{c_a} A^2 + \frac{1}{c_q} Q^2 \right) > 0. \tag{A.28}$$

Since $2\pi_R > \pi_M$, we have $V_M^{\mathrm{C}}(G_0, r_0) > V_M^{\mathrm{N}}(G_0, r_0)$.

(ii) From (3.8) and (3.14), we have

$$V_R^{\mathrm{C}}(G_0, r_0) - V_R^{\mathrm{N}}(G_0, r_0) = \frac{(2\pi_R - \pi_M)^2}{8\rho} \left(\frac{1}{c_a} A^2 + \frac{1}{c_q} Q^2 \right) > 0. \tag{A.29}$$

Hence, we get $V_R^{\mathrm{C}}(G_0, r_0) > V_R^{\mathrm{N}}(G_0, r_0)$.

(iii) Computing $V_F^{\mathrm{I}}(G_0, r_0) - V^{\mathrm{C}}(G_0, r_0)$, we have

$$V_F^{\mathrm{I}}(G_0, r_0) - V^{\mathrm{C}}(G_0, r_0) = \frac{\pi_M^2}{2\rho} \left(\frac{1}{4} \left(\frac{A^2}{c_a} + \frac{Q^2}{c_q} \right) + \frac{B^2}{c_b} \right) > 0. \tag{A.30}$$

According to (A.28), (A.29) and (A.30), we can get $V_F^{\mathrm{I}}(G_0, r_0) > V^{\mathrm{C}}(G_0, r_0) > V^{\mathrm{N}}(G_0, r_0)$. □

Proof of Proposition 4.1

Proof. The proof is similar to that of Propositions 3.1 and 3.3, hence it is omitted here. □

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