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SOLVING THE BI-OBJECTIVE ROBUST VEHICLE ROUTING PROBLEM WITH UNCERTAIN COSTS AND DEMANDS *

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Abstract. In this paper, a bi-objective Vehicle Routing Problem (bi-RVRP) with uncertainty in both demands and travel times is studied by means of robust optimization. Uncertain demands per customer are modeled by a discrete set of scenarios representing the deviations from an expected demand, while uncertain travel times are independent from customer demands. Then, traffic records are considered to get discrete scenarios to each arc of the transportation network. Here, the bi-RVRP aims at minimizing the worst total cost of traversed arcs and minimizing the maximum total unmet demand over all scenarios. As far as we know, this is the first study for the bi-RVRP which finds practical applications in urban transportation, *e.g.*, serving small retail stores. To solve the problem, different variations of solution approaches, coupled with a local search procedure are proposed: the Multiobjective Evolutionary Algorithm (MOEA) and the Non-dominated Sorting Genetic Algorithm (NSGAII). Different metrics are used to measure the algorithmic performance, the convergence, as well as the diversity of solutions for the different methods.

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1. Introduction

The Vehicle Routing Problem (VRP) is one of the most studied combinatorial optimization problem. The aim of this NP-hard problem is to determine a set of routes in which a set of customers is to be served by a fleet of vehicles with a fixed capacity, based at a depot node. Different variations on the components of the VRP have been studied over the years *e.g.*, the network, the demands, the objectives, and their nature, *i.e.*, deterministic or uncertain. In particular, we are interested in solving the bi-objective Robust Vehicle Routing Problem with uncertain demands and travel times. As presented in [5], several approaches such as stochastic programming are available to model uncertainties, where uncertain data is modeled as random variables [25,32]. However, such approaches are limited to the cases where uncertainties have a stochastic nature (which is not always the case) and when it is possible to identify the probability distribution [2]. Robust optimization is an

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alternative approach to stochastic programming, designed to avoid these drawbacks, in which the optimization seeks to protect against the worst scenarios that might arise in the future.

Robust optimization is used here to deal with the bi-objective Robust Vehicle Routing Problem with uncertainties (bi-RVRP). Readers are referred to [7] as a key-article for including robustness in multi-objective optimization. The bi-RVRP can be formally defined on a complete digraph $G = (V, A)$ with a set $V = \{0, 1, 2, \ldots, n\}$ of $n + 1$ vertices (customers), including the depot (0), and a set $A = \{(i, j)|i, j \in V, i \neq j\}$ of arcs. Uncertain data for the travel times are modeled as a set of p discrete scenarios $W = \{1, 2, \ldots, p\}$, where each scenario $k \in W$ specifies one cost $c_{ij}^k \in \mathbb{R}$ for every arc $(i, j) \in A$. In practice, one scenario can be observed in the streets of a city at a given hour (a kind of picture of the network). An expected demand d_i^* is associated with each customer $i \in V$, and its variation is defined in a set of o discrete scenarios $O = \{1, 2, \ldots, o\}$, where each scenario $b \in O$ specifies one demand d_i^b for each customer i. Split deliveries are not allowed. A fleet of identical vehicles $F = \{1, 2, \ldots, m\}$ is available at the depot, each one with a capacity equal to Q. A solution is a set of vehicle routes starting and ending at the depot, visiting customers following the service policy defined below. The bi-RVRP aims at minimizing both the worst total cost of traversed arcs and the maximum total unmet demand, over all scenarios. For a planned route, the following service policy is applied when arriving at a customer i under scenario b and discovering the real demand d_i^b: if the total amount already delivered, plus d_i^b, does not exceed vehicle capacity, then the customer is served, otherwise it is not served and the route proceeds with the next planned customer. Note that the scenarios around the expected demand in O are independent from the ones in W. Hence, to evaluate a given set of routes, we minimize the worst total travelling cost over all scenarios and the total unmet demand according to the service policy.

The following contributions are given in this study: (i) we model the bi-RVRP as a robust optimization problem taking into account uncertainties in demands and travel times. Uncertainties are addressed in the objective function following the min-max optimization criterion; (ii) the Multiobjective Evolutionary Algorithm (MOEA) and the Non-dominated Sorting Genetic Algorithm (NSGAII) together with a local search procedure are applied to solve the bi-RVRP taking into account the uncertainties in demands and travel times; (iii) uncertain travel times are handled as a bounded set of discrete scenarios on a directed network; one may note that arc costs become asymmetric and they can represent different traffic conditions for two-way roads; (iv) uncertainties over the demands are considered in a discrete set representing the possible variations for demands per client.

The remaining of this work is organized as follows: a bibliographical review is introduced in Section 2, followed by definitions and one example for the bi-RVRP in Section 3. The proposed multiobjective evolutionary metaheuristics are detailed in Section 4. Then, a local search procedure is described in Section 5. Finally, the computational experiments and concluding remarks are respectively given in Sections 6 and 7.

2. Related works

In the literature, robust optimization is considered to solve the vehicle routing problem mainly to handle uncertainty on time windows, travel times, travel costs and demands, giving a problem called the Robust Vehicle Routing Problem (RVRP). Nevertheless, just a few works have focused on uncertainties in multiple parameters of the VRPs. Moreover, there is a lack of studies dedicated to multiple uncertain parameters addressed by means of multiobjective approaches. In this section, past contributions for RVRPs are described, considering single and multiple parameters under uncertainty. Table 1 summarizes the works on RVRP where columns stand for the authors, the methods, the uncertainty representation, and the location of uncertain data in the mathematical model, respectively.

RVRPs have been mostly investigated for the case where uncertainties are associated with demands. For instance, authors in [29] consider that demand vectors are deviations from an expected demand value, belonging to different bounded sets. Their robust formulation handles the uncertain demands in the constraints following the robust approach counterpart introduced by [2]. The open source branch-and-cut-based VRP found in SYMPHONY library is implemented for solving the problem. Experiments have addressed three different sets

TABLE 1. Vehicle routing problems with uncertainty.

Authors	Methods	Uncertainty/Representation			Uncertain data model
		Time windows	Travel cost/Times	Demand	
Solano-Charris *et al.*, 2015[27]	Local Search Based Metaheuristics		Discrete scenarios [15]		Objective function
Gounaris *et al.*, In press [10]	Adaptive Memory Programming			Budget uncertainty [2]	Constraints
Cao *et al.*, 2014 [4]	Differential evolution algorithm			Bounded set [3]	Constraints/ Objective function
Toklu *et al.*, 2013 [31]	Multiple Ant Colony System		Budget uncertainty [3]		Objective function
Toklu *et al.*, 2013 [30]	Ant Colony System		Budget uncertainty [3]		Objective function
Han *et al.*, 2013 [12]	B&C algorithm		Budget uncertainty [3]		Constraints/ Objective function
Gounaris *et al.*, 2013 [11]	B&C algorithm			Bounded set [3]	Constraints
Agra *et al.*, 2013 [1]	Cutting plane Technique	Bounded set [3]			Constraints
Noorizadegan *et al.*, 2012 [20]	B&C algorithm			Bounded set [2, 3]	Constraints
Lee *et al.* 2012 [17]	Dantzig-Wolf Decomposition Approach	Budget uncertainty [3]			Demands, travel times, and time windows in constraints/ travel times in objective function
Moghadam *et al.* 2012 [18]	Particle Swarm Optimization			Interval set [2]	Constraints
Ordóñez, 2010 [21]	Analytical		Bounded set [2]		Demands and travel times in constraints/ travel times in objective function
Sungur *et al.* 2008 [29]	B&C algorithm			Bounded set [2]	Constraints

of instances, from the literature: random, clustered and modified ones, ranging from 15 to 100 customers. Results indicate that robust solutions can support the variation on the demands, while incurring a small additional cost compared to the deterministic version.

Authors in [18] consider the robust counterpart to handle uncertain distributions of the customers' demands. A hypothesis of this study is that distribution for demands are unknown. Then, a perturbation percentage from the nominal demands is computed over the constraints, while the objective is to minimize the travel distances. A Particle Swarm Optimization is presented and results are compared with the ones obtained by [29].

The VRP with heterogeneous fleet and uncertain demands is studied by [20]. The authors provide mathematical formulations based on the robust counterpart of the linear program and chance-constrained programming with uncertainties on the right side of the constraints. A Branch-and-Cut (B&C) algorithm was proposed and instances with up to 20 clients are tested using CPLEX under default parameters. Results are analyzed using the extra cost required to achieve a certain level of feasible routes, number of unmet demands and recourse costs, which is the extra cost, in case of failure, of returning to the depots for replenishment and resuming the route.

Another study of the RVRP with uncertainties on demands is presented in [11]. The robust optimization counterpart for several formulations and the robust rounded capacity inequalities are developed. The models consider customer demands as random variables on the right-hand side of constraints and it determines the minimum cost delivery plan that is feasible for all anticipated demands realization. CPLEX is used to solve 90 instances from 15 to 135 customers: its generic cuts are deactivated and replaced by Rounded Capacity Inequalities (RCI) cuts. The results show that the best robust formulation, the Two-index Vehicle Flow formulation (2VF), is improved even further if RCI cuts are used. Most instances with up to 50 nodes are solved to optimality and the average gap for the other instances is below 5%.

The Open VRP with uncertain demands is introduced by [4]. A bounded uncertainty set for customer demands is described and transportation costs and unsatisfied demands in the specific bounded uncertainty set are minimized. A differential evolution algorithm is proposed and its performance is analyzed on different strategies by considering the extra costs and unmet demand.

The most recent work considering uncertain demands is presented in [10]. The robust formulation allows uncertain customer demands, and the objective is to determine a minimum cost delivery plan that remains feasible for all demand realizations within a prespecified uncertainty set. The authors implemented an adaptive memory programming metaheuristic using two classes of uncertainty sets. Computational experiments on benchmark intances with up to 483 customers and 38 vehicles are retrieved and new best solutions for a total of 123 benchmark instances are found.

As presented in [26] and illustrated in Table 1, only a few works have dealt with uncertain travel times or travel cost. A robust scenario approach for the vehicle routing problem with uncertain travel times is studied by [12]. Multiple range forecasts in a set of time intervals are assumed. Uncertainty is handled by limiting the number of uncertain parameters on the constraints allowed to deviate from the nominal values [3]. Then for each realization (scenario) on an interval set of travel time, a robust route is identified, and the minimization of the worst case among all scenarios is applied. A two-stage recourse stochastic programme solved by a Branch-and-Bound (B&B) algorithm is used. Tests with instances with up to 25 customers are solved assuming normal and severe traffic conditions.

The works [30, 31] handled the VRP with uncertain travel costs. The total travel cost is minimized and uncertainty is expressed as intervals. The approach from [3] is implemented to control the degree of uncertainty on the model. The ant colony system and its multiple versions are introduced in [30,31], respectively, and tested on instances with up to 150 customers with different conservativeness degree configurations.

An approach for the RVRP with uncertain travel times has recently been studied by [27]. The set of arc costs is replaced by a set of discrete scenarios and the main objective is to build a set of routes using the lexicographic min-max criterion from [22]. Thus, the worst cost over all scenarios is minimized and ties are broken using the other scenarios from the worst to the best. Different methods are proposed, *i.e.*, a Greedy Randomized Adaptive

Search Procedure (GRASP), an Iterated Local Search (ILS), a Multi-Start ILS (MS-ILS), and a MS-ILS based on giant tours. Test on instances with up to 100 clients and 30 scenarios are provided.

Concerning multiple parameters under uncertainty, the article [21] takes into account uncertain demands, travel times, costs and customers. The author outlines different robust models depending on the source of uncertainty, the VRP formulation, and correlation between uncertain coefficients. Uncertainty in travel costs is introduced in the objective function, and uncertain demands and travel time coefficients in the constraints. A convex and bounded uncertainty set is estimated and the problem is solved using the robust counterpart approach. Results for small instances are provided and compared with the ones obtained by the chance constrained and the stochastic with recourse models.

Finally, uncertainties on both travel times and demands are found in [17] for a VRP with customer deadlines. The budget uncertainty from [3] is defined by limiting the sum of deviations considering travel times, and demands to their nominal values. The robustness of a solution is achieved by looking for a feasible solution for any travel time and demand defined in the uncertainty sets which minimizes the travel time. A set-partitioning formulation is proposed and solved with a column generation. The uncertainties are limited to the column generation subproblem using a robust version of a shortest path algorithm with resource constraints. Instances with 20 to 40 customers are solved to optimality. As far as we know, there are no works addressing the multiobjective approach for the RVRP with uncertainties in both demands and travel times in the objective functions.

3. Notation, definitions and example

This section introduces the notation used throughout this article which is summarized on Table 2 and describes the definitions considered for the bi-RVRP.

Without loss of generality, let us consider a minimization problem. A multiobjective optimization problem can be described in mathematical terms as follows:

$$\mathscr{P} = \min\{Z_1(x), Z_2(x), \ldots, Z_l(x)\} \tag{1}$$

Where the number of objectives (l) to minimize is higher than one ($l > 1$) and the solution x satisfies the set of constraints. Then, the "optimality" in the multiobjective sense is handled by means of Pareto optimality defined by the concept of dominance [8].

As mentioned before, the bi-RVRP is defined in a complete digraph $G = (V, A)$. Uncertain data for the travel times and demands are modeled as a set of discrete scenarios.

The proposed bi-RVRP is stated in a 2-dimensional space with the total worst cost Z_1 given in equation 2 and the worst unmet demand Z_2 presented in equation 3, over all scenarios. The objective function for the bi-RVRP is $\min(Z_1, Z_2)$.

$$Z_1 = \max_{k \in W} \sum_{(i,j) \in A} c_{ij}^k x_{ij} \tag{2}$$

$$Z_2 = \max_{b \in O} \sum_{i \in V} d_i^b y_i \tag{3}$$

The binary variable x_{ij} is equal to 1 if arc (i, j) is traversed by a vehicle, otherwise $x_{ij} = 0$. And, y_i is the boolean variable that specifies if a vehicle can attend a client $y_i = 1$ or not $y_i = 0$, according to the vehicle capacity Q and the real demand of client i. The dominance relation for Z_1 and Z_2 in the Pareto sense is defined for the bi-RVRP as follows. A solution ω' dominates solution ω, if $Z_1(\omega') \leq Z_1(\omega)$ and $Z_2(\omega') < Z_2(\omega)$, or $Z_1(\eta) < Z_1(\omega)$ and $Z_2(\eta) \leq Z_2(\omega)$. A solution is non-dominated if no other solution dominates it. The Pareto-optimal front is given by the set of non-dominated robust solutions.

Table 2. Notation for the bi-RVRP.

Notation	
G	Complete digraph
V	Set of vertices
A	Set of arcs
n	Customers
W	Set of discrete scenarios for travel times
p	Number of scenarios for travel time
k	Index of scenario for travel time
c_{ij}^k	Cost for each arc (i, j) in scenario k
d_i^*	Expected demand for customer i
O	Set of discrete scenarios for demands
o	Number of scenarios for demand
b	Index of scenario for demand
d_i^b	Real demand of customer i in scenario b
F	Set of m vehicles
Q	Vehicles capacity
x_{ij}	Binary variable equal to 1 if arc (i, j) is traversed by a vehicle, otherwise 0
y_i	Binary variable equal to 1 if a vehicle serves a client i, otherwise 0
Z_1	Worst cost
Z_2	Worst total unmet demand
ω, ω'	Solutions
$cost(\omega, k)$	Cost of solution ω for scenario k
$unmet(\omega, b)$	Unmet demand of solution ω for scenario b
r	Number of solutions or chromosomes
Δ	Cost variation of a solution for scenario k
u, v, w	Nodes
P_1, P_2	Parents
C	Offspring
Pop	Size of the population
Π	Front
ρ	ρth solution in a front
$weight$	Pseudo-weight
φ	Rate for no-systematic local search
x_i, y_i	Coordinates of each customer
d_{tot}	Total demand of customers
$dist_{ij}$	Euclidean distance for arc (i, j)
θ	Deviation factor to the baseline distance
$\bar{d}^*$	Average demand per customer
β	Global perturbation for demands
$[\tau_i^-, \tau_i^+]$	Interval values for demands
ϵ	Percentage of customers allowed to change from their expected value for the demands
Θ	Number of solutions in the Pareto front
S	Spacing
H	Hypervolume
t	Running time
vol	Hypercube
Θ'	Average value for the number of solutions in the Pareto front
S'	Average value for the Spacing
H'	Average value for the Hypervolume
t'	Average value for running time

(a) Solution with expected values (b) Robust solution

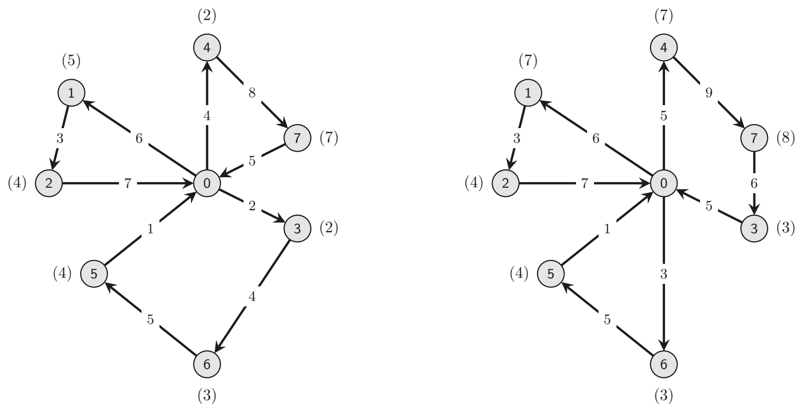

FIGURE 1. Examples of solutions for VRP and bi-RVRP.

TABLE 3. Summary of results for the VRP (*left*) and the bi-RVRP (*right*).

Route	Cost (Z_1)	Un. Demand (Z_2)
0-1-2-0	16	0
0-4-7-0	17	0
0-3-6-5-0	12	0
Total	45	0

Route	Cost (Z_1)	Un. Demand (Z_2)
0-1-2-0	16	4
0-4-7-3-0	25	8
0-6-5-0	9	0
Total	50	16

The bi-RVRP is illustrated by a simple example in Figure 1. The problem considers a single depot (0) and 7 clients to be served by 3 vehicles with capacity equal to 10. Figure 1 shows the distances between customers (indicated in the arrows) in normal traffic conditions in Figure 1a and with perturbated travel times in Figure 1b, together with the demand per client (in brackets).

The summary of results can be found in Table 3. The set of routes for a solution constructed in normal conditions uses the expected value for demands and travel times while dealing with uncertainties. The perturbated scenarios for travel times and demands are handled and the total cost Z_1 and unmet demand Z_2 are calculated. In Figure 1b, one perturbated scenario is considered. The solution is composed of the following three routes, the route sequence 0-1-2-0, a second one that follows the route 0-4-7-3-0 and the last one with route 0-6-5-0. Then, over the perturbated demands for route 0-1-2-0 the vehicle cannot satisfy demand for client 2 dealing with 4 units of unmet demand and a total cost of 16. In Route 0-4-7-3-0, the demands of clients 4 and 3 are satisfied for a total cost of 25, while client 7 is skipped (unmet demand: 8). Finally, for route 0-6-5-0 all clients are served and the total cost for the route is 9. A similar evaluation is done for the VRP (Figure 1a), but in this case a total cost of 45 is found and all clients are attended. For each solution we totalize the cost and the unmet demand obtaining a total cost of 45 and 0 unmet demand in the VRP, and in the RVRP a total cost of 50 with 16 units on the unmet demand. In the case we deal with more scenarios, the maximum values after evaluating Z_1 and Z_2 over all scenarios are minimized.

4. MULTIOBJECTIVE ALGORITHMS FOR THE BI-RVRP

After the first studies on evolutionary multiobjective optimization (EMO), different multiobjective genetic algorithms (MOGAs) have been proposed along the time. Selecting the best multiobjective algorithm for solving the bi-RVRP is not straightforward. In this paper, we consider the standard version of MOEA and NSGAII,

Table 4. Example of chromosome.

0	1	2	0	4	7	3	0	6	5	0

mainly for their flexible structure and successful applications on multiobjective combinatorial optimization problems [14,16,23]. The following subsections describe in detail the shared components and the adaptation of both methods.

4.1. Shared components of the bi-objective metaheuristics

4.1.1. Chromosome and evaluation

Each solution for the bi-RVRP is encoded as a set of routes, in which the customers appear in the order they are visited (see Tab. 4). Delimiters are used as a separator between different routes (grey color). The ranking procedure for the bi-RVRP initially evaluates the cost and the unmet demand over all scenarios for each chromosome. Let $cost(\omega, k)$ be the cost of solution ω for scenario k and $unmet(\omega, b)$ the unmet demand of solution ω for scenario b, where k stands for the index of the scenario on travel times and b the index scenario for the demands. Then, the maximum total cost, $\max\{cost(\omega, k) \mid k \in W\}$ and the maximum unmet demand, $\max\{unmet(\omega, b) \mid b \in O\}$, are stored with the chromosome. Considering these computed values for each individual, a second evaluation is performed to classify all individuals in the population, following a minimization.

4.1.2. Initial population

The generation of the initial population Pop composed of r chromosomes is built using a random generation of solutions and an adaptation of the best insertion heuristic. The first $r/2$ individuals (rounded up) to enter in Pop are built using the best insertion heuristic and the remaining individuals are randomly generated. The principle of the best insertion heuristic from [28] is to create a set of routes by selecting unserviced customers and inserting each of them in one of the partial routes already created. The heuristic builds routes in parallel, by initially selecting a customer i to be inserted in the first route, a second customer to be included in the second route, *etc.* until m initial customers (seed customers) have been assigned to each route. Then, unserviced clients are sorted in decreasing order of nominal demands and inserted one by one in the emerging routes in this order. We determine the cheapest insertion of the incumbent client v after each node u, while respecting vehicle capacity. If v is inserted between nodes u and w, the insertion cost for scenario k (or cost variation of the solution for this scenario) is $\Delta = c_{uv}^k + c_{vw}^k - c_{uw}^k$. For the bi-RVRP, the initial solutions use one cost scenario k selected at random, for the travel time, and the nominal demands (*i.e.*, the scenarios for demands are not considered). Since demands cannot be split, the insertion heuristic can fail to use only m vehicles. Thus, extra vehicles can be added to the solution in order to ensure each customer is visited by a vehicle. This violation on the number of vehicles is not repaired, and is left to be managed by MOEA and NSGAII. On the random generation, unserviced customers are assigned sequentially to a first route. When no other customer can be added, a second route is created. The procedure stops when all customers are attended by one vehicle.

4.1.3. Crossover operator

The ordered crossover (OX) introduced by [6] is applied. As the classical version of the OX does not consider trip delimiters, a modification on the procedure is implemented. Initially, the selection of parents P_1 and P_2 follow the binary tournament selection [9]. For this purpose, two random solutions are compared. Then, the solution with the smallest rank is kept as P_1. This process is repeated to select P_2. Then, the main idea of the OX is to select a sequence of customers from P_1 and insert it relating to the best insertion in P_2. Repeated customers are dropped from the offspring C to ensure that each customer i is visited once.

4.2. MOEA and NSGAII for the bi-RVRP

The standard MOEA and NSGAII have been applied to solve the bi-RVRP. For the sake of clarity, the standard MOEA developed here relies on an evolutionary genetic algorithm using the ranking operator to classify solutions in fronts at each iteration of the algorithm. Half of the initial population for both metaheuristics is randomly generated and half is created by means of the best insertion heuristic, as mentioned in Section 4.1.2. Solutions are then classified using the ranking operator. The population is renewed using the crossover OX and the binary tournament selection. The structure of the MOEA and the NSGAII are detailed below.

4.2.1. NSGAII

NSGAII proposed by [8] is presented in Algorithm 1. The general structure computes successive generations of solutions into non-dominated fronts. The non-dominated set is identified and constitutes the level 1 or front 1. The partition of the population is completed according to the level of non-domination. NSGAII uses a ranking and a crowding distance as operators to classify the solutions in the population. The crowding comparison procedure gives the density of solutions in the neighborhood, whereas the ranking allows to set the solutions in a Pareto front on the search space. At each iteration of the algorithm, r solutions are generated. The solutions are sorted and set in successive fronts of non-dominated solutions. Thus, the ranking (line 3) and the crowding distance (line 4) for each solution is computed. Considering our two objectives for the bi-RVRP and a front $\mathit{\Pi}$, the crowding distance is computed as follows:

$$\mathit{Crowding}(\mathit{\Pi}(\rho)) = \frac{Z_1(\mathit{\Pi}(\rho)+1) - Z_1(\mathit{\Pi}(\rho)-1)}{Z_1^{\max} - Z_1^{\min}} + \frac{Z_2(\mathit{\Pi}(\rho)-1) - Z_2(\mathit{\Pi}(\rho)+1)}{Z_2^{\max} - Z_2^{\min}} \tag{4}$$

Where $\mathit{\Pi}(\rho)+1$ and $\mathit{\Pi}(\rho)-1$ are respectively the successor and the predecessor values for the ρth solution in the sorted front. For the extreme points in all fronts, the $\mathit{crowding}(\mathit{\Pi}(\rho)) = \infty$. The goal is to favor the extreme points on the selection.

In order to generate a new solution, two parents P_1 and P_2 are selected (line 8) considering the binary tournament selection. Then, the ordered crossover (OX) is applied to obtain an offspring C (line 9). The new offspring C is added to the population if it is unique (line 10). At the end of each iteration, the population contains $2r$ solutions. After a non-dominated ranking on the $2r$ solutions, only the best r solutions are kept for the next iteration (line 15). The procedure is repeated until a prespecified number of iterations.

Algorithm 1. Non-dominated sorting genetic algorithm for the bi-RVRP.

1: **Require** $G = (V, A), m, Q, W, d_i^* \; \forall i \in V, O$
2: Initialize Pop with r
3: $Pop \leftarrow$Non-Dominated Ranking(Pop)
4: Get crowding distance(Pop)
5: **repeat**
6: //Generation of r new solutions
7: **while** $\mid Pop \mid < 2r$ **do**
8: Choose P_1 and P_2
9: $C \leftarrow$ Crossover(P_1, P_2)
10: Add C to Pop if C is not a clone
11: **end while**
12: //Create a new population
13: $Pop \leftarrow$Non-Dominated Ranking(Pop)
14: Get crowding distance(Pop)
15: Reduce Pop to its best r solutions
16: **until** stopping condition

4.2.2. The MOEA

A modified version of the standard MOEA introduced by [13] is proposed for solving the bi-RVRP. The general structure of this MOEA is provided in Algorithm 2. The structure is similar to NSGA-II (Algorithm 1), but in the standard MOEA solutions are sorted using only the ranking procedure. Then, lines 4 and 17 in Algorithm 1 are removed, and in the selection process between two solutions with different non-domination ranks, the MOEA prefers the one with the lower rank. Otherwise, if both solutions belong to the same front the selection is indifferent on which one is selected.

Algorithm 2. Multiobjective evolutionary algorithm for the bi-RVRP.

Require $G = (V, A), m, Q, W, d_i^* \; \forall i \in V, O$
  Initialize Pop with r
  $Pop \leftarrow$Non-Dominated Ranking(Pop)
repeat
  //Generation of r new solutions
  while $\mid Pop \mid < 2r$ **do**
    Choose P_1 and P_2
    $C \leftarrow$ Crossover(P_1, P_2)
    Add C to Pop if C is not a clone
  end while
  //Create a new population
  $Pop \leftarrow$Non-Dominated Ranking(Pop)
  Reduce Pop to its best r solutions
until stopping condition

5. Local search

The general definition of the standard MOEA and the NSGAII does not include local search procedures. Nevertheless, a local search procedure is added to improve the results and to accelerate the convergence. The cost and the unmet demand of a solution ω are computed for each scenario if the move is performed and a first improvement strategy is applied. The whole local search stops when all neighborhoods are explored without finding any improvement. *Relocation*, *Interchanges*, *2-opt*, and a modification of *2-opt* moves are developed for the bi-RVRP. For instance, *relocation* applied intra-routes moves one or two continuous customers to a different position. *Interchange* permutates 1 or 2 consecutive customers from the same route. Moreover, *2-opt* inverts the sequence of customers from i to j when it is applied to intra-routes (j must be after i). When the *2-opt* is applied to two different routes, the vehicle capacity constraint must be ensured before computing the variations on the cost and the unmet demand. In addition, *a modification over* 2-opt is also applied to consider the asymmetric cost when a route is inverted. For this move the customers sequence in a route before i and after j are inverted.

5.1. Acceptance criteria of moves in the local search

Deciding to perform a move on the previous presented neighbourhoods is a little more complicated for the bi-RVRP than for a mono-objective RVRP, since the evaluation relies on a two-dimentional space considering minimizing both, the worst total cost and the worst total unmet demand over all scenarios. Thus, this section clarifies how to efficiently perform such evaluation. In the following, the criteria used to decide if a move $\omega \rightarrow \omega'$ improves the incumbent solution are as follows:

LS1: Pareto dominance. Solution ω' dominates solution ω if ($Z_1(\omega') \leq Z_1(\omega)$ and $Z_2(\omega') < Z_2(\omega)$, or $Z_1(\omega') < Z_1(\omega)$ and $Z_2(\omega') \leq Z_2(\omega)$).

LS2: Convex combination. A move to be accepted must improve the function $weight \cdot [Z_1(\omega') - Z_1(\omega)] + (1 - weight) \cdot [Z_2(\omega') - Z_2(\omega)]$ with $weight \in [0, 1]$. The main objective is to improve solutions in the first front

TABLE 5. Versions for the evolutionary metaheuristics.

Method	Description	Acceptance criteria
MOEA(1)	without local search	n.a.
MOEA(2)	local search on new solutions	LS1
MOEA(3)	local search on first front	LS2
MOEA(4)	local search on new solutions+local search on first front	LS1+LS2
NSGAII(1)	without local search	n.a.
NSGAII(2)	local search on new solutions	LS1
NSGAII(3)	local search on first front	LS2
NSGAII(4)	local search on new solutions+local search on first front	LS1+LS2

conducting the search by descending values with emphasis on the extreme solutions, while preserving the space between solutions [19]. Therefore, the pseudo-weight *weight* must be computed for a given solution ω. Authors in [19] use Equation 5 in which $Z_1^{\min}$, $Z_1^{\max}$, $Z_2^{\min}$ and $Z_2^{\max}$ are respectively the minimum and the maximum values of each criterion. This formula gives diverging directions when applied to non-dominated solutions.

$$weight = \frac{\dfrac{Z_1(\omega) - Z_1^{\min}}{Z_1^{\max} - Z_1^{\min}}}{\dfrac{Z_1(\omega) - Z_1^{\min}}{Z_1^{\max} - Z_1^{\min}} + \dfrac{Z_2(\omega) - Z_2^{\min}}{Z_2^{\max} - Z_2^{\min}}} \tag{5}$$

Since this criterion may change the fronts, it must be followed by a second call of the Non-Dominated Ranking(Pop).

5.2. Integrating the local search on multiobjective heuristics

The local search can be applied in different positions of the multiobjective evolutionary metaheuristics according to a systematic or no-systematic search. The systematic search implies that the local search procedure is applied to all iterations, and the no-systematic is applied according to a fixed rate. Table 5 summarizes the variations on the position of the local search together with the acceptance criterion applied. Regarding MOEA(2) and NSGAII(2), a no-systematic local search on children is applied with a fixed rate φ and introduced after line 9 in Algorithm 1. In MOEA(3) and NSGAII(3), a systematic local search is incorporated after line 13 in Algorithm 1 and applied on solutions in front 1. The last consideration found in MOEA(4) and NSGAII(4) consists of combining the no-systematic and the systematic local search from version 2 and 3 of the MOEA and the NSGAII. This means that the no-systematic local search applied on children with LS1 as acceptance criterion and the systematic local search applied on the solutions in front 1 with LS2 as acceptance criterion are both applied.

6. COMPUTATIONAL EXPERIMENTS

The computational experiments were performed on an HP ZBook, Intel Core i7-4800MQ, 2.7 GHz with 16GB of RAM. The proposed metaheuristics were developed in C++ on Visual Studio Professional 2013. The following subsections describe the instances used and the results.

6.1. Test instances

The computational experiments are based on two sets of random-instances called set A and set B. Set A contains instances for which the total demand for all clients sligtly differs from the total expected demand d_{tot} for all clients. Thus, after computing the total deviation from the total expected demand, a portion is randomly selected for individual clients. The two sets contain 24 instances with $n = \{30, 40, 50\}$ customers, $m = \{3, 4, 5\}$,

$p = \{20, 30\}$ and $o = \{3, 5\}$. They are generated as follows: the customers are randomly placed according to an uniform distribution in the main square, *i.e.*, the coordinates x_i and y_i of each customer i are randomly selected in $[0, 1000]$. We ensure that $\mid x_i - x_j \mid \geq 1$ and $\mid y_i - y_j \mid \geq 1$. All node coordinates are recorded with the instance, as real numbers. Concerning the depot node 0, the main square is divided into 3×3 smaller squares and this node is randomly placed in the central square, *i.e.*, its coordinates x_0 and y_0 are drawn in the interval $[333.33, 666.67]$. For each scenario on travel time k and each arc (i, j), the arc cost is randomly drawn in $[dist_{ij}, \theta \times dist_{ij}]$, where $dist_{ij}$ denotes the Euclidean distance and $\theta \in \{1.1, 1.5, 2\}$ is a maximum deviation factor to this baseline distance. The arc costs are rounded to the closest integer.

The nominal or ideal values of the demands are generated assuming a vehicle capacity Q. The average demand per customer is first computed as $\bar{d}^* = Q/m$ and then the demand d_i^* of each customer i is randomly set as an integer in $[1, 2\bar{d}^*]$. These demands are finally adjusted to obtain a total demand d_{tot} such that $d_{\text{tot}} \cong 90\% \times mQ$, *i.e.*, 90% of the total capacity available. Then, a customer is randomly selected and its demand is diminished (if $d_{\text{tot}} > 90\% \times mQ$) or augmented (if $d_{\text{tot}} < 90\% \times mQ$) by a random integer between 1 and 5 (included). This process is repeated until 10% of spare capacity is obtained. The file names recall the values n–m–p–θ–o.

In particular, for set A, the discrete scenarios on demands are generated according to a global perturbation β of the total demand d_{tot} in percent, where $\beta = \{5\%, 25\%\}$. Thus, the individual variation for the demand on each client i is random selected and limited by the global perturbation such as $\sum_{i \in V} d_i = \beta\ d_{\text{tot}}$.

For set B uncertainties on demands are defined between a lower and an upper value $[\tau_i^-, \tau_i^+]$, respectively, per each customer i. Here, the interval variation is fixed according to a range of $[-50\%, 100\%]$ over d_i^* and the number of clients allowed to change from their expected value of the demand is limited by a percentage $\epsilon = 50\%$.

6.2. Evaluation criteria

The performance of the proposed metaheuristics is gives through four metrics: the number of solutions in the Pareto front (Θ), the distribution of solutions in the Pareto front called spacing (S), the hypervolume (H) and the running time (t) in seconds. The spacing proposed by [24] estimates the relative distance between two consecutive solutions obtained in the non-dominated set, as follows:

$$S = \sqrt{\frac{1}{|\Theta|} \sum_{i=1}^{|\Theta|} (\gamma_i - \bar{\gamma})^2} \tag{6}$$

Where γ_i minimizes the sum of the absolute difference in objective function values between the ith solution and any other solution in the non-dominated set. Moreover, $\bar{\gamma}$ is the mean value of the above distance measure. When solutions are near uniformly spaced, the corresponding distance measure will be small. Thus, an heuristic having a smaller spacing S is better.

The hypervolume introduced by [33] is the volume covered by members of Θ. Mathematically, for each solution $\omega \in \Theta$, a hypercube vol_i is constructed with a reference point. The reference point is defined by the maximal value of Z_1 and Z_2 in the initial population. Thereafter, a union of all hypercubes is found and its hypervolume H is calculated. An heuristic with a large value of H is desirable.

$$H = \text{volume}\left(\bigcup_{i=1}^{|\Theta|} \text{vol}_i\right) \tag{7}$$

The spacing and the hypervolume are not free from arbitrary scaling objectives. The above metrics are evaluated by using normalized objective functions in the interval [0, 1].

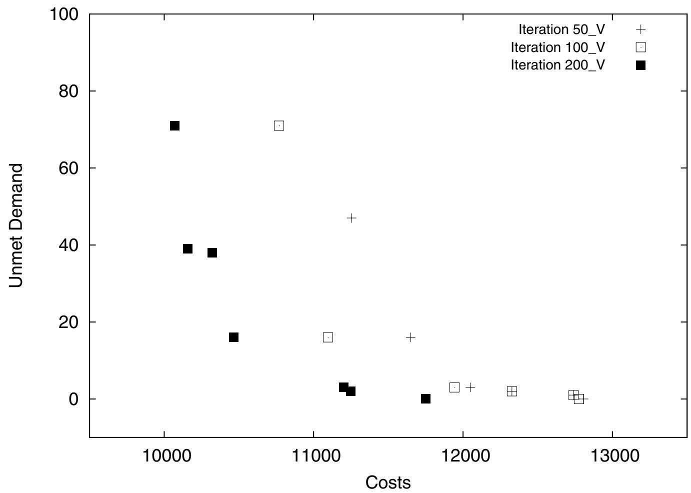

FIGURE 2. Example of convergence on NSGAII(1) for instance 30-3-30-100-3, $\beta = 5\%$ on set A.

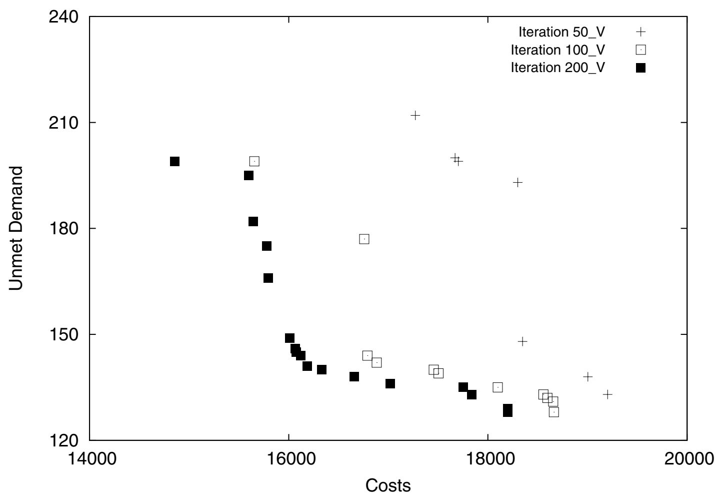

FIGURE 3. Example of convergence on NSGAII(1) for instance 50-5-20-100-5, $\beta = 25\%$ on set A.

6.3. Parameter tuning

For the set of instances, calibration with different parameter configurations is used to see which one produces the best results from the metaheuristics. To select the parameters, a fine-tuning procedure is used, beginning from a promising configuration using 100 and 200 iterations. The population size was defined according to the number of customers per each instance [23]. Moreover, the rate of local search is defined in $\varphi = 10\%$ when the no-systematic local search is applied.

Results showed that in general the best compromise between solution quality and running time is obtained with $r = |V|$ and 200 iterations achieving lower values for S and prevailing in average high values for Θ and H. According to these results, the same configuration of parameters is considered on the different variations of NSGAII and MOEA in order to have a fair comparison between both methods. Figures 2, 3, 4 present an example of the convergence for the NSGAII(1) on three representative instances, instance 30-3-30-100-3 with $\beta = 5\%$, instance 50-4-30-50-5 with $\beta = 25\%$ for the set A and instance 50-5-20-100-5 with $\epsilon = 50\%$ for the set B, showing best Pareto front after 50, 100, 200 iterations for a population size of $r = |V|$.

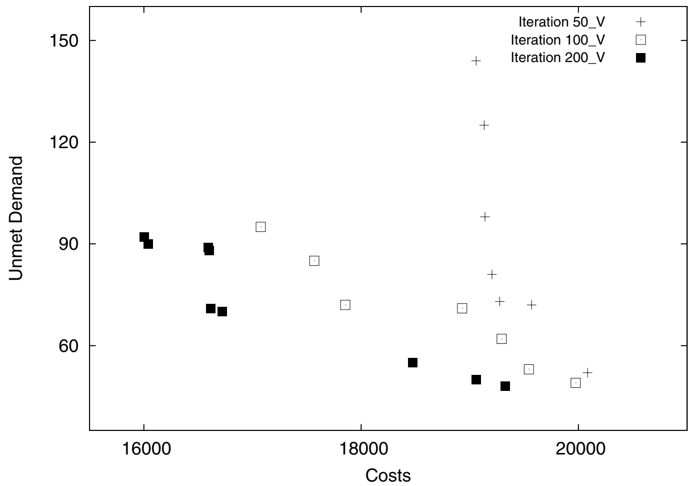

FIGURE 4. Example of convergence on NSGAII(1) for instance 50-5-20-100-5, $\epsilon = 50\%$ on set B.

TABLE 6. Average results for the set A.

Method	$\beta = 5\%$				$\beta = 25\%$			
	Θ'	S'	H'	t'	Θ'	S'	H'	t'
MOEA(1)	5.2	0.18	0.99	190.33	7.7	0.15	0.99	193.25
MOEA(2)	4.7	0.17	0.99	209.04	7.2	0.12	0.99	206.50
MOEA(3)	4.8	0.22	0.99	213.75	7.0	0.15	0.99	211.50
MOEA(4)	4.8	0.16	0.99	217.75	5.8	0.15	0.99	215.50
NSGAII(1)	5.6	0.15	0.99	193.33	8.3	0.10	0.98	196.00
NSGAII(2)	5.1	0.16	0.99	212.04	8.1	0.18	0.99	211.50
NSGAII(3)	4.5	0.20	0.99	215.75	6.9	0.18	0.99	213.50
NSGAII(4)	4.5	0.15	0.99	220.75	6.5	0.16	0.99	218.50

TABLE 7. Average results for the set B.

Method	$\epsilon = 50\%$			
	Θ'	S'	H'	t'
MOEA(1)	9.7	0.12	0.98	188.92
MOEA(2)	8.8	0.17	0.98	220.79
MOEA(3)	7.3	0.19	0.99	209.42
MOEA(4)	8.1	0.16	0.99	232.79
NSGAII(1)	9.7	0.12	0.98	190.92
NSGAII(2)	6.6	0.15	0.99	224.79
NSGAII(3)	6.6	0.19	0.99	222.79
NSGAII(4)	7.8	0.13	0.99	240.75

6.4. Results

The detailed results for each set of instances are listed from Tables A.1 to A.6 (see Appendix). Each line on the detailed results corresponds to an instance then, the metrics Θ, S, H and t are shown for MOEA and NSGAII. The detailed results are summarized in Tables 6 and 7, using the average values for the metrics (Θ, S, H, t) defined respectively by the headings Θ', S', H' and t', and graphic examples are also provided in order to see the behavior of methods with a fixed number of iterations and they are presented in Figures 5 and 6.

As can been seen in Tables 6 and 7, in terms of Θ, generally NSGAII found a higher number of solutions in the non-dominated Pareto front for set A giving to the decision maker a set of solutions with a good compromise

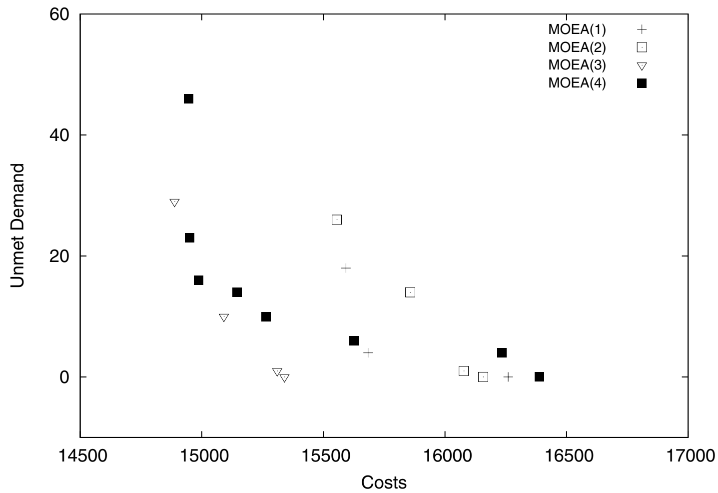

FIGURE 5. Impact of the different methods with a fixed number of iterations for instance 50-5-20-100-3 with $\beta = 5\%$ on set A.

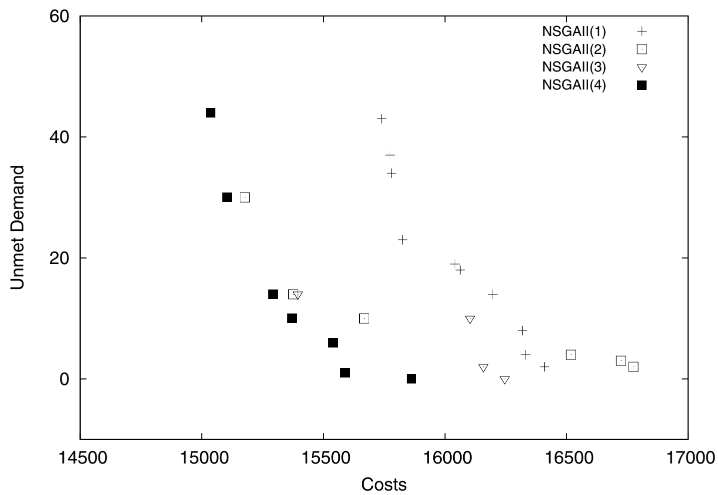

FIGURE 6. Impact of the different methods with a fixed number of iterations for instance 50-5-20-100-3 with $\beta = 5\%$ on set A.

between the worst total cost and the total unmet demand. Regarding the metric H, both methods have a good coverage in the objective space by solutions from the non-dominated fronts. In most of the cases, NSGAII has got better results for the metric S than MOEA over its different variations which induces a better distribution on the non-dominated pareto fronts. It is important to mention that the position of the solutions in the search space has a direct impact on the metric S. Even when the metrics H and Θ improve their values, that does not guarantee better values for S. As was expected the running time t differs on MOEA and NSGAII, due to their different operators to classify the population. Concerning the results achieved with $\beta = 5\%$, NSGAII(1) has obtained the most interesting results with $H' = 0.99$,the highest value for $\Theta' = 5.6$, and the lowest value found over all methods for S'. Regarding the results with $\beta = 25\%$, MOEA(2) produces relevant results, with $H' = 0.99$, Θ'=7.2, and S'=0.12. And with set B, significant results has retrieved by NSGAII(4) with $H' = 0.99$, $\Theta' = 7.8$, and $S' = 0.13$. However, the difference on the proposed methods is not significant,

TABLE 8. Average results for Time budget with set A.

Method	$\beta = 5\%$				$\beta = 25\%$			
	Θ'	S'	H'	$niter'$	Θ'	S'	H'	$niter'$
MOEA(1)	5.3	0.16	0.99	290	6.8	0.13	0.99	264
MOEA(2)	5.1	0.18	0.99	271	7.1	0.17	0.99	239
MOEA(3)	5.1	0.16	0.99	262	7.1	0.15	0.99	230
MOEA(4)	4.4	0.14	0.99	253	5.9	0.15	0.99	224
NSGAII(1)	5.8	0.13	0.99	282	7.9	0.12	0.98	258
NSGAII(2)	5.6	0.16	0.99	264	8.3	0.14	0.98	230
NSGAII(3)	4.6	0.20	0.99	257	7.2	0.17	0.99	227
NSGAII(4)	4.6	0.19	0.99	247	7.1	0.15	0.99	220

TABLE 9. Average results for Time budget with set B.

Method	$\epsilon = 50\%$			
	Θ'	S'	H'	$niter'$
MOEA(1)	9.5	0.13	0.98	181
MOEA(2)	8.4	0.14	0.98	184
MOEA(3)	7.0	0.19	0.99	116
MOEA(4)	7.1	0.18	0.99	148
NSGAII(1)	8.5	0.10	0.98	140
NSGAII(2)	7.9	0.15	0.98	135
NSGAII(3)	6.5	0.20	0.99	178
NSGAII(4)	7.1	0.15	0.99	162

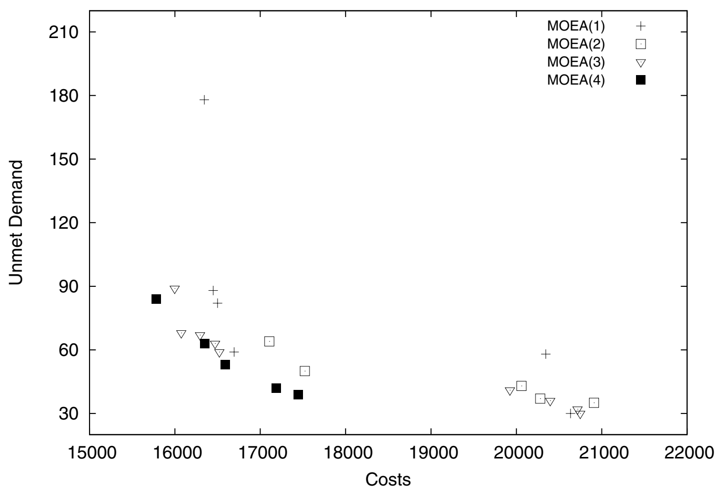

FIGURE 7. Impact of the different methods with a limited running time for instance 50-5-20-100-3 with $\epsilon = 50\%$ on set B.

therefore the behavior of the metaheuristics under a common time limit (200 s) has been evaluated using set A and B, to see their behavior the limited time. The results are summarized in Tables 8 and 9 considering the average values for the performance metrics and two graphic examples are also included considering the Time Budget (see Figs. 7 and 8). Apropos of the results, for set A with $\beta = 5\%$ NSGAII(1) is still a better than all the performance metrics with $H' = 0.99$, $\Theta' = 5.8$, and $S' = 0.13$. About results for instances with $\beta = 25\%$, NSGAII(4) produces interesting results ($H' = 0.99$, $\Theta' = 7.1$, and $S' = 0.13$) but comparable results

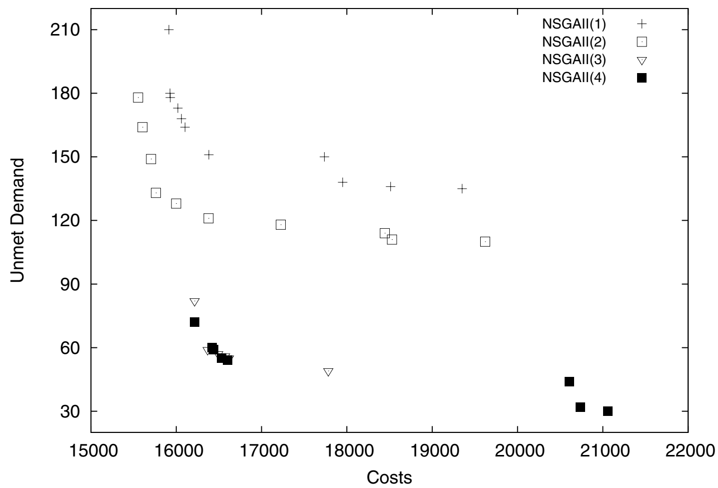

FIGURE 8. Impact of the different methods with a limited running time for instance 50-5-20-100-3 with ϵ=50% on set B.

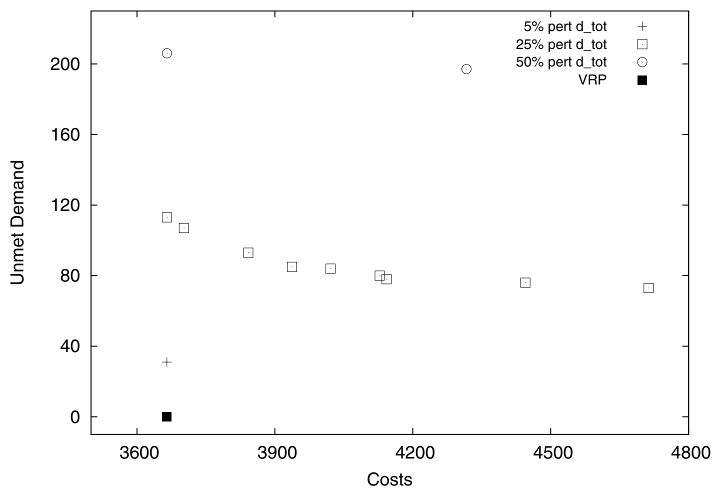

FIGURE 9. Impact of β for instance 10-2-20-50-3 on the NSGAII(1).

are also obtained with MOEA(1). In regard to set B, the method with the most promising behavior is found by NSGAII(4) which in average could obtained a high value for $H' = 0.99$, a good value for $\Theta' = 7.1$ and a low value for $S' = 0.15$.

These results indicate that if an individual perturbation on demands is considered, the local search is helpful. However when the global perturbation is applied for small percentage values (*i.e.*, within 5%), the local search procedure seems not to be useful. On the contrary, whenever perturbations increase to up to 25%, the local search is interesting to be applied.

In order to see the quality of the solutions, the evaluation of the bi-RVRP with 10 and 20 clients for set A using β = 5% and 25% is considered and compared with the deterministic VRP *i.e.*, nominal values for the demands and travel times. Figures 9, 10, 11 and 12 show the results obtained with NSGAII(1). One can notice

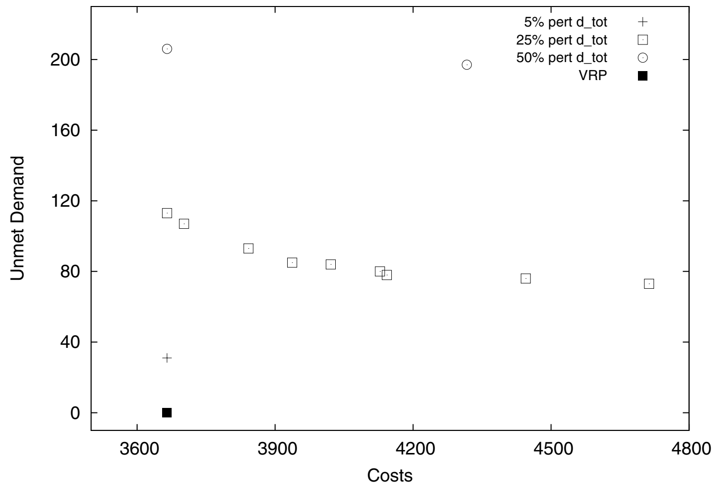

FIGURE 10. Impact of β for instance 10-2-20-50-5 on the NSGAII(1).

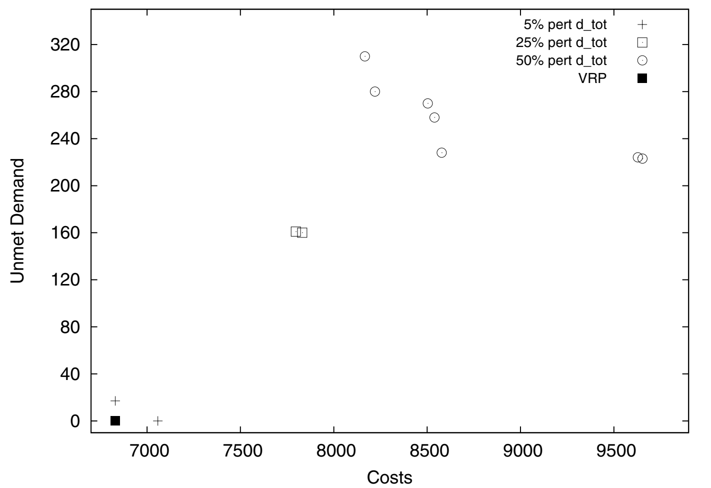

FIGURE 11. Impact of β for instance 20-3-20-100-3 on the NSGAII(1).

the price to pay for a robust solution against the deterministic case and the impact of the perturbation level β on the final solutions. As expected, even when we can achieve the same costs found on the VRP we can fall into an increasing value for the total unmet demand that varies according to the percentage β and the number of scenarios.

It can be also seen that when the number of scenarios increases there is a clear enlargement on the Pareto front due to the increment on the worst total cost or the maximum total unmet demand, which is more evident to see in larger instances. It is important to emphasise that when we are dealing with a multiobjective problem, the improvement that could be obtained in one of the objective functions will affect the other one. This situation is observed in the cases when the worst total cost of the deterministic case is reached but with a higher value for the maximum total unmet demand, or in the opposite way when the maximum total unmet demand is improved and the worst total cost becomes worst.

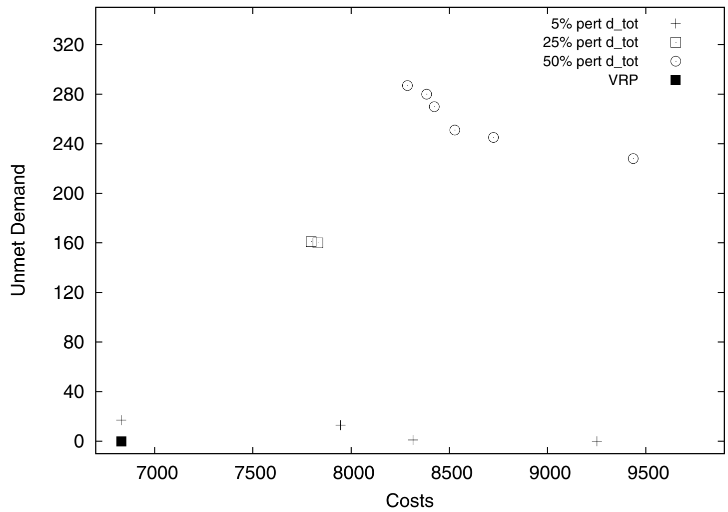

FIGURE 12. Impact of β for instance 20-3-20-100-5 on the NSGAII(1).

7. CONCLUDING REMARKS

In this paper the bi-RVRP with uncertainty in both demands and travel times is studied by means of robust optimization. Here, the bi-RVRP aims at minimizing the worst total cost of traversed arcs and minimizing the maximum total unmet demand over all scenarios. To the best of our knowledge, this version of the bi-RVRP has never been studied before and finds practical applications in urban transportation. MOEA and NSGAII are adapted to solve the bi-RVRP together with a local search. Different variations for both metaheuristics were considered on the location of the proposed local procedure in the MOEA and NSGAII structure. Different analyses are computed including the impact that the level of perturbation have on the worst total cost and the total unmet demand, the impact of the local search on the MOEA and NSGAII, and the convergence as well as the diversity for the different methods. The computational experiments are based on two sets of random-instances in order to see the impact that the application of different representations of uncertain demands has on the bi-RVRP, one that specifies a perturbation over the total demand of clients, while the other considers for each client an individual variation defined in an interval. Regarding the impact of the local search procedure on the different variations for the MOEA and the NSGAII, It can be seen that the local search is sensitive to the perturbation rate. In some cases, it is relevant to apply a local search, *i.e.*, when perturbation is up to 25%. On the contrary, if perturbation is small, *i.e.*, within 5%, the local search seems not to be helpful. In terms of the number of non-dominated solutions the NSGAII found better solutions than MOEA. With respect to the hypervolume both metaheuristics, the MOEA and the NSGAII, have a good coverage on the solutions. However, when the hypervolume and the number of non-dominated solutions improve their values, that does not guarantee a good spread of the solution on the Pareto front. Nevertheless results show that in general our proposed methods have a good behavior on all the performance metrics. Although in the cases where the deterministic version of the VRP gives solutions with no unmet demands our proposed method can find solutions that support the variations which might arise in the future with a good comprise between the cost and unmet demand. Regarding future research, there is room for including other representations for the uncertainties and to design exact approaches handling the bi-RVRP. An interesting option would be to accept split deliveries, for instance by delivering a maximum amount to each customer when its demand cannot be completely satisfied.

Appendix A. Detailed results for each instance

Table A.1. Results for NSGAII(1), NSGAII(2), MOEA(1) and MOEA(2) with $o = 3$, $o = 5$ and $\beta = 5\%$ for set A.

Instance	NSGAII(1)				NSGAII(2)				MOEA(1)				MOEA(2)			
n–m–p–θ–o	Θ	S	H	t	Θ	S	H	t	Θ	S	H	t	Θ	S	H	t
30-3-20-50-3	2	0.00	1.00	73	2	0.00	1.00	78	4	0.26	0.99	70	3	0.17	1.00	75
30-3-20-50-5	7	0.16	0.99	72	4	0.27	0.99	79	4	0.35	1.00	69	4	0.00	0.99	76
30-3-20-100-3	4	0.04	1.00	72	2	0.00	0.99	76	3	0.48	0.99	69	2	0.00	1.00	73
30-3-20-100-5	1	0.00	1.00	85	4	0.20	1.00	88	3	0.32	0.99	82	5	0.25	0.99	85
30-3-30-50-3	2	0.00	1.00	82	1	0.00	1.00	84	1	0.00	1.00	79	2	0.00	1.00	81
30-3-30-50-5	4	0.11	0.99	83	3	0.01	1.00	86	1	0.00	1.00	80	3	0.03	1.00	83
30-3-30-100-3	4	0.48	1.00	110	7	0.17	0.98	111	2	0.00	0.99	107	2	0.00	0.99	108
30-3-30-100-5	2	0.00	1.00	85	5	0.14	0.99	87	2	0.00	1.00	82	5	0.38	1.00	84
40-4-20-50-3	2	0.00	0.99	161	11	0.08	0.99	168	8	0.12	0.98	158	6	0.10	0.99	165
40-4-20-50-5	14	0.08	0.98	164	9	0.19	0.97	169	5	0.29	0.99	161	6	0.23	0.97	166
40-4-20-100-3	8	0.10	0.98	183	6	0.04	0.99	172	7	0.25	1.00	180	5	0.21	0.99	169
40-4-20-100-5	5	0.22	1.00	172	7	0.09	0.99	174	6	0.40	0.99	169	6	0.13	0.99	171
40-4-30-50-3	9	0.10	0.99	179	4	0.02	1.00	190	10	0.09	0.99	176	6	0.24	1.00	187
40-4-30-50-5	7	0.12	0.99	176	7	0.04	1.00	239	10	0.09	0.98	173	8	0.09	1.00	236
40-4-30-100-3	2	0.00	1.00	171	3	0.38	0.97	227	4	0.24	1.00	168	6	0.07	0.98	224
40-4-30-100-5	9	0.10	0.98	180	4	0.41	0.97	229	6	0.34	0.96	177	9	0.11	0.95	226
50-5-20-50-3	7	0.29	1.00	303	6	0.02	0.99	333	6	0.17	0.99	300	7	0.20	0.99	330
50-5-20-50-5	7	0.26	0.99	317	5	0.23	1.00	347	9	0.09	0.98	314	5	0.09	0.99	344
50-5-20-100-3	11	0.19	0.99	319	7	0.24	0.99	349	3	0.08	1.00	316	4	0.37	1.00	346
50-5-20-100-5	5	0.32	1.00	313	4	0.47	0.97	343	9	0.06	0.97	310	4	0.12	0.99	340
50-5-30-50-3	7	0.12	0.99	331	6	0.21	0.98	361	6	0.08	0.99	328	5	0.25	0.99	358
50-5-30-50-5	4	0.45	0.99	335	6	0.30	0.99	365	6	0.22	1.00	332	6	0.23	1.00	362
50-5-30-100-3	4	0.36	0.99	334	5	0.25	1.00	364	5	0.26	0.99	331	3	0.75	0.99	361
50-5-30-100-5	8	0.12	0.99	340	4	0.07	0.98	370	5	0.10	1.00	337	1	0.00	1.00	367
Average	5.6	0.15	0.99	193.33	5.1	0.16	0.99	212.04	5.2	0.18	0.99	190.33	4.7	0.17	0.99	209.04

TABLE A.2. Results for NSGAII(1), NSGAII(2), MOEA(1) and MOEA(2) with $o = 3$, $o = 5$ and $\beta = 25\%$ for set A.

Instance	NSGAII(1)				NSGAII(2)				MOEA(1)				MOEA(2)			
n–m–p–θ–o	Θ	S	H	t	Θ	S	H	t	Θ	S	H	t	Θ	S	H	t
30-3-20-50-3	2	0.00	1.00	101	5	0.19	1.00	108	2	0.00	1.00	98	5	0.17	0.99	102
30-3-20-50-5	10	0.07	0.97	103	2	0.00	1.00	104	1	0.00	1.00	100	2	0.00	1.00	103
30-3-20-100-3	7	0.07	0.99	97	8	0.12	0.98	98	13	0.23	0.98	94	5	0.17	0.99	98
30-3-20-100-5	2	0.00	1.00	97	7	0.11	0.98	99	4	0.39	0.99	94	8	0.16	0.99	98
30-3-30-50-3	10	0.12	0.98	99	4	0.20	1.00	126	5	0.31	0.99	96	6	0.32	0.99	106
30-3-30-50-5	8	0.32	0.95	85	10	0.14	0.99	116	7	0.18	0.96	82	5	0.09	0.99	110
30-3-30-100-3	8	0.07	0.98	108	5	0.29	0.98	112	11	0.12	0.99	105	7	0.17	0.98	107
30-3-30-100-5	7	0.29	0.98	84	12	0.04	0.97	111	3	0.30	1.00	81	4	0.05	0.99	110
40-4-20-50-3	10	0.10	0.99	166	10	0.08	0.99	216	7	0.24	0.99	163	7	0.17	1.00	214
40-4-20-50-5	7	0.15	0.99	167	6	0.24	1.00	214	6	0.05	0.99	164	2	0.00	1.00	212
40-4-20-100-3	8	0.05	0.99	167	7	0.24	0.99	217	7	0.19	0.98	164	11	0.07	0.97	214
40-4-20-100-5	7	0.12	0.98	168	4	0.40	1.00	214	7	0.10	0.99	165	4	0.36	1.00	182
40-4-30-50-3	9	0.05	0.98	183	6	0.40	0.99	224	6	0.15	0.99	180	8	0.16	0.97	181
40-4-30-50-5	1	0.00	1.00	178	3	0.40	1.00	181	1	0.00	1.00	175	2	0.00	1.00	180
40-4-30-100-3	12	0.07	0.97	175	6	0.20	0.99	180	8	0.13	1.00	172	8	0.12	0.99	182
40-4-30-100-5	12	0.06	0.95	178	12	0.07	0.95	180	7	0.23	0.98	175	11	0.09	0.96	182
50-5-20-50-3	11	0.07	0.98	309	13	0.09	0.98	311	11	0.20	0.99	308	8	0.12	0.97	309
50-5-20-50-5	13	0.04	0.95	308	6	0.28	1.00	311	12	0.20	0.98	306	14	0.05	0.98	314
50-5-20-100-3	9	0.10	0.98	307	11	0.13	0.98	314	12	0.15	1.00	304	11	0.04	1.00	310
50-5-20-100-5	13	0.11	0.99	307	17	0.07	0.98	311	12	0.09	0.98	304	10	0.05	0.99	308
50-5-30-50-3	9	0.08	0.98	332	11	0.09	0.99	335	15	0.14	0.98	329	9	0.10	0.98	333
50-5-30-50-5	10	0.04	0.97	328	7	0.20	0.99	327	6	0.02	0.99	325	5	0.17	1.00	339
50-5-30-100-3	7	0.16	0.99	326	13	0.15	0.99	332	10	0.09	0.96	325	11	0.05	0.98	329
50-5-30-100-5	8	0.15	0.99	331	10	0.15	0.99	335	12	0.09	0.99	329	9	0.10	0.99	333
Average	8.3	0.10	0.98	196.00	8.1	0.18	0.99	211.50	7.7	0.15	0.99	193.25	7.2	0.12	0.99	206.50

TABLE A.3. Results for NSGAII(3), NSGAII(4), MOEA(3) and MOEA(4) with $o = 3$, $o = 5$ and $\beta = 5\%$ for set A.

Instance	NSGAII(3)				NSGAII(4)				MOEA(3)				MOEA(4)			
n–m–p–θ–o	Θ	S	H	t	Θ	S	H	t	Θ	S	H	t	Θ	S	H	t
30-3-20-50-3	1	0.00	1.00	81	1	0.00	1.00	86	2	0.00	1.00	79	1	0.00	1.00	83
30-3-20-50-5	2	0.00	1.00	82	5	0.27	0.98	87	5	0.38	0.99	80	4	0.47	1.00	84
30-3-20-100-3	4	0.21	0.99	79	2	0.00	1.00	84	2	0.00	1.00	77	2	0.00	1.00	81
30-3-20-100-5	4	0.22	1.00	89	2	0.00	1.00	94	4	0.40	0.99	87	1	0.00	1.00	91
30-3-30-50-3	2	0.00	1.00	87	1	0.00	1.00	92	1	0.00	1.00	85	2	0.00	1.00	89
30-3-30-50-5	1	0.00	1.00	89	1	0.00	1.00	94	1	0.00	1.00	87	3	0.38	0.99	91
30-3-30-100-3	2	0.00	1.00	118	5	0.55	1.00	123	5	0.53	1.00	116	6	0.01	1.00	120
30-3-30-100-5	3	0.18	0.99	89	3	0.30	0.99	94	4	0.19	1.00	87	2	0.00	0.99	91
40-4-20-50-3	2	0.00	0.99	169	5	0.18	0.98	174	5	0.08	0.99	167	7	0.16	0.99	171
40-4-20-50-5	6	0.30	0.99	171	8	0.21	0.97	176	4	0.52	0.99	169	6	0.14	0.99	173
40-4-20-100-3	6	0.14	1.00	173	7	0.13	0.99	178	3	0.20	1.00	171	5	0.08	1.00	175
40-4-20-100-5	7	0.22	0.99	174	6	0.20	0.99	179	11	0.09	0.99	172	5	0.29	0.99	176
40-4-30-50-3	6	0.19	0.99	194	4	0.02	1.00	199	12	0.11	0.99	192	8	0.16	0.99	196
40-4-30-50-5	8	0.30	0.98	243	6	0.11	0.97	248	13	0.07	0.98	241	6	0.09	0.99	245
40-4-30-100-3	5	0.37	1.00	232	1	0.00	1.00	237	4	0.31	0.99	230	3	0.20	1.00	234
40-4-30-100-5	8	0.18	0.97	235	8	0.10	0.94	240	4	0.18	1.00	233	10	0.16	0.98	237
50-5-20-50-3	4	0.43	1.00	335	5	0.27	0.99	340	3	0.08	1.00	333	2	0.00	1.00	337
50-5-20-50-5	8	0.09	0.99	352	5	0.21	1.00	357	4	0.35	1.00	350	5	0.14	0.99	354
50-5-20-100-3	4	0.36	1.00	358	7	0.10	1.00	363	4	0.44	1.00	356	8	0.12	1.00	360
50-5-20-100-5	4	0.35	1.00	346	6	0.18	0.99	351	6	0.44	0.96	344	7	0.32	1.00	348
50-5-30-50-3	7	0.24	0.98	368	5	0.30	0.98	373	5	0.05	0.99	366	5	0.16	1.00	370
50-5-30-50-5	4	0.38	1.00	371	3	0.20	1.00	376	9	0.15	0.98	369	8	0.16	0.99	373
50-5-30-100-3	3	0.47	1.00	369	6	0.06	0.99	374	4	0.55	1.00	367	4	0.43	1.00	371
50-5-30-100-5	7	0.11	0.98	374	6	0.11	0.99	379	1	0.10	0.99	372	5	0.33	0.98	376
Average	4.5	0.20	0.99	215.75	4.5	0.15	0.99	220.75	4.8	0.22	0.99	213.75	4.8	0.16	0.99	217.75

TABLE A.4. Results for NSGAII(3), NSGAII(4), MOEA(3) and MOEA(4) with $o = 3$, $o = 5$ and $\beta = 25\%$ for set A.

Instance	NSGAII(3)				NSGAII(4)				MOEA(3)				MOEA(4)			
n–m–p–θ–o	Θ	S	H	t	Θ	S	H	t	Θ	S	H	t	Θ	S	H	t
30-3-20-50-3	5	0.18	1.00	110	5	0.34	1.00	115	3	0.46	1.00	108	6	0.15	1.00	112
30-3-20-50-5	5	0.15	1.00	106	1	0.00	1.00	111	5	0.08	1.00	104	3	0.14	1.00	108
30-3-20-100-3	4	0.38	0.98	100	6	0.18	1.00	105	4	0.33	0.99	98	2	0.00	1.00	102
30-3-20-100-5	6	0.26	0.99	101	5	0.07	1.00	106	6	0.22	0.99	99	6	0.07	0.99	103
30-3-30-50-3	7	0.10	0.99	128	5	0.29	1.00	133	6	0.11	0.99	126	4	0.13	1.00	130
30-3-30-50-5	3	0.26	1.00	118	8	0.19	0.98	123	3	0.05	1.00	116	2	0.00	1.00	120
30-3-30-100-3	10	0.08	0.98	114	6	0.18	0.99	119	4	0.01	0.99	112	5	0.17	0.99	116
30-3-30-100-5	8	0.12	1.00	113	5	0.20	1.00	118	5	0.13	0.99	111	5	0.23	0.99	115
40-4-20-50-3	4	0.43	1.00	218	4	0.25	0.99	223	9	0.15	0.98	216	8	0.20	0.99	220
40-4-20-50-5	8	0.09	1.00	216	6	0.35	1.00	221	9	0.14	1.00	214	7	0.16	1.00	218
40-4-20-100-3	9	0.12	0.99	219	7	0.13	0.99	224	5	0.34	1.00	217	9	0.12	1.00	221
40-4-20-100-5	5	0.24	1.00	216	3	0.32	1.00	221	4	0.28	1.00	214	4	0.38	1.00	218
40-4-30-50-3	5	0.22	0.99	226	8	0.08	0.99	231	8	0.13	0.99	224	6	0.23	0.99	228
40-4-30-50-5	2	0.00	1.00	183	1	0.00	1.00	188	1	0.00	1.00	181	1	0.00	1.00	185
40-4-30-100-3	9	0.19	0.98	182	7	0.19	0.98	187	9	0.04	0.98	180	5	0.24	1.00	184
40-4-30-100-5	10	0.10	0.97	182	8	0.20	0.97	187	10	0.10	0.94	180	10	0.08	0.97	184
50-5-20-50-3	15	0.07	0.97	313	7	0.10	0.99	318	13	0.14	0.99	311	6	0.29	0.99	315
50-5-20-50-5	10	0.12	0.97	313	10	0.04	0.99	318	9	0.10	1.00	311	10	0.03	0.99	315
50-5-20-100-3	4	0.12	1.00	316	8	0.06	0.99	321	9	0.13	0.99	314	5	0.25	1.00	318
50-5-20-100-5	4	0.57	1.00	313	8	0.16	0.99	318	9	0.14	0.99	311	10	0.12	0.99	315
50-5-30-50-3	7	0.07	1.00	337	11	0.09	0.98	342	10	0.20	0.99	335	5	0.21	1.00	339
50-5-30-50-5	8	0.11	1.00	329	6	0.28	1.00	334	9	0.09	0.98	327	10	0.19	1.00	331
50-5-30-100-3	9	0.13	1.00	334	8	0.13	0.99	339	6	0.20	0.99	332	6	0.16	1.00	336
50-5-30-100-5	8	0.22	0.99	337	13	0.10	0.97	342	11	0.14	0.97	335	5	0.09	0.99	339
Average	6.9	0.18	0.99	213.50	6.5	0.16	0.99	218.50	7.0	0.15	0.99	211.50	5.8	0.15	0.99	215.50

TABLE A.5. Results for NSGAII(1), NSGAII(2), MOEA(1) and MOEA(2) with $o = 3$, $o = 5$ and ϵ= 50% for set B.

Instance	NSGAII(1)				NSGAII(2)				MOEA(1)				MOEA(2)			
n–m–p–θ–o	Θ	S	H	t	Θ	S	H	t	Θ	S	H	t	Θ	S	H	t
30-3-20-50-3	11	0.10	0.97	76	2	0.00	0.99	102	4	0.10	0.99	74	8	0.18	0.99	98
30-3-20-50-5	9	0.17	0.98	78	10	0.06	0.97	103	5	0.18	1.00	76	6	0.16	0.98	99
30-3-20-100-3	7	0.09	0.99	75	4	0.38	0.99	99	5	0.29	0.99	73	6	0.10	0.97	95
30-3-20-100-5	11	0.17	0.97	73	5	0.15	0.98	100	7	0.23	0.98	71	9	0.18	0.97	96
30-3-30-50-3	8	0.14	0.97	83	8	0.09	0.98	109	6	0.17	0.99	81	5	0.08	0.99	105
30-3-30-50-5	7	0.15	0.99	83	10	0.15	0.99	114	11	0.11	0.98	81	7	0.13	0.98	110
30-3-30-100-3	8	0.07	0.99	86	7	0.15	1.00	115	8	0.14	0.98	84	7	0.22	0.99	111
30-3-30-100-5	6	0.21	0.97	83	5	0.27	0.99	115	5	0.21	1.00	81	3	0.70	0.96	111
40-4-20-50-3	8	0.12	0.99	160	5	0.19	0.99	219	12	0.07	0.99	158	6	0.18	1.00	215
40-4-20-50-5	14	0.08	0.98	166	12	0.10	0.98	223	13	0.06	0.99	164	19	0.06	0.97	219
40-4-20-100-3	10	0.10	0.99	166	3	0.31	1.00	230	12	0.07	0.99	164	10	0.17	0.99	226
40-4-20-100-5	8	0.13	0.99	166	4	0.29	0.99	228	14	0.16	0.99	164	4	0.21	0.99	224
40-4-30-50-3	14	0.05	0.99	185	5	0.12	0.99	253	16	0.12	0.97	183	11	0.15	0.98	249
40-4-30-50-5	12	0.06	0.97	185	5	0.12	0.99	252	17	0.05	0.96	183	8	0.19	0.97	248
40-4-30-100-3	6	0.25	0.98	173	5	0.19	1.00	236	10	0.07	0.99	171	5	0.18	0.99	232
40-4-30-100-5	9	0.13	0.99	179	4	0.26	0.99	242	7	0.19	0.98	177	9	0.07	0.96	238
50-5-20-50-3	8	0.12	0.98	306	4	0.04	0.96	411	9	0.05	0.98	304	13	0.12	0.97	407
50-5-20-50-5	6	0.18	1.00	310	4	0.07	0.98	349	11	0.11	0.99	308	14	0.04	0.99	345
50-5-20-100-3	15	0.04	0.99	307	10	0.06	0.98	306	8	0.12	0.97	305	8	0.19	0.98	302
50-5-20-100-5	12	0.17	0.99	311	11	0.03	0.98	304	12	0.06	0.96	309	11	0.14	0.99	300
50-5-30-50-3	9	0.19	0.99	331	10	0.10	0.98	325	19	0.07	0.98	329	17	0.04	0.98	321
50-5-30-50-5	16	0.11	0.97	333	9	0.11	1.00	319	7	0.12	0.99	331	8	0.27	0.99	315
50-5-30-100-3	10	0.11	0.97	330	7	0.20	0.99	317	4	0.04	0.99	328	8	0.20	0.98	313
50-5-30-100-5	9	0.08	0.98	337	9	0.16	0.99	324	11	0.12	0.98	335	9	0.07	0.98	320
Average	9.7	0.12	0.98	190.92	6.6	0.15	0.99	224.8	9.7	0.12	0.98	188.92	8.8	0.17	0.98	220.79

TABLE A.6. Results for NSGAII(3), NSGAII(4), MOEA(3) and MOEA(4) with $o = 3$, $o = 5$ and ϵ= 50% for set B.

Instance	NSGAII(3)				NSGAII(4)				MOEA(3)				MOEA(4)			
n–m–p–θ–o	Θ	S	H	t	Θ	S	H	t	Θ	S	H	t	Θ	S	H	t
30-3-20-50-3	5	0.08	0.98	100	4	0.18	0.99	108	4	0.18	0.98	81	3	0.49	1.00	104
30-3-20-50-5	4	0.19	1.00	101	6	0.15	0.97	107	4	0.35	0.98	80	5	0.12	0.99	107
30-3-20-100-3	3	0.04	0.99	97	4	0.08	0.99	102	3	0.68	0.99	78	7	0.11	0.98	104
30-3-20-100-5	4	0.36	0.99	98	5	0.33	0.99	104	6	0.08	0.99	81	5	0.19	0.98	107
30-3-30-50-3	4	0.37	0.99	107	5	0.23	0.99	116	3	0.04	0.99	87	4	0.28	0.99	113
30-3-30-50-5	5	0.29	0.99	112	4	0.26	1.00	117	4	0.10	1.00	94	5	0.14	1.00	121
30-3-30-100-3	5	0.35	0.99	113	7	0.15	0.99	123	7	0.06	1.00	96	4	0.66	0.99	117
30-3-30-100-5	5	0.21	1.00	113	6	0.17	0.98	120	6	0.28	0.99	92	14	0.10	0.98	126
40-4-20-50-3	6	0.18	0.99	217	9	0.15	0.99	224	8	0.11	0.99	178	4	0.38	0.99	223
40-4-20-50-5	4	0.24	1.00	221	12	0.07	0.97	234	7	0.21	1.00	183	10	0.13	0.98	238
40-4-20-100-3	8	0.13	0.99	228	8	0.08	0.99	231	5	0.29	0.99	178	6	0.09	1.00	228
40-4-20-100-5	5	0.12	1.00	226	11	0.07	1.00	233	6	0.12	1.00	180	8	0.05	1.00	231
40-4-30-50-3	9	0.25	1.00	251	12	0.03	0.99	261	11	0.12	0.99	252	15	0.04	0.99	272
40-4-30-50-5	5	0.19	0.99	250	8	0.10	0.99	259	6	0.24	1.00	263	13	0.08	0.99	268
40-4-30-100-3	4	0.23	1.00	234	9	0.10	0.98	242	3	0.60	1.00	241	2	0.00	1.00	236
40-4-30-100-5	6	0.25	0.98	240	9	0.08	0.97	255	8	0.16	0.99	247	6	0.20	0.98	227
50-5-20-50-3	8	0.17	0.99	409	13	0.05	0.97	432	7	0.08	0.99	406	16	0.21	0.97	336
50-5-20-50-5	12	0.08	0.98	347	10	0.08	0.99	403	9	0.16	0.99	344	11	0.08	0.99	336
50-5-20-100-3	5	0.24	1.00	304	4	0.29	1.00	322	10	0.05	0.97	301	6	0.09	0.99	322
50-5-20-100-5	11	0.06	0.98	302	12	0.11	0.99	328	12	0.22	0.99	299	10	0.12	1.00	330
50-5-30-50-3	9	0.22	0.99	323	7	0.11	1.00	355	12	0.06	1.00	320	8	0.05	0.99	360
50-5-30-50-5	11	0.04	0.98	317	6	0.28	1.00	358	21	0.04	0.97	314	11	0.12	0.99	363
50-5-30-100-3	6	0.12	0.99	315	1	0.00	1.00	369	6	0.18	0.99	312	6	0.13	1.00	358
50-5-30-100-5	15	0.06	0.98	322	14	0.03	0.99	375	7	0.24	1.00	319	15	0.04	0.98	360
Average	6.6	0.19	0.99	222.79	7.8	0.13	0.99	240.75	7.3	0.19	0.99	209.42	8.1	0.16	0.99	232.79

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