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AUTOMATED CREDIT RATING PREDICTION IN A COMPETITIVE FRAMEWORK

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Abstract. Automated credit rating prediction (ACRP) algorithms are used to predict the ratings of bonds without having to trust one rating agency, like Moody's, Fitch or S&P. Nevertheless, for the moment, the accuracy of ACRP algorithms is investigated by empirical tests. In this paper, the framework for a competitive analysis is set and afterwards in this framework, the definition of competitive ACRP algorithms and its demonstration is given. In this way, for a competitive ACRP algorithm, a worst-case guarantee concerning the misclassification error is offered. Furthermore, several ACRP algorithms from the literature are compared according their competitiveness.

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1. Introduction

Companies and governments finance their projects and services by issuing financial obligations, as debts in the form of bonds. The evaluation of credit risk concerning the issuer and/or the issued bonds is the main task of the rating agencies (as Moody's, Fitch and S&P). The credit risk realizes when the issuer is not able to service the debts. This state is considered as a default. The probability of the default is expressed by a rating based on a specific rating scale. For example, Moody's rating scale contains 21 ratings where "Aaa" shows the lowest risk and "C" the highest risk. Nevertheless, there exist several bonds without any rating which is mainly due to the fact of two reasons. In the case of sovereign bonds, rating agencies do not analyze the credit risk of countries for several causes. For example, the countries are not interesting enough for investors or their political and economical system is too unstable to determine a rating. By this reason, the rating agencies evaluate these bonds as too risky for an average investor. In the case of corporate bonds, the evaluation process has to be paid by the company itself. Each company has to decide by itself whether accepting the benefit of a rating or rejecting the payment of the rating fees [16].

To handle the problem of unrated bonds, automated credit rating prediction (ACRP) algorithms are employed. ACRP is defined as follow: The prediction of a credit rating for at least one unrated bond based on an algorithm which uses only quantitative information from the attributes of a set of rated bonds. The set of rated bonds is used to calibrate the ACRP algorithm. Afterwards, the algorithm is utilized to predict the rating

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of an unrated bond. Examples of attributes of a bond are: value-at-risk, conditional value-at-risk or duration. Different techniques, like artificial neural networks [6], support vector machines [11] and support vector domain description [10], are used to develop an ACRP algorithm.

The accuracy of ACRP algorithms are currently analyzed with the help of experiments. From the best of our knowledge a competitive analysis of ACRP algorithms in the literature is missing. This paper contributes to closure this gap. Competitive analysis is the analytical performance evaluation of an algorithm (ALG) in comparison to an optimal benchmark (OPT) based on a worst-case situation [4, 13]. The analysis conducts to the analytical determination of the competitive ratio c which is a constant value. c is identified by solving the following equality: $c = \max_{\mathbf{a}\in\mathcal{A}} |ALG(\mathbf{a}) - OPT(\mathbf{a})|$, where $\mathbf{a}$ is the input vector given by the bond's attributes. Then a misclassification guarantee can be stated as

$$c \leq |ALG(\mathbf{a}) - OPT(\mathbf{a})| \quad \forall \mathbf{a} \in \mathcal{A} \tag{1.1}$$

with $\mathcal{A}$ as the whole sets or a partition of the sets of possible inputs. Thus, after some transformations, the subsequent guarantee, which always holds, is stated as:

$$OPT - c \leq ALG \leq OPT + c. \tag{1.2}$$

Each algorithm, for which such a competitive ratio c can be determined, is called competitive.

In this paper, different types of risk information are introduced. In each worst-case scenario, an additional type of information is assumed to be known. Thus, the competitive analysis represents an estimation of the worst-case performance of an arbitrary ACRP algorithm if only different stages of risk information are known. A competitive analysis of one concrete algorithm is not executed. However, the analysis undertaken in this paper provides the framework to conduct the competitive analysis of one concrete ACRP algorithm. A first estimation for the concrete competitive ratio of ACRP algorithms and a rough classification of them is given.

In other words, this paper contributes to the community of Operation Research, regarding the development of ACRP algorithms. Till now, the accuracy of ACRP algorithms is tested through the help of empirical analysis and strongly dependents on the applied dataset. This paper offers a new perspective on the testing of ACRP algorithms. More precisely, the algorithms are tested on a theoretical basis, *e.g.*, competitive analysis. The analytical performance is determined by considering worst-case scenarios. The resulting performance evaluation is independent of a dataset and a mis-classification guarantee can be stated. This guarantee indicates that the distance between a potential wrong prediction of the algorithm and the correct rating is bounded. Therefore, an objective distinction between various ACRP algorithms is possible. Additionally, the paper helps developers to create new ACRP algorithms. Furthermore, the provided framework helps to determine the level of risk information included in the attributes, when three types of risk information are exactly defined. Consider the case that one may ask how much risk information can be extracted from the observed attributes. Thus, one can calculate the empirical performance and compare it with the competitive performance. Based on this comparison, additional attributes to improve the practicability of an ACRP algorithm can be easily identified.

In the next section, the notations are introduced. The framework for the competitive analysis is provided in Section 3. Section 4 presents the competitive analysis. The comparison of different ACRP algorithms according to their competitiveness is given in Section 5. Finally, Section 6 concludes the paper and gives some outlooks for future work.

2. Literature review

The history of ACRP algorithms is already very long. In 1968, a first ACRP algorithm was developed to predict if companies defaults or not [1]. A company defaults if it is unable to refinance their debt and to pay the contractual payments to the investors. [1] establishes a linear relationship between the balance sheet of companies and their financial healthiness. Nevertheless, the ACRP algorithm of [1] remains still simple and is only able to determine if a company has a higher risk to default or not. In 1977, the ACRP algorithm of [1] is improved by extending the linear regression by discriminant analysis [2]. In 1989, a study to investigate the default risk of high yields bonds is undertaken without providing any new ACRP algorithm [5]. In 1994, with the emerging of artificial neural networks (ANN), ACRP algorithm based on ANN are developed and compared to the traditional methods based on discriminant analysis [3]. The obtained results show that ANN-based ACRP algorithms outperform in terms of accuracy the traditional methods. In 2004, an ACRP algorithm was developed based on support vector machines (SVM) and the obtained algorithm was tested against an ACRP algorithm based on ANN [11]. The test shows that ACRP algorithms based on SVM obtain a better performance than using ANN. Nevertheless, the algorithm of [11] is only able to predict the rating group of bonds. A rating group represents a merging of rating degrees. For example, using Moody's rating scale, Aaa equals to the first rating degree and C to the last rating degree. The exact definition concerning rating degrees and rating groups are given in the course of this paper. In 2006, an ACRP algorithm based on ANN is designed by [6]. The focus of this algorithm are the rating degrees of bonds. In this way, it represents one of the first ACRP algorithms which tries to predict the correct rating degrees of the bonds. Nevertheless, the algorithm of [6] attempts to define a direct relation between the employed attributes and the rating degrees of the bonds without taking other possible information like the rating groups into consideration. In 2013, [10] introduced a new ACRP algorithm based on support vector domain description (SVDD). The ACRP algorithm addresses its focus on the prediction of the correct rating group. Thus, the rating degrees are completely ignored by this algorithm. Finally, in 2014, a new ACRP algorithm based on SVDD and linear regression (LR) was developed by [7]. The idea of the development of this algorithm was to overcome the mentioned drawbacks of the other ACRP algorithms. The algorithm predicts the rating degrees of the bonds but takes the information about their affiliation to one rating group into account. Briefly, the major steps of the algorithm are described. In the training phase, the rating groups are calibrated with the help of SVDD and then, for each rating group the relationship between the bonds' attributes and their rating degrees is established with the help of LR. In the prediction phase, SVDD is used to determine the correct rating group of the unrated bonds and thereafter, the rating degrees are identified by LR.

From the best of our knowledge, the performance evaluation of ACRP algorithms is mainly based on empirical studies [6, 10, 11]. Nevertheless, the results of each empirical analysis always dependent on the used datasets. Using different datasets may generate different results. Therefore, to determine the average performance of ACRP algorithms on real bonds, all possible bonds and combination of bonds have to be applied which is not applicable due to time constraints. [13] describe competitive analysis as a dataset-free way of performance evaluation for algorithms. In competitive analysis, the ACRP algorithms are investigated in a theoretical concept with worst-case scenarios. The ACRP algorithm is compared to an optimal benchmark (OPT). OPT always takes the best possible decision on a given input. Then, competitive analysis identifies how far the analyzed ACRP algorithm is away from OPT [13]. This distance allows to determine the behavior of the algorithm even in the worst possible cases. In this way, one knows what is the maximal error which could occur if the respective algorithm is applied [13]. Due to the problem setting of ACRP, the relative competitive ratio, as described and discussed in [13], can not be applied. According to [4, 12, 15], the absolute competitive ratio is applied. In expression, these algorithms are called approximative algorthims. The difference between the absolute and the relative competitive ratio is that in the first one, the difference between the ACRP algorithm and OPT is determine in absolute terms and in the second one, the ratio is computed by building the quotient of OPT and the respective ACRP algorithm. At this moment, the performance of ACRP algorithms is only investigated by empirical analysis, there does not exist any literature dealing explicitly with this topic. For this reason, this paper closes this gap and introduces a new concept of performance evaluation of ACRP algorithms.

3. Notations

The following list introduces the employed main symbols in the paper and is orientated of the symbols by [7,8].

Unrated bonds

- $\tilde{B}$: one unrated bond.
- $\hat{R}^D$: predicted rating degree of $\tilde{B}$.

- $\hat{R}^G$: predicted rating group of $\tilde{B}$.
- $\hat{R}^C$: predicted rating characteristic of $\tilde{B}$.

Rating degree

- Ω_d^D: rating degree d, with $d = 1, \ldots, D$.
- ϵ^D: threshold degree value – first non-investment rating degree.

Rating group

- Ω_g^G: rating group g, with $g = 1, \ldots, G$.
- g': index of the last investment grade rating group.
- ϵ^G: boundary to separate one rating group into bonds with rating characteristic investment grade and non-investment rating.
- r_g^D: degree value to separate two adjacent groups Ω_g^G and $\Omega_{(g+1)}^G$, with $g = 1, \ldots, (G-1)$.

Extra notations

- $V_1^{3\to1}$: relation between type of information 3 and type of information 1.
- $V_2^{3\to2}$: relation between type of information 3 and type of information 2.
- $V_1^{2\to1}$: relation between type of information 2 and type of information 1.
- Π: set of possible rating degrees.
- δ: distance between two adjacent rating degrees.

4. Framework for the competitive analysis

First, several types of information are introduced and formally defined. Finally, the relations between the different types of information is established.

4.1. Risk information

A credit rating incorporates three different types of information. These types are:

(1) Rating characteristic.
(2) Rating group, Ω_g^G.
(3) Rating degree, Ω_d^D.

Figure 1 illustrates the three types of information and the relations between them.

Rating characteristic	investment										non-investment										
Rating group, Ω_g^G	1			2				3						4			5				
Rating degree, Ω_d^D	1	1.3	1.5	1.8	2	2.3	2.5	2.8	3	3.3	3.5	3.8	4	4.3	4.5	4.8	5	5.3	5.5	5.8	6
Moody's rating	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3	Ba1	Ba2	Ba3	B1	B2	B3	Caa1	Caa2	Caa3	Ca	C

Figure 1. Types of information.

Before explaining the different types of information, the encoding rule is defined. With the help of the encoding rule, the ratings from an arbitrary rating scale are mapped into equidistant rating degrees.

Definition 4.1. The encoding rule is given as follows:

$$\Omega_d^D = 1 + (d-1)\delta, \qquad \forall d = 1, \ldots, D$$

where δ is the distance between two adjacent ratings and D is the total number of ratings.

Additionally, the prediction process for an arbitrary ACRP algorithm is given by its rating function, $f(\cdot)$, which is based on the employed methods of the concrete ACRP algorithms. More details on the formal description of the rating function can be found in the original paper [8]. Type of information 1, rating characteristic, is binary information. The predicted rating characteristic, $\hat{R}^C$, indicates if $\tilde{B}$ is of investment or non-investment grade. Thus, 0 signifies non-investment grade and 1 indicates investment grade. Then, this type of information is formally described in the following way:

$$\begin{aligned} \mathbb{R} &\to \{0,1\} \\ \mathbf{a} &\mapsto f_C(\mathbf{a}) = \hat{R}^C. \end{aligned} \tag{4.1}$$

Furthermore, the set of possible rating degrees corresponding to type of information 1 is defined.

Definition 4.2. Under the assumption that the bonds' attributes only incorporate type of information 1, the sets of possible rating degrees is:

- $\Pi_1 = \left\{ \Omega_1^D, \ldots, \Omega_{(\epsilon^D - 1)}^D \right\}$ if the bonds are of investment grade.
- $\Pi_2 = \left\{ \Omega_{\epsilon^D}^D, \ldots, \Omega_D^D \right\}$ if the bonds are of non-investment grade.

Example 4.3. For example, lets use Moody's rating scale. The different rating degrees used by Moody's are shown in Figure 1. In all, there exists $D = 21$ rating degrees. Let set the distance between two adjacent rating degree, δ, equal to 0.25. According to the encoding rule, given in Definition 4.1, the following mapping between the alphanumerical rating scale used by Moody's and a numerical rating scale can be defined.

- $Aaa \mapsto 1 + (1-1)0.25 = 1$.
- $Aa1 \mapsto 1 + (2-1)0.25 = 1.25$.
- $\ldots$.
- $C \mapsto 1 + (21-1)0.25 = 6$.

For the given rating degrees, investment and non-investment grade are clearly defined in [14]. Thus, the threshold degree value, ϵ^D, equals 11. After applying Definition 4.2, the following two set of rating degrees are established:

- $\Pi_1 = \{1, \ldots, 10\} = \{Aaa, \ldots, Baa3\}$: investment grade.
- $\Pi_2 = \{11, \ldots, 21\} = \{Ba1, \ldots, C\}$: non-investment grade.

The type of information 2 expresses the rating group affiliation of $\tilde{B}$. Rating groups are a merging of bonds with similar credit risk. Due to the relation between type of information 1 and 2, which is formally demonstrated in the next subsection, the following definition is stated.

Definition 4.4. Let d_l be the starting rating degree and d_r the finishing rating degree of a rating group. Then, under the assumption that types of information 1 and 2 are known, the following sets are defined. Let d_l^1 be equal to 1 and d_r^1 equals some d', with d' greater than d_l^1. Then, the first set is given by: $\Pi_1 = \left\{ \Omega_{d_l^1}^D, \ldots, \Omega_{d_r^1}^D \right\}$.

For each additional set, the following recursive rule is taken: $d_l^g = d_r^{g-1} + 1$. Thus, the additional sets are defined as follows: $\Pi_g = \left\{ \Omega_{d_l^g}^D, \ldots, \Omega_{d_r^g}^D \right\}$, with $g = 2, \ldots, G$. G is the total number of sets. Naturally, d_r^G has to be equal to D.

Example 4.5. Based on Example 4.3, Moody's rating scale is again used. Assume that five rating groups have to be identified. In this way, the number of rating groups, G, is fixed to 5. Furthermore, let d_l^g and d_g^r the starting, respectively finishing, rating degree of the identified rating groups. In this example, the groups are determined in such a way that the following interpretation about the included credit risk can be made:

1. Π_1: very little risk.

2. Π_2: little risk.
3. Π_3: medium risk.
4. Π_4: high risk.
5. Π_5: very high risk.

Therefore, applying Definition 4.4, the following starting and finishing rating degrees are fixed: $d_l^1 = 1$, $d_r^1 = 3$, $d_r^2 = 7$, $d_r^3 = 13$, $d_r^4 = 16$ and $d_r^5 = 21$. In this way, the following rating groups are asserted:

- $\Pi_1 = \{1, \ldots, 3\} = \{Aaa, \ldots, Aa2\}$.
- $\Pi_2 = \{4, \ldots, 7\} = \{Aa3, \ldots, A3\}$.
- $\Pi_3 = \{8, \ldots, 13\} = \{Baa1, \ldots, Ba3\}$.
- $\Pi_4 = \{14, \ldots, 16\} = \{B1, \ldots, B3\}$.
- $\Pi_5 = \{17, \ldots, 21\} = \{Caa1, \ldots, C\}$.

Finally, type of information 3 expresses the exact predicted rating degree, $\hat{R}^D$, for the analyzed bond. Basically, this type is also of ordinal character. However, a relaxation to a cardinal rating scale is undertaken because some ACRP algorithms outputs their credit rating prediction with the help of cardinal rating degrees. Nevertheless, a mapping to the well-known ordinal rating scale used by the rating agencies is always feasible. Thus, the relaxation can be made without loss of generality. Hence, type of information 3 is formally defined as follows:

$$\begin{aligned} \mathbb{R} &\to [1, \ldots, (D-1)\delta] \\ \mathbf{a} &\mapsto f_D(\mathbf{a}) = \hat{R}^D. \end{aligned} \tag{4.2}$$

Figure 1 illustrates the case for Moody's rating scale with $\delta = 0.25$ and where D is equal to 21. Concluding, all three types of information are formally described and the respective sets of possible rating degrees are defined.

4.2. Relations between the different types of information

Starting with the following question: can an ACRP algorithm, which focuses on the type of information 3, offer the two other types of information to an investor? Thus, the relations on the basis of type of information 3 have to be analyzed.

Proposition 4.6. *Type of information 1 can be deduced from type of information 3.*

Proof. Let ϵ^D be the first non-investment rating degree, called threshold rating. Thus, type of information 1 can be extracted from type of information 3 as follows:

$$V_1^{3\to 1} = \begin{cases} 0 & \text{if } \hat{R}^D \geq \epsilon^D \quad \text{“non-investment grade”} \\ 1 & \text{if } \hat{R}^D < \epsilon^D \quad \text{“investment grade”}. \end{cases} \tag{4.3}$$

□

The relation between type of information 2 and type of information 3 is given in the following proposition:

Proposition 4.7. *Type of information 2 can be extracted from type of information 3.*

Proof. A merging of rating degrees is undertaken to restore the rating groups. Let $r_1^D, \ldots, r_{(G-1)}^D$ be the distinctive rating degrees. Using these distinctive degrees, type of information 2 is deduced from type of information 3 in the following way:

$$V_1^{3\to 2} = \begin{cases} 1 & \text{if } \hat{R}^D < r_1^D \\ 2 & \text{if } r_1^D \leq \hat{R}^D < r_2^D \\ & \vdots \\ G & \text{if } \hat{R}^D \geq r_{(G-1)}^D \end{cases} \tag{4.4}$$

□

Consequently, type of information 3 allows reconstituting the other two types of information. By this way, an ACRP algorithm, focusing on type of information 3, can also offer an investor the other two types (rating characteristic and rating group). The relation between type of information 2 and type of information 1 is established by the subsequent proposition.

Proposition 4.8. *Type of information* 1 *can be extracted from type of information* 2.

Proof. Two possible cases can occur:

(i) Each rating group consists of investment **or** non-investment grade bonds.
(ii) There is one rating group which consists of investment **and** non-investment grade bonds.

Case (i)
Let $\bar{g}$ be the index of the last rating group containing rating bonds with rating characteristic investment grade. Recall that for calibrating an ACRP algorithm, the information from a set of rated bonds is used. Thus, the relation is given by:

$$V_1^{2\to 1} = \begin{cases} 0 & \text{if } \hat{R}^G \in \Omega_g^G \quad \text{for } g \in \{(\bar{g}+1), \ldots, G\} \\ 1 & \text{if } \hat{R}^G \in \Omega_g^G \quad \text{for } g \in \{1, \ldots, \bar{g}\}\,. \end{cases} \tag{4.5}$$

Case (ii)
Assume the only rating group composed of investment and non-investment grade bonds is the rating group of index g'. Thus, in the rating groups Ω_g^G with $g = 1, \ldots, (g'-1)$, all the bonds have the rating characteristic investment grade and all bonds out of the rating groups Ω_g^G with $g = (g'+1), \ldots, G$, are of non-investment grade. A precise analysis of $\Omega_{g'}^G$ has to be undertaken to split the bonds into investment – and non-investment grade. The separation of investment – and non-investment grade bonds, ϵ^G, is identifiable by applying the ACRP algorithm on $\Omega_{g'}^G$. Thus, $\Omega_{g'}^G$ is split into two subgroups:

$$\Omega_{g'}^G = \begin{cases} \text{“investment grade”} & \text{if } \hat{R}^G < \epsilon^G \\ \text{“non-investment grade”} & \text{if } \hat{R}^G > \epsilon^G. \end{cases} \tag{4.6}$$

Recall, type of information 1 is extracted from 2. It follows:

$$V_1^{2\to 1} = \begin{cases} 0 & \text{if } \hat{R}^G \in \Omega_{g'}^G \quad \text{and } \hat{R}^G > \epsilon^G \\ 0 & \text{if } \hat{R}^G \in \Omega_g^G \quad \text{for } g \in \{(g'+1), \ldots, G\} \\ 1 & \text{if } \hat{R}^G \in \Omega_{g'}^G \quad \text{and } \hat{R}^G < \epsilon^G \\ 1 & \text{if } \hat{R}^G \in \Omega_g^G \quad \text{for } g \in \{1, \ldots, g'\}\,. \end{cases} \tag{4.7}$$

□

To finish the setting of the framework, the benchmark algorithm, noted as OPT, is defined and the formal definition of the performance measure is given.

4.3. Performance measure and benchmark algorithm

Let **a** be the vector of one single bond's attributes. The unrated bond is given by $\tilde{B}$. Additionally, let $\mathcal{A}$ be the set of all possible attributes combinations for one single bond. This set $\mathcal{A}$ is restricted by allowing the attributes incorporating more and more information. In competitive analysis, the predicted result is compared to an optimal benchmark ACRP algorithm, as noted as OPT.

Definition 4.9. The benchmark algorithm OPT is considered as the adversary player of the analyzed ACRP algorithm ALG. Thus, ALG will always try to minimize the damage which OPT can perpetrate. Naturally, OPT wants to maximize this damage. The damage is the difference between the rating degree obtained by ALG and OPT.

Due to the incorporated types of information in different stages, in the bond's attributes, the credit risk is inputted into the ACRP algorithm. The performance measure is given as follows:

Definition 4.10. Let $\mathbf{a} \in \mathcal{A}$ the attributes of $\tilde{B}$, then the deviation between ALG and OPT is computed as follows:

$$c(\mathbf{a}) = |ALG(\mathbf{a}) - OPT(\mathbf{a})| \quad \forall \mathbf{a} \in \mathcal{A} \tag{4.8}$$

In competitive analysis, the worst-case is considered. In this way, the maximum possible deviation between ALG and OPT is determined.

$$c_{\max} = \max_{\mathbf{a}\in\mathcal{A}} c(\mathbf{a}) = \max_{\mathbf{a}\in\mathcal{A}} |ALG(\mathbf{a}) - OPT(\mathbf{a})| \tag{4.9}$$

The competitive ratio is defined in absolute terms similar to [4,9,15]. The absoluteness of the ratio is due to the following reason. The maximum deviation is searched with respect to the rating degrees are given by the encoding rule in Definition 4.1. The division, to obtain a relative value is not possible because the relative ratio considers differently two identical deviations. The following example illustrates the reason. Using the subsequent rating degrees: $\Omega_1^D = 1$, $\Omega_3^D = 1.5$, $\Omega_4^D = 1.75$ and $\Omega_6^D = 2.25$. Then, the following absolute and relative values are obtained:

- absolute value: $\Omega_3^D - \Omega_1^D = 0.5$ and $\Omega_6^D - \Omega_4^D = 0.5$
- relative value: $\frac{\Omega_3^D}{\Omega_1^D} = \frac{1.5}{1} = \frac{3}{2}$ and $\frac{\Omega_6^D}{\Omega_4^D} = \frac{2.25}{1.75} = \frac{9}{7}$

The potential deviations for this two presented cases are identical. However, the relative values deform this view on the problem. Thus, only the absolute competitive ratio is usable to determine the maximum deviation on a set of rating degrees.

After the framework of the competitive analysis is stated, the next section furnishes the analysis and defines a competitive ACRP algorithm.

5. Competitive analysis

The competitive analysis is set in the following way. With each worst-case scenario, which is investigated, more information are assumed to be known.

(1) No additional information are given and the ACRP algorithm (ALG) has to predict the rating characteristic of $\tilde{B}$.
(2) The rating characteristic is known and the correct rating group has to be determined by ALG.
(3) The rating characteristic as well as the rating group are known and ALG has to identify the correct rating degree of $\tilde{B}$.

A first view on the problem will give the following conclusion: ALG predicts rating degree Ω_1^D and the correct rating degree given by OPT is Ω_D^D. In this way, the competitive ratio equals: $c = \left|\Omega_1^D - \Omega_D^D\right| = D - 1$. This competitive ratio, c, represents an upper bound. The setting of ALG focuses to achieve OPT. It is assumed that a minimal information which can be treated by every ACRP algorithm is known. This assumption is plausible as each ACRP algorithm is trained with a set of rated bonds before the algorithm is used for the prediction task. During the training phase, this minimal information is introduced to the ACRP algorithm. Therefore, for the rest of the paper, assume that if ALG mis-classify $\tilde{B}$ then ALG miss the optimal result in the lowest possible way.

5.1. Scenario 1

In Scenario 1, the task for ALG is to predict if $\tilde{B}$ is of investment – or non-investment grade. OPT, as the adversary player of ALG, will always declare the exact opposite of the prediction which ALG is making. Therefore, the following lemma is stated.

Lemma 5.1. *The maximum deviation between ALG and OPT, in case of Scenario* 1*, equals* $c_{\max} = \max\left\{(D - (\epsilon^D - 1))\delta, (\epsilon^D - 1)\delta\right\}$.

Proof. Two different cases can occur:

(i) ALG predicts $\tilde{B}$ as investment grade.
(ii) ALG preditcs $\tilde{B}$ as non-investment grade.

The set of possible rating degrees are given in Definition 4.2.

Case (i)
In Case (i), ALG identifies $\tilde{B}$ as an investment grade bond. Thus, ALG outputs a rating degree out of the set, Π_1. Nevertheless, ALG always wants to minimize the damage to its adversary player, OPT. Therefore, ALG issues the last possible investment rating degree, $\Omega^D_{(\epsilon^D-1)}$. As OPT is the adversary player, OPT identifies $\tilde{B}$ as a non-investment grade bond. Thus, a rating degree from the set, Π_2 is assigned to $\tilde{B}$ by OPT. In the context to maximize the deviation, OPT chooses the last existing rating degree, *i.e.*, Ω^D_D, Thus, the maximum deviation for this case can be determined in the following way:

$$\begin{aligned} c^{(i)}_{\max} &= \max_{\mathbf{a}\in\mathcal{A}} |ALG(\mathbf{a}) - OPT(\mathbf{a})| \\ &= \left|1 + ((\epsilon^D - 1) - 1)\delta - (1 + (D-1)\delta)\right| \\ &= (D - (\epsilon^D - 1))\delta. \end{aligned} \tag{5.1}$$

Case (ii)
ALG determines $\tilde{B}$ as a non-investment grade bond and OPT dictates it as an investment grade bond. The respective rating degrees are assigned to obtain the maximum deviation. This deviation is determined as follows:

$$\begin{aligned} c^{(ii)}_{\max} &= \max_{\mathbf{a}\in\mathcal{A}} |ALG(\mathbf{a}) - OPT(\mathbf{a})| \\ &= \left|1 + (\epsilon^D - 1)\delta - (1 + (1-1)\delta)\right| \\ &= (\epsilon^D - 1)\delta. \end{aligned} \tag{5.2}$$

Now after the analysis of the two cases, the (global) maximum deviation for Scenario 1 can be computed. By taken the maximum, the (global) maximum deviation is determined by

$$c^1_{\max} = \max\left\{c^{(i)}_{\max}, c^{(ii)}_{\max}\right\}. \tag{5.3}$$

□

Example 5.2. Lets take again Moody's rating scale in this example to explain the computation of the (global) maximum deviation under the assumption of Scenario 1. In Example 4.3, the threshold degree value, ϵ^D, is identified and equals 11. Additionally, the maximal number of existing rating degrees, D, is equal to 21 and the distance between two adjacent rating degrees, δ is fixed to 0.25. Applying Lemma 5.1, the maximal deviation for Scenario 1 is determinable. To explain the computation steps. First, if the rating degree of the analyzed bond, $\tilde{B}$, is predicted by the ACRP algorithm, ALG, as investment grade, then the deviation is determined as follows:

$$\begin{aligned} c^{(i)}_{\max} &= |1 + ((\epsilon^D - 1) - 1)\delta - (1 + (D-1)\delta)| \\ &= |1 + ((11-1)-1)0.25 - (1 + (21-1)0.25)| \\ &= |3.25 - 6| \\ &= |-3.75| = 3.75. \end{aligned}$$

In the same logic, if ALG predicts a non-investment grade rating degrees for $\tilde{B}$, the deviation is as follows:

$$\begin{aligned} c_{\max}^{(ii)} &= |1 + (\epsilon^D - 1)\delta - (1 + (1-1)\delta)| \\ &= |1 + (11-1)0.25 - (1 + (1-1)0.25)| \\ &= |3.5 - 1| \\ &= |2.5| = 2.5. \end{aligned}$$

After identifying the maximum over $c_{\max}^{(i)}$ and $c_{\max}^{(ii)}$, the global maximum deviation for Scenario 1 is established:

$$c_{\max}^1 = \max\left(c_{\max}^{(ii)}, c_{\max}^{(ii)}\right) = 3.75.$$

This value indicates that if $\tilde{B}$ would have a rating degree equal to 1, so according to Moody's rating scale, corresponding to Aaa, then if an error in the prediction process occurs, then the predicted rating degree does not exceed the following rating degree: $1 + 3.75 = 4.75$. Thus, using Moody's rating scale, the predicted rating degree does not exceed $B3$ if an error has arisen. In the Appendix, the example is also illustrated in Figure A.1 to simplify the understanding of Lemma 5.1.

5.2. Scenario 2

Now, the distinction between investment – and non-investment grade is known. Thus, ALG has to determine the right rating group of $\tilde{B}$. Let assume G rating groups. $\Omega_{g'}^G$ is the rating group containing investment – and non-investment grade bonds. Then the subsequent lemma states the maximum deviation.

Lemma 5.3. *Under the assumption of Scenario* 2*, the maximum deviation between ALG and OPT is equal to:* $c_{\max}^2 = \max\{A1, A2, B1, B2\}$

with:
$$\begin{aligned} A1 &= ((\epsilon^D - 1) - d_r^1)\delta \\ A2 &= d_r^{g'-1}\delta \\ B1 &= (D - d_r^{g'})\delta \\ B2 &= ((d_r^{G-1} + 1) - \epsilon^D)\delta. \end{aligned}$$

Proof. It is known if $\tilde{B}$ is of investment – or non-investment grade. Therefore, two cases can be distinguished: (i) investment – and (ii) non-investment grade.

Case (i)
There exists g' investment grade rating groups. Nevertheless, only rating groups Ω_1^G and $\Omega_{g'}^G$ have to be considered because examining the intermediate rating groups always results in a smaller deviation. Therefore, two sub-cases remain:

- ALG sets $\tilde{B}$ in rating group Ω_1^G.
- ALG sets $\tilde{B}$ in rating group $\Omega_{g'}^G$.

Sub-Case (i-1):
ALG predicts a rating degree out of the set of Π_1 (see Def. 4.4) and OPT issues a rating degree out of the set of $\Pi_{g'}$. Thus, the following result is obtained:

$$\begin{aligned} c_1 &= \max_{\mathbf{a}\in\mathcal{A}} |ALG(\mathbf{a}) - OPT(\mathbf{a})| \\ &= \left|d_r^1\delta - (\epsilon^D - 1)\delta\right| = ((\epsilon^D - 1) - d_r^1)\delta. \end{aligned} \tag{5.4}$$

Sub-Case (i-2):
Similar to the previous sub-case, the deviation between ALG and OPT is directly computed:

$$\begin{aligned} c_2 &= \max_{\mathbf{a}\in\mathcal{A}} |ALG(\mathbf{a}) - OPT(\mathbf{a})| = \left| d_l^{g'}\delta - d_l^1\delta \right| \\ &= (d_l^{g'} - d_l^1)\delta = ((d_r^{g'-1}+1) - d_l^1)\delta. \end{aligned} \tag{5.5}$$

Hence, as $d_l^1 = 1$, c_2 can be rewritten in the following way:

$$c_2 = ((d_r^{g'-1}+1) - d_l^1)\delta = (d_r^{g'-1}+1-1)\delta = (d_r^{g'-1})\delta.$$

Concluding, the maximum deviation for Case (i) equals to: $c_{\max}^{(i)} = \max\{c_1, c_2\}$.

Case (ii)
$G - g' + 1$ non-investment grade rating groups are admissible. Similar to the previous case, the intermediate rating groups do not have to be considered. Thus, the following sub-cases are possible:

- ALG designates $\tilde{B}$ in rating group $\Omega_{g'}^G$.
- ALG designates $\tilde{B}$ in rating group Ω_G^G.

Sub-Case (ii-1):
ALG designates $\tilde{B}$ in rating group $\Omega_{g'}^G$ and OPT issues the last existing rating degree to $\tilde{B}$. The maximum deviation is given by:

$$\begin{aligned} c_3 &= \max_{\mathbf{a}\in\mathcal{A}} |ALG(\mathbf{a}) - OPT(\mathbf{a})| \\ &= \left| d_r^{g'}\delta - d_r^G\delta \right| = (D - d_r^{g'})\delta. \end{aligned} \tag{5.6}$$

Sub-Case (ii-2):
Similar to the previous sub-case, the maximum deviation between ALG and OPT is directly given:

$$\begin{aligned} c_4 &= \max_{\mathbf{a}\in\mathcal{A}} |ALG(\mathbf{a}) - OPT(\mathbf{a})| = \left| d_l^G\delta - \epsilon^D\delta \right| \\ &= (d_l^G - \epsilon^D)\delta = ((d_r^{G-1}+1) - \epsilon^D)\delta. \end{aligned} \tag{5.7}$$

Concluding, the maximum deviation for Case (ii) is given by: $c_{\max}^{(ii)} = \max\{c_3, c_4\}$.

Finally, the determination of the (global) maximum deviation is given by:

$$c_{\max}^2 = \max\left\{c_{\max}^{(i)}, c_{\max}^{(ii)}\right\}. \tag{5.8}$$

□

Example 5.4. Reusing the $G = 5$ rating groups introduced in Example 4.5 to illustrate Scenario 2. Furthermore, Moody's rating scale is again employed and the distance between two adjacent rating degrees, δ is fixed to 0.25. In this way, the threshold degree value equals $\epsilon^D = 11$. Recall the third rating group: $\Pi_3 = \{8, \ldots, 13\} = \{Baa1, \ldots, Ba3\}$. Thus, this rating group is composed of investment - and non-investment grade rating degrees. For this reason, the index of the last investment grade rating group equals: $g' = 3$. The starting and finishing rating degrees for the used 5 rating groups are given in Example 4.5. Now, applying Lemma 5.3, the following deviations are identified:

- $c_1 = ((\epsilon^D - 1) - d_r^1)\delta = (10 - 3)0.25 = 1.75.$

- $c_2 = ((d_r^{g'-1} + 1) - d_l^1)\delta = ((7+1) - 1)0.25 = 1.75.$
- $c_3 = (D - d_r^{g'})\delta = (21 - 13)0.25 = 2.$
- $c_4 = ((d_r^{G-1} + 1) + 1) - \epsilon^D)\delta = ((16+1) - 11)0.25 = 1.5.$

Thus, the (global) maximum deviation is given by:

$$c^2_{\max} = \max\{1.75, 1.75, 2, 1.5\} = 2.$$

The obtained deviation indicates that if the ACRP algorithm, ALG, produces an error by predicting Aaa as rating degree of the analyzed bond $\tilde{B}$, then the predicted rating degree does not exceed the following rating degree: $1 + 2 = 3$ which equals $Baa2$. Figure A.2 illustrates this example to help the understanding.

5.3. Scenario 3

Types of information 1 and 2 are known. ALG has to predict the right rating degree of $\tilde{B}$. Let $N_g = d_r^g - d_l^g + 1$ the number of rating degrees in rating group Ω_g^G with $g = 1, \ldots, G$ and d_m^g be the rating degree which is the center of Ω_g^G. g' represents the index of the mixed rating group. Additionally, let d_{m-i}, respectively d_{m-ni}, be the rating degree which is in the middle of the respective part (investment or non-investment) of rating group $\Omega_{g'}^G$.

Lemma 5.5. *The (global) maximal deviation between ALG and OPT, under the assumptions of Scenario* 3*, is equal to:*

$$c^3_{\max} = \max\{A1, B1, B2, C1, C2\}$$

with: $A1 = \max_{g=1,\ldots,G \quad g\neq g'} \left\{(d_r^g - d_m^g)\delta, (\lceil \frac{d_r^g - d_l^g}{2} \rceil)\delta\right\}$
$B1 = (\lceil \frac{(\epsilon^D - 1) + d_l^{g'}}{2} \rceil)\delta$
$B2 = ((\epsilon^D - 1) - d_{m-i})\delta$
$C1 = (\lceil \frac{d_r^{g'} - \epsilon^D}{2} \rceil)\delta$
$C2 = (d_{m-ni} - \epsilon^D)\delta.$

Proof. As type of information 1 and 2 are known, two cases have to be investigated:

(i) $\tilde{B}$ belongs to rating group Ω_g^G, with $g \neq g'$.
(ii) $\tilde{B}$ belongs to rating group $\Omega_{g'}^G$.

Case (i)
$\tilde{B}$ belongs to one rating group containing either investment – or non-investment grade bonds. ALG always issues a rating degree which is in the middle of the rating group because OPT always assigns a boundary rating degree. Now two sub-cases can occur: (i-1) N_g is odd or (i-2) N_g is even.

Sub-Case (i-1):
If N_g is odd, then d_m^g exists. OPT always assigns either d_l^g or d_r^g, so a boundary degree, which would represent a symmetric choice. Thus, the deviation will be:

$$c_1 = \max_{\mathbf{a}\in\mathcal{A}} |ALG(\mathbf{a}) - OPT(\mathbf{a})| = |d_m^g\delta - d_r^g\delta| = (d_r^g - d_m^g)\delta \tag{5.9}$$

Sub-Case (i-2):
Now, N_g is even. The integral result of the division of N_g by 2 is given by $d_{m'}^g$ which gives a rating degree which is approximately the center of Ω_g^G. If ALG assigns $d_{m'}^g$ to $\tilde{B}$, then OPT will issue the boundary rating degree which is the farthest away from $d_{m'}^g$. Without loss of generality, let assume that it will be the finishing rating degree, given by d_r^g. ALG tries to react on this situation without leaving the center of Ω_g^G. Therefore, ALG

tries to minimize the damage by assigning $d^g_{m'}+1$. However, OPT also reacts on this situation and issues the opposite boundary degree d^g_l. Due to symmetry, the deviation is as follows:

$$\begin{aligned} c_2 &= |d^g_{m'}\delta - d^g_r\delta| = (d^g_r - d^g_{m'})\delta \\ &= \left(d^g_r - \frac{d^g_r - d^g_l + 1}{2}\right)\delta = \left(\left\lceil \frac{d^g_r - d^g_l}{2} \right\rceil\right)\delta \end{aligned} \tag{5.10}$$

Concluding, the maximum deviation for Case (i) is determined in the following way:

$$c^{(i)}_{\max} = \max\{c_1, c_2\} \tag{5.11}$$

To obtain $c^{(i)}_{\max}$, the maximum over all rating groups, excepted $\Omega^G_{g'}$, is taken.

Case (ii)
Rating group $\Omega^G_{g'}$ contains investment – as well as non-investment grade bonds. Thus, two sub-cases are examined:

(i) $\tilde{B}$ is of investment grade.
(ii) $\tilde{B}$ is of non-investment grade.

Sub-Case (ii-1):
Similar to the previous case, with the only change that the last admissible rating degree is $\epsilon^D - 1$. d_{m-i} is in the middle of the remaining part of the group. If d_{m-i} does not represent a real rating degree, then the maximum deviation equals:

$$\begin{aligned} c_3 &= \left|d_{m-i}\delta - (\epsilon^D - 1)\delta\right| = ((\epsilon^D - 1) - d_{m-i})\delta \\ &= \left((\epsilon^D - 1) - \left\lceil \frac{(\epsilon^D - 1) - d^{g'}_l}{2} \right\rceil\right)\delta = \left(\left\lceil \frac{(\epsilon^D - 1) + d^{g'}_l}{2} \right\rceil\right)\delta. \end{aligned} \tag{5.12}$$

Otherwise, the maximum deviation is as follows:

$$c_4 = \left|d_{m-i}\delta - (\epsilon^D - 1)\delta\right| = ((\epsilon^D - 1) - d_{m-i})\delta. \tag{5.13}$$

Sub-Case (ii-2):
Now, the first permitted rating degree is ϵ^D and d_{m-ni} is in the middle of the non-investment part of the group. If d_{m-ni} does not represent a real rating degree, then the maximum deviation is as follows:

$$\begin{aligned} c_5 &= \left|d_{m-ni}\delta - \epsilon^D\delta\right| = (d_{m-ni} - \epsilon^D)\delta \\ &= \left(\left\lceil \frac{d^{g'}_r - \epsilon^D}{2} \right\rceil - \epsilon^D\right)\delta = \left(\left\lceil \frac{d^{g'}_r - \epsilon^D}{2} \right\rceil\right)\delta. \end{aligned} \tag{5.14}$$

Otherwise, the maximum deviation equals:

$$c_6 = \left|d_{m-ni}\delta - \epsilon^D\delta\right| = (d_{m-ni} - \epsilon^D)\delta. \tag{5.15}$$

Closing, the maximal deviation for Case (ii) is determined:

$$c^{(ii)}_{\max} = \max\{c_3, c_4, c_5, c_6\}. \tag{5.16}$$

Finally, the (global) maximal deviation for scenario 3 is computed as follows:

$$c^3_{\max} = \max\left\{c^{(i)}_{\max}, c^{(ii)}_{\max}\right\}. \tag{5.17}$$

□

Example 5.6. Reusing the $G = 5$ rating groups introduced in Example 4.5. Furthermore, Moody's rating scale is again used and δ, the distance between two adjacent rating degrees, is set to 0.25. In this way, the threshold degree value equals $\epsilon^D = 11$. Recall the third rating group: $\Pi_3 = \{8, \ldots, 13\} = \{Baa1, \ldots, Ba3\}$. Thus, the index of the mixed rating group equals $g' = 3$. The starting and finishing rating degrees are described in Example 4.5 and are: $d_l^1 = 1$, $d_r^1 = 3$, $d_r^2 = 7$, $d_r^3 = 13$, $d_r^4 = 16$ and $d_r^5 = 21$. First, the rating groups excluded the mixed one, are investigated to determined the maximum deviations in each group. Let d_m^g be the rating degree which is the center of rating group Ω_g^G. Dependent if d_g^m represents a real rating degree or not, either $(d_r^g - d_m^g)\delta$ or $(\lceil \frac{d_r^g - d_l^g}{2} \rceil)\delta$ is used. Therefore, the following deviation are obtained:

- $c_{g=1} = (d_r^g - d_m^g)\delta = (3 - 2)0.25 = 0.25$.
- $c_{g=2} = (\lfloor \frac{d_r^g - d_l^g}{2} \rfloor)\delta = (\lceil \frac{7-4}{2} \rceil)0.25 = 2 \times 0.25 = 0.5$.
- $c_{g=4} = (d_r^g - d_m^g)\delta = (16 - 15)0.25 = 0.25$.
- $c_{g=5} = (d_r^g - d_m^g)\delta = (21 - 19)0.25 = 0.5$.

Second, the mixed rating group, $\Omega_{g'}^G$ is analyzed to determine its maximum deviation. Let d_{m-i} and d_{m-ni} be the rating degree which lies in the middle of the respective part of the rating group: investment – or non-investment. Regarding the composition of the group, d_{m-i} and d_{m-ni} represents real rating degrees. Thus, the obtained deviations are as follows:

- $c'_{g'=3} = ((\epsilon^D - 1) - d_{m-i})\delta = (10 - 9)0.25 = 0.25$: investment grade
- $c''_{g'=3} = (d_{m-ni} - \epsilon^D)\delta = (12 - 11)0.25 = 0.25$: non-investment grade

Hence, the (global) maximum deviation for Scenario 3, which is illustrated in this example, is given by taken the maximum over the received deviations:

$$\begin{aligned} c_{\max}^3 &= \max\{c_{g=1}, c_{g=2}, c_{g=4}, c_{g=5}, c'_{g'=3}, c''_{g'=3}\} \\ &= \max\{0.25, 0.5, 0.25, 0.5, 0.25, 0.25\} \\ &= 0.5. \end{aligned}$$

The interpretation of the maximum deviation is as follow. If the ACRP algorithm, ALG, predicts 1 as rating degree of the analyzed bond, $\tilde{B}$ and has made an error in its prediction, then the predicted rating degree does not exceed the degree $1 + 0.5 = 1.5$ which equals $Aa2$ in the rating scale employed by Moody's. This example is illustrated in Figure A.3 for its better understanding.

After the investigation of all three scenarios, competitive ACRP algorithms are defined by the subsequent theorem.

Theorem 5.7. *An ACRP algorithm (ALG) is competitive if a guarantee concerning the misclassification error for each scenario can be provided. Each full ACRP algorithm is competitive.*

Proof. ALG has to be able to handle all three types of information. In the opposite case, for the worst-case scenarios based on the missing types of information, a guarantee can not be stated. Thus, ALG is a full ACRP algorithm. In this way, the previous stated Lemmas 5.1, 5.3 and 5.5 is applicable and the guarantees for the respective scenarios follow. The reasoning is based on the relation between type of information 3 and the two remaining types of information, stated in Section 3. □

6. Comparison of ACRP algorithms

The four ACRP algorithms introduced in the literature review, are analyzed according to their ability to handle the different types of risk information. Afterwards, Theorem 5.7 is applied on the ACRP algorithms to compare them and to determine which of the mentioned ACRP algorithms is competitive. The ACRP algorithm developed by [6] is designed to predict the rating degrees of unrated bonds. However, one has to mention that this ACRP algorithm directly focuses on type of information 3. In this way, the additional information given by types of information 1 and 2 is not taken into account during the prediction process. In contrast, the ACRP algorithms of [11], respectively [10], are able to predict the rating group affiliation of unrated bonds. Type of information 2 is the highest risk information which is used by these algorithms. Even if type of information 3 would be available, the algorithms ignore it during the prediction of the rating groups. Finally, the ACRP algorithm developed by [7] utilizes all three type of risk information if they are available and it predicts the rating degrees of the bonds. The following table indicates on which types of information the different ACRP algorithms focus.

TABLE 1. Comparison of ACRP algorithms based on the types of information.

	Type of information 1	Type of information 2	Type of information 3
HCH04	X	X	
BCT06			X
GZS12	X	X	
GDS14	X	X	X

According to Theorem 5.7, the different ACRP algorithms, introduced in this paper, can be divided into competitive and non-competitive algorithms. The subsequent table gives an overview of their competitiveness.

TABLE 2. Competitiveness of ACRP algorithms.

	Competitive	Non-competitive
HCH04	-	X
BCT06	-	X
GZS12	-	X
GDS14	X	-

As a result, the only real competitive algorithm is our algorithm, as it is the only algorithm which is able to utilize all three types of information to predict the rating degrees. The algorithm of [6] is the only algorithm, except ours, which also tries to predict the right rating degrees of the bonds. The other two algorithms only tries to identify the right rating group, therefore these two algorithms are by definition non-competitive. For the algorithm of [6], the competitiveness can not be stated as possible mis-classifications are not bounded by the information about the rating group which are completely ignored during the prediction process.

7. Conclusion

This paper offers the framework for a competitive analysis of automated credit rating prediction (ACRP) algorithms. Additionally, it is shown that every full ACRP algorithm is competitive. A full ACRP algorithm is an algorithm which handles all three types of information (rating characteristic, rating group, rating degree). The given maximal deviations limit the misclassification error of the algorithm. Even if the competitive analysis is done in the view of the different risk information which are available and not for a concrete ACRP algorithm, several findings are revealed. First, instead of measuring only the performance of ACRP algorithms by empirical tests, an estimation of their concrete competitive ratios in a worst-case situation is given. Secondly, a first rough classification of the algorithms is offered and in this way, an investors has an objective criterion to select an ACRP algorithm.

Additionally, the provided framework for competitive analysis offers some ideas for future work. First, the introduced framework represents the foundation for a competitive analysis of a concrete ACRP algorithm. In this way, the concrete competitive ratio of the algorithm can be determined and compared to the estimate given by the maximum deviations in this work. Thus, the level of risk information used by the algorithm can be identified. Secondly, the result of our framework can be used to develop a new ACRP algorithm with the intention to improve its competitive ratio. In this way, the new developed algorithm should behave, in worst-case situations, similar as the established rating agencies, like Moody's, Fitch and S&P.

Appendix A.

Worst-case situations for Scenario 1 if no previous information is known

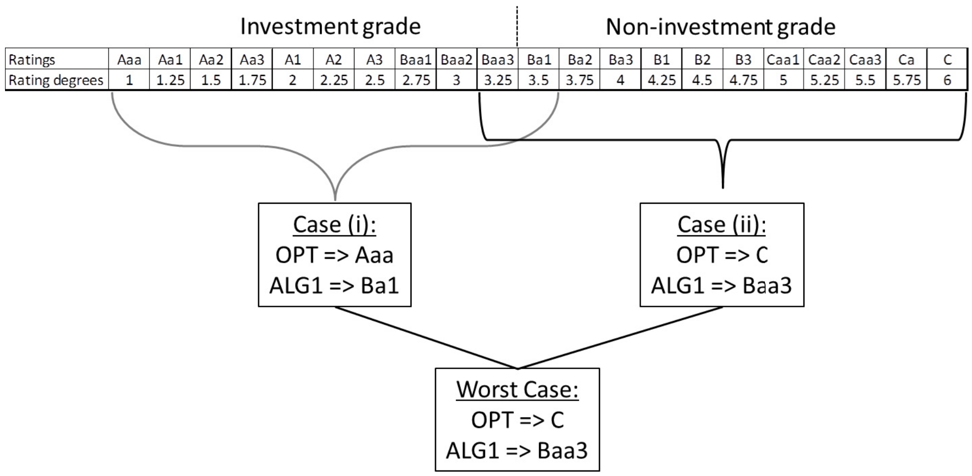

Figure A.1. Illustration of Scenario 1.

Worst-case situations for Scenario 2 if type of information 1 is given

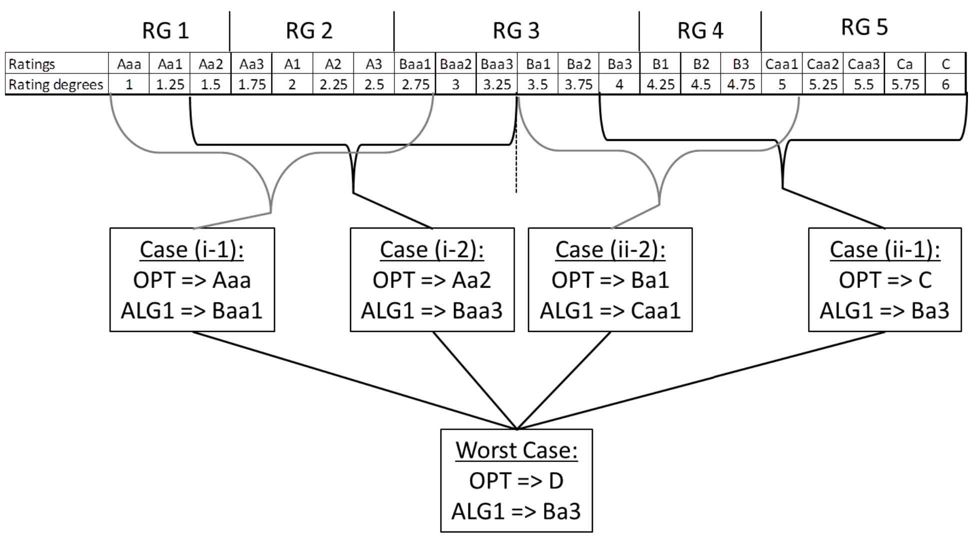

Figure A.2. Illustration of Scenario 2.

Worst-case situations for Scenario 3 if types of information 1 and 2 are known

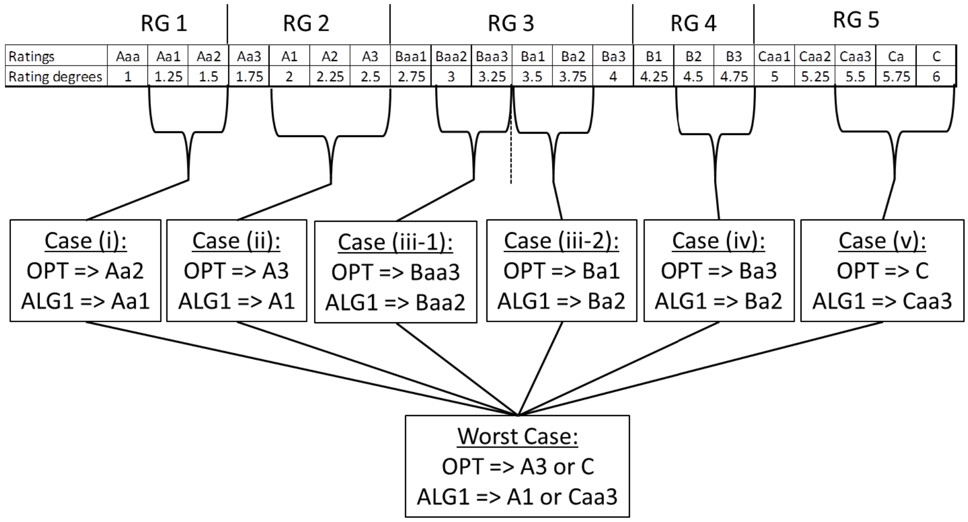

FIGURE A.3. Illustration of Scenario 3.

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