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TRANSSHIPMENT AND COORDINATION IN A TWO-ECHELON SUPPLY CHAIN *

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Abstract. To better match supply and demand, many distributors need to make the strategic decision on whether to collaborate with their competing distributors by adopting the transshipment strategy. In this paper, we mainly aim to answer the following questions: whether and when distributors should adopt the transshipment strategy in the presence of inventory competition? If the transshipment strategy is adopted, how supply chains can be coordinated? To answer these questions, in this paper, we model a supply chain with one manufacturer selling to two competing distributors. For the first question, we find that regardless of centralization or decentralization, the transshipment strategy is better when transshipment cost is lower or competition is less intense. Moreover, under decentralization, there always exists a threshold of transshipment cost. When transshipment cost is lower than the threshold, regardless of the competitive intensity, the distributors should always adopt the transshipment strategy. We further extend the model from symmetric distributors to asymmetric distributors and show our results are robust using numerical studies. For the second question, we design a buyback with sale rebate and penalty contract which can achieve coordination as well as win-win outcomes for all supply chain members.

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1. INTRODUCTION

Nowadays, many products have an increasingly shorter life cycle. As a result, the risk of holding excess inventory is much larger than ever before. For example, once a new mobile phone model enters the market, the customer demand of its old version often drops significantly. On the other hand, if a distributor or a retailer purchases and/or holds insufficient inventory, stock-outs may happen. For example, Ton [36] finds stock-outs lead to lost sales of 7 to 12 billion dollars annually in the supermarket industry alone. To solve this issue, many scholars begin to study what inventory levels can lead to maximize profits or minimize cost (see, *e.g.*, [12, 25, 28–30]). To better match supply and demand, Krishnan and Rao [13] first propose a lateral transshipment strategy which means distributors or retailers set up relationship with competitors to transfer excess inventory to satisfy excess demand.

More recently, a lot of studies have confirmed that the transshipment strategy can lead to win-win outcomes for all supply chain members (see, *e.g.*, [9,10,21,32]). Interestingly, in business practice, though the transshipment

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strategy has been used in many industries, such as automobile, fashion and oil, there are still many industries which have not adopted this strategy. A potential major reason is that the above-mentioned studies do not consider inventory competition when studying the transshipment strategy. Inventory competition means that if one distributor is out of stock, a fraction of the unsatisfied customers may visit another distributor for the product [14, 22]. So in the presence of inventory competition, a distributor needs to decide whether to tranship the product to another distributor or wait for customers shift. It is worthwhile noting that inventory competition is a common business practice.

Therefore, the following questions are of great practical importance: in the presence of inventory competition, whether and when distributors should adopt the transshipment strategy? Moreover, if distributors adopt the transshipment strategy, how can we design a coordinating contract for this supply chain? To answer these questions, in this paper, we model a supply chain with one manufacturer (she) selling to two distributors (he). If the transshipment strategy is adopted, a distributor can sell its excess inventory, if any, to the other and charges a transshipment price. If the no-transshipment strategy is adopted, inventory selling/buying between the two distributors is not allowed but inventory competition or substitution may occur. That is, if one distributor is out of stock, a fraction of his customers whose demands are not met will turn to the other distributor.

Our paper has the similar supply chain structures with Shao *et al.* [35], but Shao *et al.* [35] do not consider inventory competition between the retailers. To the best of our knowledge, only Zhao and Atkins [46] conduct a comparison between transshipment and no-transshipment considering inventory competition. This paper differs from Zhao and Atkins [46] from three aspects. First, on the issue of whether competing distributors should adopt a transshipment strategy, Zhao and Atkins [46] present only numerical, but not analytical, results. On the other hand, we obtain some analytical results which can help the distributors make decisions easier. For example, we find that under a decentralized supply chain, if transshipment cost is lower than a threshold, the distributors should always adopt the transshipment strategy. Second, Zhao and Atkins [46] limit themselves to a one-echelon supply chain. We consider a two-echelon supply chain and study whether allowing transshipments between two competing distributors is optimal when the supply chain is either centralized or decentralized. Third, almost all extant studies on transshipment assume that the manufacturer is just as wholesale-price taker. This paper further studies how the manufacturer to coordinate the supply chain under transshipment.

The main findings of this paper are the following. First, we find that regardless of supply chain centralization or decentralization, the transshipment strategy is better when transshipment cost is lower or competition is weaker. However, under decentralization, there always exists a threshold of transshipment cost. When transshipment cost is lower than the threshold, regardless of the competitive intensity, the distributors should always adopt the transshipment strategy. Second, decentralization and centralization may lead to different optimal strategy choices and the transshipment strategy is more attractive under decentralization (*vs.* centralization). Finally, we develop a buyback with sale rebate and penalty contract which can coordinate the supply chain under the transshipment strategy and achieve win-win outcomes for all supply chain members.

The rest of this paper is organized as follows. Section 1 provides a brief literature review. Section 2 introduces model assumptions and notations. Sections 3 and 4 investigate the transshipment strategies under the centralized and decentralized supply chains, respectively. Section 5 proposes a coordinating buyback with sale rebate and penalty contract whose optimal contractual parameters are also determined. Section 6 employs numerical studies to illustrate our results and gain more managerial insights. Section 7 provides concluding remarks and suggests future research directions.

2. Literature review

This paper is closely related to two streams of literature: transshipment and supply chain coordination. We discuss each of these two streams in turn.

In the research stream of transshipment, a number of studies have confirmed that the transshipment strategy can benefit the participants under different settings. Herer *et al.* [9] find that transshipment can enhance both agility and leanness. Slikker *et al.* [32] establish a model with multiple newsvendor-type retailers and conclude that they can improve their expected joint profit by transshipment. Olsson [21] studies a continuous review inventory system where transshipment is allowed only in one direction and also proves that transshipment can improve performance for all participants. This rapid development of e-commerce has prompted some researcher to study transshipment under dual-channel supply chains. Seifert *et al.* [33] investigate transshipment decisions under a dual-channel model, and show that the profits of dual-channels can be improved by transshipment. He *et al.* [10] study unidirectional transshipment policies in a dual-channel supply chain and show that transshipment can be beneficial to both retailers simultaneously. However, those above-mentioned papers omit competition. Zhao and Atkins [46] incorporate competition by studying a one-echelon supply chain with competing retailers and find that the transshipment strategy may not be optimal. Liao *et al.* [15] consider two retailers who engage in inventory competition under a centralized control system. They compare lateral transshipment and emergency order options and find that the transshipment strategy is not always better.

Meanwhile, quite a few studies focus on how to improve the effectiveness of the transshipment strategy. Sošic [34] investigates how much of retailers' unsold inventory or unsatisfied demand they want to share with other retailers under the transshipment strategy. Nasr *et al.* [19] obtain the optimal safety stock at each location and the optimal amount to be transshipped from a location to the other under a two-echelon supply chain. Research has also been done to improve the effectiveness through means other than transshipment prices and quantities. Axsäter [1] establishes decision rules which can minimize the expected costs when transshipment takes place. Zhao *et al.* [43] analyze the optimal production and transshipment policy for a two-location make-to-stock system with exponential production. Archibald *et al.* [2] develop an approximate solution method to determine the optimal transshipment policy in multi-location inventory systems. Paterson *et al.* [23] propose a quasi-myopic approach to develop a performance-enhancing reactive transshipment policy. Noham and Tzur [20] further consider a multi-item situation and develop a simple heuristic algorithm to derive a better transshipment policy.

Moreover, some studies focus on the contributing factors on the effectiveness of transshipment. Dong and Rudi [7] extend the traditional transshipment problem to include a manufacturer, and find that retailers' order quantities are insensitive to the wholesale price charged by the manufacturer, who in turn benefits from transshipments by charging a higher wholesale price. Zhang [44] extends the model of Dong and Rudi [44] to general demand distributions. Burton and Banerjee [4] examine the cost effects of two lateral transshipment approaches in a two-echelon supply chain.

This paper is also related to the second research stream of supply chain coordination with contracts. The extant transshipment research mainly focuses on the lateral coordination under one-echelon supply chain. Rudi *et al.* [27] obtain transshipment prices which can induce multiple locations to choose inventory levels consistent with joint-profit maximization. Hezarkhani and Kubiak [8] derive transshipment prices that can give rise to a coordinating contract for a supply chain. Li *et al.* [16] develop a bidirectional revenue sharing contract that can coordinate transshipment quantities of the two locations. Yan and Zhao [42] first propose a mechanism which can coordinate multi-retailers considering asymmetric information and transshipment. For two-echelon supply chain coordination, many contracts have been extensively studied in the literature. They include revenue sharing contracts (*e.g.*, [3, 45]), buyback contracts (*e.g.*, [40, 41]), quantity flexibility contracts (*e.g.*, [37, 38]), sales rebate contracts (*e.g.*, [39]), and quantity discount contracts (*e.g.*, [17, 18]). See Cachon [5] for an excellent and comprehensive review. However, some researchers find that under some complex environments, it is difficult to coordinate a supply chain if the above-mentioned contracts are used alone. Therefore, they develop some combined contracts to deal with those complex environments (*e.g.*, [6, 11]).

3. Model notation and assumptions

3.1. Model description

In this paper, we consider a single period model where a manufacturer (she), indexed by m, produces a single product and distributes it through two identical distributors (he), indexed by i and j. The manufacturer and distributors are blessed with full information. The two distributors face newsboy problem. It means the distributors face random demand for the product and need to know how many units of the product to stock in order to maximize expected profit [31]. The demands of the two distributors are statistically independent random variables. The two distributors decide whether to adopt the transshipment and if so, to determine the transshipment price, and pay transshipment cost when the transshipment occurs. If the two distributors adopt the no-transshipment, a proportion of distributor i's unsatisfied demand will switch to distributor j with a fixed demand switching rate. The model structure is shown in Figure 1.

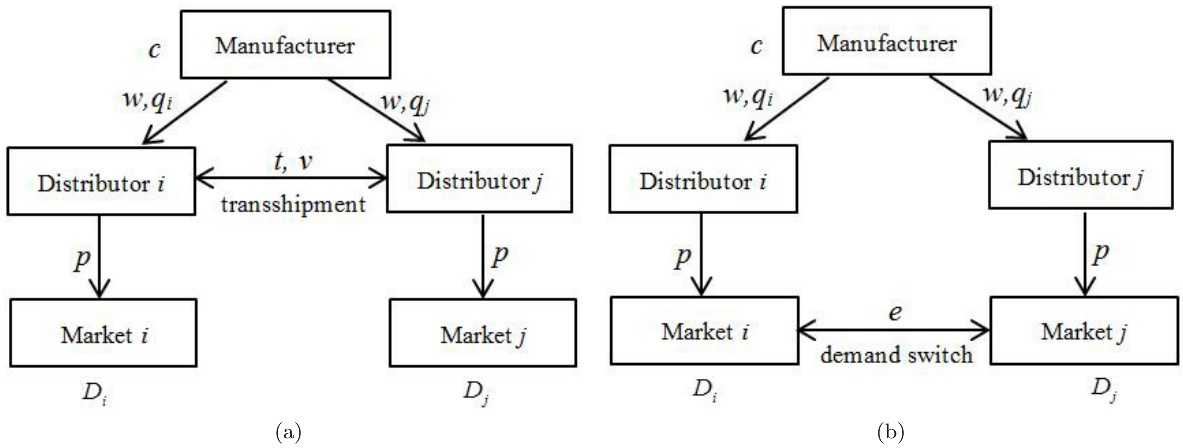


Figure 1. Model structure under transshipment and no-transshipment.

3.2. Basic notations

The basic notations are shown as follows:

c	a unit production cost for manufacturer;
w	wholesale price which is exogenous;
p	a unit sale price which is exogenous;
q_x	order quantity of distributor x, $x = i, j$;
D_x	demand of distributor x, $x = i, j$;
$F_x(\cdot)$	cumulative probability distribution for the demand of distributor x, $x = i, j$;
$f_x(\cdot)$	probability distribution function for the demand of distributor x, $x = i, j$;
t	transshipment price, $t \in (0, p)$;
v	a unit transshipment cost, $v \in (0, t)$;
e	demand switching rate, $e \in [0, 1]$;
Z_{xy}	the expected transshipment quantity from distributor x to y, $x, y = i, j$; equal to $E[\min\{(q_i - D_i)^+, (D_j - q_j)^+\}]$;
U_{xy}	the expected switched demand from distributor x to y, $x, y = i, j$; equal to $E[\min\{e(D_i - q_i)^+, (q_j - D_j)^+\}]$;
R_x	distributor x's expected sales quantity from its own-inventory and own-market, $x = i, j$.

3.3. Sequence of the events

The sequence of game stages is shown as follows:

(1) the two distributors decide on whether to adopt transshipment;
(2) if the two distributors adopt transshipment, they determine the transshipment price;
(3) the two distributors decide on ordering quantities from the manufacturer in advance of a selling season with random demand simultaneously.

After above game process, the manufacturer starts to produce and delivers products to the distributors. Then, the random customer demands are realized. If transshipment is adopted, the distributors transship excess inventory to match unsatisfied demand. If not, a proportion of one distributor's unsatisfied demand switches to another.

4. Centralized supply chain

To establish a performance benchmark, we first analyze the problem of a centralized supply chain. In the centralized supply chain, the manufacturer and two distributors act like they belong to the same parent-company. So the common goal is to maximize the total profit of the entire supply chain.

4.1. Ordering quantities under centralized supply chain

First, we analyze the optimal order quantities under the transshipment strategy and the no-transshipment strategy.

With transshipment, as the supply chain is centralized, the transshipment price should be zero. Let superscript $(\cdot)^T$ denote the transshipment scenario. The centralized supply chain's expected profit Π^{CT} is:

$$\Pi^{CT}(q_i^{CT}, q_j^{CT}) = pR_i^{CT} + pR_j^{CT} + pZ_{ji}^{CT} - vZ_{ij}^{CT} + pZ_{ij}^{CT} - vZ_{ji}^{CT} - c\left(q_i^{CT} + q_j^{CT}\right). \tag{4.1}$$

Similar to Robinson [26], we can verify that Π^{CT} as in (4.1) is concave in (q_i^{CT}, q_j^{CT}) and the optimal ordering quantities $(q_i^{CT^*}, q_j^{CT^*})$ are characterized by

$$F_i(q_i^{CT^*}) = \frac{p-c}{p} + \frac{p-v}{p}(h_1 - h_2). \tag{4.2}$$

where h_1 and h_2 denote $\frac{\partial Z_{ij}}{\partial q_i}$ and $(-\frac{\partial Z_{ij}}{\partial q_j})$ respectively.

On the other hand, if no-transshipment strategy is adopted by the centralized supply chain, a proportion of the unsatisfied demand of one distributor, if any, will switch to the other. Let superscript $(\cdot)^S$ denote this no-transshipment scenario. Then the centralized supply chain's expected profit Π^{CS} is

$$\Pi^{CS}(q_i^{CS}, q_j^{CS}) = pR_i^{CS} + pR_j^{CS} + pU_{ji}^{CS} + pU_{ij}^{CS} - c\left(q_i^{CS} + q_j^{CS}\right). \tag{4.3}$$

Similar to Parlar [24], we can verify that (4.3) is concave in (q_i^{CS}, q_j^{CS}) and the optimal ordering quantities $(q_i^{CS^*}, q_j^{CS^*})$ are characterized by

$$F_i(q_i^{CS^*}) = \frac{p-c}{p} + g_1 - g_2 \tag{4.4}$$

where g_1 and g_2 denote $\frac{\partial U_{ij}}{\partial q_j}$ and $(-\frac{\partial U_{ij}}{\partial q_i})$ respectively.

4.2. Strategy choice under centralized supply chain

In the centralized supply chain, v and e affect $\Pi^{CT^*}(q_i^{CT^*}, q_j^{CT^*})$ and $\Pi^{CS^*}(q_i^{CS^*}, q_j^{CS^*})$, so they will also affect the strategy choice. We start by studying how v and e impact profits.

Proposition 4.1. *Π^{CT^*} decreases in v and Π^{CT^*} increases in e.*

Proposition 4.1 confirms the intuition that the transshipment strategy is more attractive when the transshipment cost is lower, and the no-transshipment strategy is better when inventory competition is more intense.

We next aim to find the optimal strategy to maximize the supply chain's profit. To this end, we compare the optimal profits under the transshipment and no-transshipment strategies. Because the profits are impacted by the v and e, we start by comparing the minimal and maximal Π^{CT^*} and Π^{CS^*}. Based on our assumptions and Proposition 4.1, we know that when $v = 0$ and $e = 1$, Π^{CT^*} and Π^{CS^*} will be maximal respectively, denoted as $\Pi^{CT^*}_{\max}$ and $\Pi^{CS^*}_{\max}$. When $v = p$ and $e = 0$, Π^{CT^*} and Π^{CS^*} and will be minimal respectively, denoted as $\Pi^{CT^*}_{\min}$ and $\Pi^{CS^*}_{\min}$. We then can have the following proposition,

Proposition 4.2. $\Pi^{CT^*}_{\max} = \Pi^{CS^*}_{\max}$, $\Pi^{CT^*}_{\min} = \Pi^{CS^*}_{\min}$.

From Propositions 4.1 and 4.2, we can conclude that no such threshold exists. Therefore, neither the transshipment strategy nor the no-transshipment strategy is optimal for the centralized supply chain under any circumstances. The implication is that supply chain managers should pay closer attention to changes in competitive intensity and transshipment cost when deciding whether to allow for transshipment.

5. Decentralized supply chain

In the decentralized supply chain, the manufacturer and the two distributors are independent entities and each maximizes his/her own profit. So whether the transshipment strategy or the no-transshipment strategy is better depends on not only the transshipment cost and demand switching rate, but also on the transshipment price. Using backward deduction to solve the game, we first analyze the distributors' ordering decisions, and then study the transshipment price decision. Finally, we derive the optimal strategy choice. Because we assume two symmetric distributors, we will discuss the focal distributor i only.

5.1. Ordering quantities under decentralization

Similar to the case of the centralized supply chain, here we also analyze the ordering decisions under the transshipment strategy as well as the no-transshipment strategy.

With transshipment, distributor i's expected profit, denoted as $\Pi_i^T(q_i^T, q_j^T)$, is

$$\Pi_i^T(q_i^T, q_j^T) = pR_i^T + (t - v)Z_{ij}^T + (p - t)Z_{ji}^T - wq_i^T. \tag{5.1}$$

According to Shao *et al.* [35], a unique Nash equilibrium exists. The equilibrium ordering quantity of distributor i satisfies:

$$F_i(q_i^{T*}) = \frac{p - w}{p} + \frac{t - v}{p}h_1 - \frac{p - t}{p}h_2. \tag{5.2}$$

Without transshipment, distributor i's expected profit, denoted as Π_i^S, is

$$\Pi_i^S(q_i^S, q_j^S) = pR_i^S + pU_{ji}^S - wq_i^S. \tag{5.3}$$

According to Parlar [24], there exists a unique Nash equilibrium. The equilibrium ordering quantity of distributor i satisfies:

$$F_i(q_i^{S^*}) = \frac{p - w}{p} + g_1. \tag{5.4}$$

5.2. Transshipment price under decentralization

In this subsection, we study how the two distributors decide the transshipment price to maximize their profits. Though we cannot get an analytic solution for $(q_i^{T^*}, q_j^{T^*})$ from (6), we know $(q_i^{T^*}, q_j^{T^*})$ are functions of the transshipment price t. Therefore, with transshipment, distributor i's expected profit $\Pi_i^T(t)$ is

$$\Pi_i^T(t) = pR_i^{T^*} + (t-v)Z_{ij}^{T^*} + (p-t)Z_{ji}^{T^*} - wq_i^{T^*} \tag{5.5}$$

where $R_i^{T^*}$, $Z_{ij}^{T^*}$, and $Z_{ji}^{T^*}$ denote $R_i^T(q_i^{T^*})$, $Z_{ij}^T(q_i^{T^*}, q_j^{T^*})$, and $Z_{ji}^T(q_i^{T^*}, q_j^{T^*})$ respectively.

Because we cannot get clear solution of $(q_i^{T^*}, q_j^{T^*})$ if we do not know the specific parameters of demand, we cannot obtain the best response function and use it to get the clear solution for optimal transshipment price. However, based on (6), we can use the Implicit Function Theorem to get the following proposition:

Proposition 5.1. *Distributor i's expected profit $\Pi_i^T(t)$ is quasi-concave in t, and exist an unique optimal transshipment price t^*, which is given by*

$$t^*(h_2^* + h_1^*) - vh_2^* - ph_1^* = 0 \tag{5.6}$$

where h_1^* and h_2^* are functions of t^*.

Figure 2 can illustrate the Proposition 5.3. From Figure 2, we can find that when $t = 12$, the distributor i can obtain the maximum profit; when $t < 12$, distributor i's profit is increasing in transshipment price; when $t > 12$, distributor i's profit is decreasing in transshipment price.

In real life, if we know specific parameters of demand, we can use (5.6) to get the value of optimal transshipment price. From (5.6), We can easy to check $v \leqslant t^* \leqslant p$. Substituting t^* into (6), we can get that distributor i's optimal ordering quantity $q_i^{T^*}$ is given by

$$F_i(q_i^{T^*}) = \frac{p-w}{p} + \frac{p-v}{p}(h_1 - h_2). \tag{5.7}$$

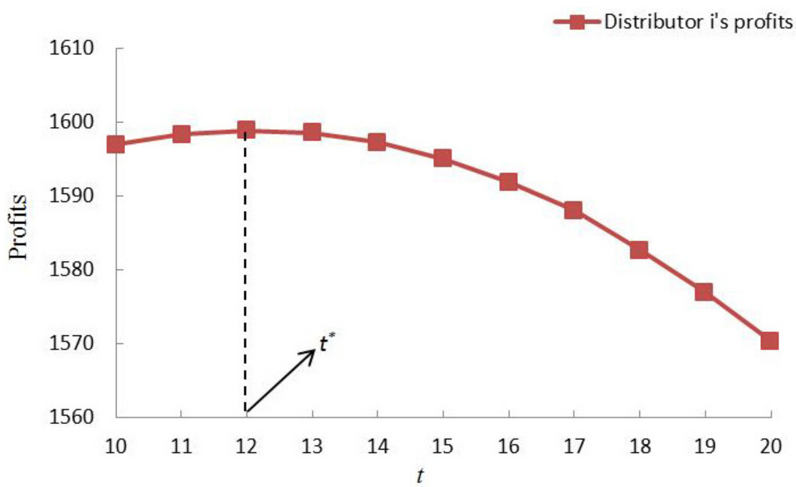


FIGURE 2. Impact of t on Π_i^T. Note. The parameter values: $p = 20$, $c = 4$, $w = 8$, $v = 10$, $e = 0.8$, $D_j = D_i \sim U[0, 400]$.

5.3. Strategy choice under decentralization

In this subsection, we mainly study the following questions: is the transshipment strategy or the no-transshipment strategy better when the supply chain is decentralized? Will supply chain centralization and decentralization lead to different optimal strategy choices? Similar to the case of centralized supply chain, we start by studying how v and e impact the distributors' profits. And we have the following result.

Proposition 5.2. *$\Pi_i^{T^*}$ decreases in v and $\Pi_i^{S^*}$ increases in e.*

Proposition 5.2 implies that under the decentralized supply chain, the transshipment strategy is better when transshipment cost is lower, and the no-transshipment strategy is more attractive when competition is more intense. This result is consistent with that for the centralized supply chain. We next compare the minimal and maximal $\Pi_i^{T^*}$ and $\Pi_i^{S^*}$, for which we have the following:

Proposition 5.3. $\Pi_{i\max}^{T^*} > \Pi_{i\max}^{S^*}$, $\Pi_{i\min}^{T^*} = \Pi_{i\min}^{S^*}$.

From Proposition 5.3, we can see that a transshipment cost $\overline{v}^*$ always exists to satisfy $\Pi_i^{T^*}(\overline{v}^*) = \Pi_{i\max}^{S^*}$. Thus we can obtain the following proposition.

Proposition 5.4. *Under the decentralized supply chain, when $v \leqslant \overline{v}^*$, regardless of the competitive intensity e, the distributors should always adopt the transshipment strategy.*

Comparing Propositions 5.4 and 4.2, we can see that the transshipment strategy is more effective under supply chain decentralization (*vs.* centralization). A major cause is that under decentralization, the transshipment strategy provides a coordination tool, *i.e.*, the transshipment price. The distributors can choose an appropriate transshipment price which leads to appropriate ordering quantities and achieve horizontal coordination. On the other hand, when $v > \overline{v}^*$, no dominant strategy exists and the distributors need to choose an appropriate strategy based on competitive intensity and transshipment cost. We next compare the optimal strategy choices between centralization and decentralization.

Proposition 5.5. *Supply chain decentralization and centralization lead to different choices between the transshipment strategy and the no-transshipment strategy.*

From Proposition 5.1, we can see that distributors' strategy choice is not necessarily optimal for the entire supply chain under some conditions. Therefore, if the manufacturer wants to coordinate the supply chain, she must take into account both strategy choice and ordering quantity simultaneously.

6. Supply chain coordination

According to Cachon [5], flexible coordinating contracts should first provide incentives for the members of the decentralized supply chain to make decisions consistent with chain-wide profit maximization. Second, arbitrary divisions of the supply chain's profit should be able to be implemented by adjusting contractual parameters. In this section, we aim to find such contracts for the two-echelon supply chain with one manufacturer and two competing distributors under the transshipment strategy.

Comparing (5.7) with (4.2), we can see $q_i^{T^*} \neq q_i^{CT^*}$. This means that wholesale-price-only contracts cannot coordinate the supply chain under transshipment. To coordinate the supply chain, now we first try to use buyback contract which is widely used in practice.

6.1. Buyback contract

With a buyback contract the manufacturer pays the distributor b for each unsold unit at the end of the selling season. Let $\overline{X}$ to denote the condition under buy back contract. Under the buyback contract, the distributor's profit function is

$$\overline{\Pi}_i^T\left(q_i^T, q_j^T, b\right) = pR_i^T + (t-v)\, Z_{ij}^T + (p-t)\, Z_{ji}^T + b\left(q_i^T - R_i^T - Z_{ij}^T\right) - (w-b)\, q_i^T. \tag{6.1}$$

Using the same proof method of Shao *et al.* [35], we can show that there exists a unique Nash equilibrium. The equilibrium ordering quantity of distributor i satisfies:

$$F_i(\overline{q}_i^{T^*}) = \frac{p-w}{p-b} + \frac{t-v-b}{p-b}h_1 - \frac{p-t}{p-b}h_2. \tag{6.2}$$

We then study the transshipment price decision. Using the same method in the proof of Proposition 5.1, we can get that there exists an unique optimal transshipment price $\overline{t}^*$, which is given by

$$\overline{t}^* = \frac{(v+b)h_2^* + ph_1^*}{h_2^* + h_1^*}. \tag{6.3}$$

Substituting (6.3) in to (6.2), we can have

$$F_i(\overline{q}_i^{T^*}) = \frac{p-w}{p-b} + \frac{p-v-b}{p-b}(h_1 - h_2). \tag{6.4}$$

Comparing (6.4) with (4.2), we can see that to have $\overline{q}_i^{T^*} = q_i^{CT^*}$, the following equations should be satisfied simultaneously

$$\begin{cases} \dfrac{p-w}{p-b} = \dfrac{p-c}{p} \\ \dfrac{p-v-b}{p-b} = \dfrac{p-v}{p}. \end{cases} \tag{6.5}$$

From (6.5), we find that there does not exist any value of b which can satisfy the above two equations simultaneously. So buyback contract cannot coordinate the supply chain.

6.2. Buyback with srp contract

In this subsection, we combine the buyback contract with a sale rebate and penalty (SRP) contract. The combined contract are often used in supply chain coordination under the complex environment (*e.g.*, [6, 11]). Under buyback with SRP contract, the manufacturer provides buyback contract and SRP contract simultaneously. Under a SRP contract, the manufacturer sets up a sales target G for a distributor. If the distributor's sales are beyond the target, the manufacturer will give the retailer a rebate: a reward of s for each unit of product sold above G. Otherwise, the distributor will need to pay the manufacturer a penalty: a payment of s for each unit of product unsold below G. Let $\hat{X}$ to denote this condition.

Under this contract, Distributor i and the manufacturer's profits function are

$$\hat{\Pi}_i^T(q_i^T, q_j^T, s, b) = pR_i^T + (t-v)Z_{ij}^T + (p-t)Z_{ji}^T + s(R_i^T + Z_{ji}^T - G) + b\left(q_i^T - R_i^T - Z_{ij}^T\right) - wq_i^T \tag{6.6}$$

$$\hat{\Pi}_m^T(w, s, b) = (w-c)(q_i^T + q_j^T) - s(R_i^T + R_j^T + Z_{ji}^T + Z_{ij}^T - 2G) - b\left(q_i^T + q_j^T - R_i^T - R_j^T - Z_{ij}^T - Z_{ji}^T\right). \tag{6.7}$$

From (6.6), we firstly see that the ordering quantity has a unique Nash equilibrium and satisfies:

$$F_i(\hat{q}_i^{T^*}) = \frac{p-w+s}{p+s-b} + \frac{t-v}{p+s-b}h_1 - \frac{p-t+s}{p+s-b}h_2. \tag{6.8}$$

Hence, there exists an unique optimal transshipment price $\hat{t}^*$, which is given by

$$\hat{t}^* = \frac{(v+b)h_2^* + (p+s)h_1^*}{h_2^* + h_1^*}. \tag{6.9}$$

Substituting (6.9) in to (6.8), we can get

$$F_i(\hat{q}_i^{T^*}) = \frac{p-w+s}{p+s-b} + \frac{p-v+s-b}{p+s-b}(h_1 - h_2). \tag{6.10}$$

Comparing (6.10) with (4.2), we can see to have $\hat{q}_i^{T^*} = q_i^{CT^*}$, the following equations should be satisfied simultaneously

$$\begin{cases} \dfrac{p-w+s}{p+s-b} = \dfrac{p-c}{p} \\ \dfrac{p-v+s-b}{p+s-b} = \dfrac{p-v}{p}. \end{cases} \tag{6.11}$$

From (6.11), we can see that when $s^* = b^* = w - c$, the two equations of (6.11) are satisfied simultaneously. Therefore, the buyback with SRP contract can coordinate the supply chain under transshipment.

Substituting $s^* = b^* = w - c$ into (6.6) and (6.7), we get the total profit of the two distributors and the profit of the manufacturer:

$$\hat{\Pi}_i^T(q_i^T, q_j^T, s^*, b^*) + \hat{\Pi}_j^T(q_i^T, q_j^T, s^*, b^*) = \Pi^{CT}(q_i^T, q_j^T) - 2sG \tag{6.12}$$

$$\hat{\Pi}_m^T(w, s^*, b^*) = 2sG. \tag{6.13}$$

From (6.12) and (6.13), we can see that the total supply chain profit can be split as $[2sG, (\Pi^{CT} - 2sG)]$ between the manufacturer and the distributors. Any profit allocation can be realized by changing G. To sum up, we can get the following proposition:

Proposition 6.1. *When $s^* = b^* = w - c$, The buyback with SRP contract can coordinate the supply chain under transshipment, and arbitrary profit allocation can be achieved by varying G.*

In reality, members of a supply chain are willing to accept a coordination contract only when this contract leads to win-win outcomes. Therefore, the manufacturer should choose G to ensure that everyone is better off with the coordinating contract.

7. Numerical studies

To supplement our analytical results and gain more managerial insights, we conduct numerical studies in this section. We mainly focus on the impact of system parameters (v, e) on the optimal strategy choice under two cases: symmetric distributors and asymmetric distributors.

7.1. Impact of (v, e) on strategy choice under symmetric

We use the following base parameter values: $p = 20$, $c = 4$, $w = 8$, $v = 4$, $e = 0.8$, and D_i and D_j follow a uniform distribution with lower bound 0 and upper bound 400. For exposure clarity, we plot the profit ratio, namely, the profit with the transshipment strategy divided by the profit with the no-transshipment strategy. Obviously, when the profit ratio is more than one, the transshipment strategy should be adopted. In Figures 3 and 4, we vary the transshipment cost v and demand switching rate e.

Comparing Figures 3 and 4, we can see that the slope of the profit ratio under decentralization is larger than that under centralization. Meanwhile, the spacing between the profit ratios for different demand switching rates

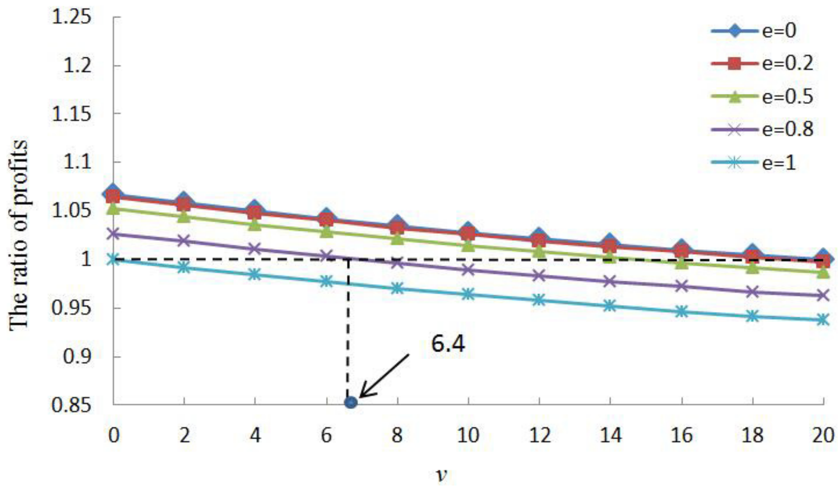


FIGURE 3. The impact of v and e on the strategy choice under centralization.

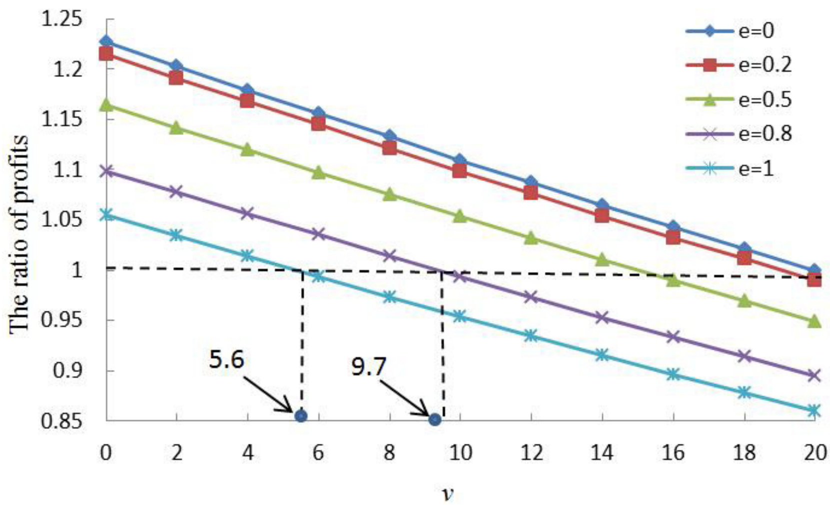


FIGURE 4. The impact of v and e on the strategy choice under decentralization.

under decentralization is also larger than that under centralization. This is summarized in Observation 1:

Observation 1. *The strategy choice is more sensitive to transshipment cost and degree of competition under decentralization.*

Observation 1 indicates that the distributors should pay closer attention to the strategy choice in the decentralized supply chain when competition or transshipment cost changes.

In addition, these numerical examples also can illustrate some our findings. Firstly, from Figures 3 and 4, we can find that the transshipment strategy is better when competition is weaker (smaller e) and the no-transshipment strategy is more attractive when competition is more intense (larger e). The results are consistent with Propositions 4.1 and 5.3. Secondly, from Figure 4, we can also observe that when $v \leqslant 5.6$, the distributors should always adopt the transshipment strategy, which is consistent with Proposition 5.4. Lastly, Comparing Figures 3 and 4, we can see that the no-transshipment strategy is optimal under centralization when $v > 6.4$ and

$e = 0.8$, while the transshipment strategy is optimal under decentralization when $6.4 < v < 9.7$ and $e = 0.8$. We can find that decentralization and centralization lead to different optimal strategy choices when $6.4 < v < 9.7$ and $e = 0.8$, which is consistent with Proposition 5.5.

7.2. Impact of (v, e) on strategy choice under asymmetric

Our basic model considers symmetric distributors. In this subsection, we consider the case where the distributors are asymmetric. We assume the distributor i and distributor j's demands are not identical, and $E(D_i) > E(D_j)$. We then mainly investigate the impact of this asymmetry on strategic choice under decentralized supply chain using numerical studies. To this end, we first need to get the distributors' ordering decisions under the asymmetric condition.

Under transshipment, according to Rudi *et al.* [27], a unique Nash equilibrium exists. The equilibrium ordering quantity of distributors satisfies:

$$\begin{cases} F_i(q_i^{T*}) = \dfrac{p-w}{p} + \dfrac{t-v}{p}h_1 - \dfrac{p-t}{p}h_2 \\ F_j(q_j^{T*}) = \dfrac{p-w}{p} + \dfrac{t-v}{p}\hat{h}_1 - \dfrac{p-t}{p}\hat{h}_2 \end{cases} \tag{7.1}$$

where $\hat{h}_1$ and $\hat{h}_2$ denote $\frac{\partial Z_{ji}}{\partial q_j}$ and $(-\frac{\partial Z_{ji}}{\partial q_i})$ respectively.

Under no-transshipment strategy, according to Parlar [24], there exists a unique Nash equilibrium. The equilibrium ordering quantity of distributors satisfies:

$$\begin{cases} F_i(q_i^{S^*}) = \dfrac{p-w}{p} + \hat{g}_1 \\ F_j(q_j^{S^*}) = \dfrac{p-w}{p} + g_1 \end{cases} \tag{7.2}$$

where $\hat{g}_1$ denotes $\frac{\partial U_{ji}}{\partial q_i}$.

Because the distributors are asymmetric, they have different optimal transshipment prices. We next study the impact of the transshipment prices on the distributors' profits using numerical simulation. The basic parameter values are given as: $p = 20$, $c = 4$, $w = 8$, $v = 4$, $e = 0.8$, and $D_i \sim Uniform[0, 400]$ and $D_j \sim Uniform[0, 200]$.

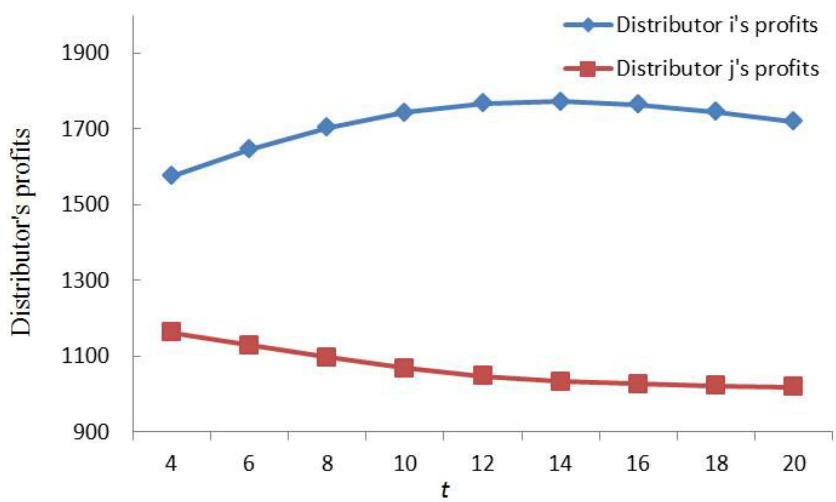
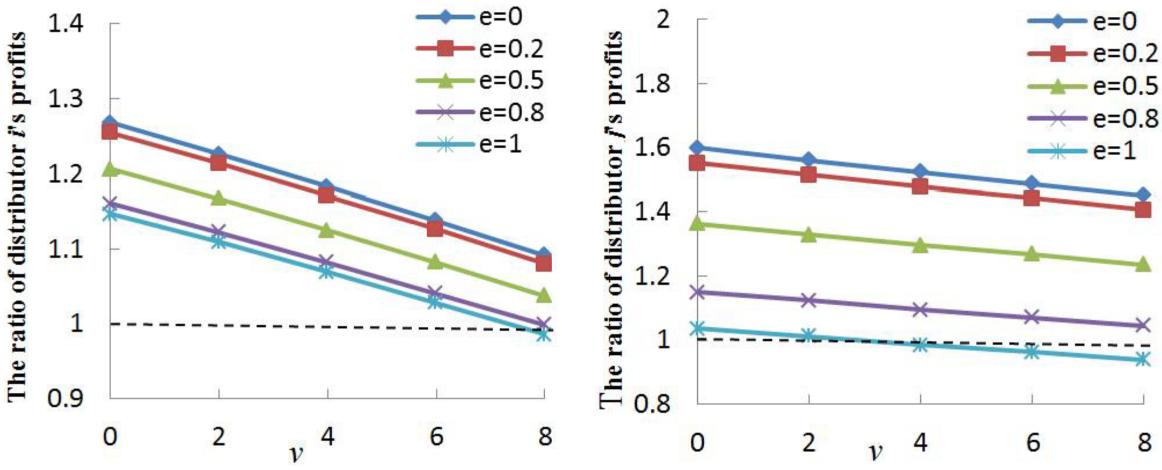
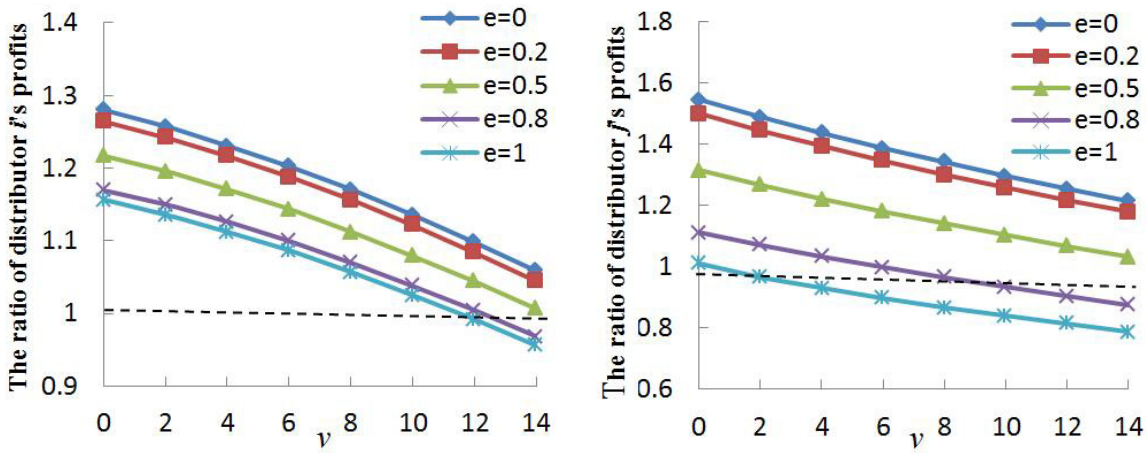


FIGURE 5. The impact of t on profits.



(a) (b)

FIGURE 6. The impact of v and e on strategy choice ($t = 8$).



(a) (b)

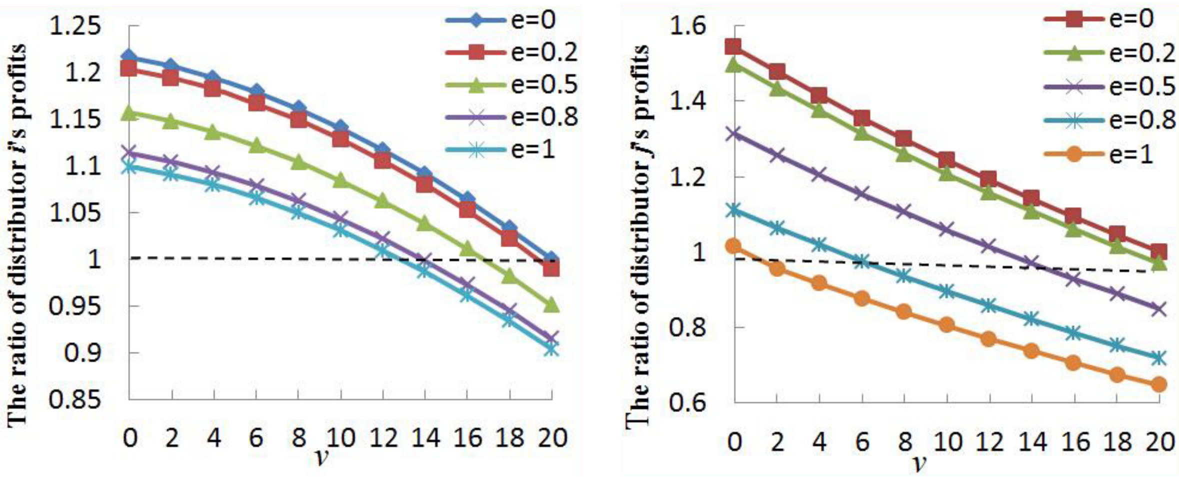
FIGURE 7. The impact of v and e on strategy choice ($t = 14$).

From Figure 5, we can have the following observation:

Observation 2. *For the distributor with low expected demand (LED),* i.e., *distributor j, his profit decreases with the transshipment price, and for the distributor with high expected demand (HED),* i.e., *distributor i, his profit first increases and then decreases with the transshipment price.*

From Observation 2, we can derive some useful insights. Firstly, both distributors, regardless of their expected demands, should not set high transshipment price which will hurt their profits. Secondly, for the distributor with LED, the low transshipment price can bring more profit to him. However, for the distributor with HED, the medium transshipment price is more suitable for him.

We next study the impact of transshipment cost and degree of competitive intensity on optimal strategy choice. From above, we know that there does not exist a unique optimal transshipment price and the transshipment price has different impacts on the distributors' profits. Therefore, we will study the strategy choice under



(a) (b)

FIGURE 8. The impact of v and e on strategy choice ($t = 20$).

different transshipment prices, *i.e.*, low price ($t = 8$), medium price ($t = 14$) and high price ($t = 20$). From Figures 6–8, we can have the following observations:

Observation 3. *Under the asymmetric distributors condition, there exists $\overline{v}_L^*(\overline{v}_H^*)$ for the distributor with LED (HED). When $v \leqslant \overline{v}_L^*(v \leqslant \overline{v}_H^*)$, regardless of the competitive intensity e, the distributor with LED (HED) always prefers the transshipment strategy.*

Observation 3 is similar to Proposition 5.4 under the symmetric case. The transshipment strategy also dominates for the distributor with LED (HED) when $v \leqslant \overline{v}_L^*(v \leqslant \overline{v}_H^*)$. A major reason is that under the asymmetric case, the transshipment strategy can not only lead to inventory transfer, but also induce them to choose more suitable ordering quantities and to achieve partial horizontal coordination. In addition, from Figures 6–8, we can find that $\overline{v}_L^*$ is less than $\overline{v}_H^*$. Of course, the distributors are both willing to accept the transshipment strategy only when this strategy leads to win-win outcomes. So only when $v \leqslant \overline{v}_L^*$, regardless of the competitive intensity, the distributors will always adopt the transshipment strategy. From $\overline{v}_L^* < \overline{v}_H^*$, we also can get that comparing to the distributor with LED, the transshipment strategy is more attractive for the distributor with HED.

Observation 4. *The transshipment strategy is better when competition is weaker* (*smaller* e) *and the non-transshipment strategy is more attractive when competition is more intense* (*larger* e).

Competition is weaker means that more unsatisfied consumers will be lost. The transshipment strategy can effectively reduce this potential loss. When competition is intense, most of the unsatisfied consumers will transfer to another distributor. In this process of transfer, the distributors do not need to pay extra fee. However, under the transshipment strategy, the inventory transfer occurs at a certain cost. So under this case, the non-transshipment strategy is better. In addition, Observation 4 is also consistent with the symmetric case.

8. CONCLUSIONS

In almost all extant transshipment literatures, they do not consider the inventory competition and supply chain coordination. To fill these gaps, this paper studies the effectiveness of the transshipment strategy under inventory competition and how to coordinate a supply chain in the presence of transshipment. To answer these questions, we establish a model which includes a manufacturer and two competing distributors.

For the first question, we find that under the centralized supply chain, the transshipment is not always beneficial for the supply chain. The transshipment strategy is better only when transshipment cost is lower or competition is weaker. Under the decentralized supply chain, when the transshipment cost is more than a threshold, we can have the same results as those under centralization. However, when the transshipment cost is lower than the threshold, we show that the transshipment strategy is always better than the no-transshipment. Moreover, we extend the model from symmetric distributors to asymmetric distributors and obtain similar results using numerical studies.

For the second question, we find that the order quantity decisions of the distributors are different under centralization and decentralization. We then design a buyback with sale rebate and penalty contract which can coordinate the supply chain and achieve win-win outcomes for all supply chain members.

There are some limitations in this paper which can also be future research directions. Firstly, in this paper, we assume that each distributor's sale price is exogenous. However, in reality, a distributor often can modify his sale price within a reasonable range. Hence a future research direction is to study the transshipment strategy when a distributor can decide on sale price. Secondly, following most extant research, we assume that the demands of the two distributors are independent. However, in some conditions, demands may be correlated. So a next step is to examine how this correlation may impact the transshipment strategy choice. Lastly, in this paper, we just consider a single period. So a future direction is to establish a multi-period model to study the transshipment strategy.

Appendix A.

A.1. Proof of Proposition 4.1

$$\frac{\mathrm{d}\Pi^{CT}}{\mathrm{d}v} = \frac{\partial \Pi^{CT}}{\partial v} + \frac{\partial \Pi^{CT}}{\partial q_i^{CT}}\frac{\partial q_i^{CT}}{\partial v} + \frac{\partial \Pi^{CT}}{\partial q_j^{CT}}\frac{\partial q_j^{CT}}{\partial v}\cdot \tag{A.1}$$

Because we obtain the $(q_i^{CT^*}, q_j^{CT^*})$ by solving the equation $\frac{\partial \Pi^{CT}}{\partial q_i^{CT}} = 0, \frac{\partial \Pi^{CT}}{\partial q_j^{CT}} = 0$, we can rearranging (A.1)

$$\frac{\mathrm{d}\Pi^{CT}}{\mathrm{d}v} = \frac{\partial \Pi^{CT}}{\partial v} = -Z_{ij}^{CT} - Z_{ji}^{CT} < 0$$

Therefore, $\Pi^{CT^*}(q_i^{CT^*}, q_j^{CT^*})$ decreases as v increases.

$$\frac{\mathrm{d}\Pi^{CS}}{\mathrm{d}e} = \frac{\partial \Pi^{CS}}{\partial e} + \frac{\partial \Pi^{CS}}{\partial q_i^{CS}}\frac{\partial q_i^{CS}}{\partial e} + \frac{\partial \Pi^{CS}}{\partial q_j^{CS}}\frac{\partial q_j^{CS}}{\partial e} \tag{A.2}$$

Because we obtain the $(q_i^{CT^*}, q_j^{CT^*})$ by solving the equation $\frac{\partial \Pi^{CT}}{\partial q_i^{CS}} = 0, \frac{\partial \Pi^{CT}}{\partial q_j^{CS}} = 0$, we can rearranging (A.2)

$$\frac{\mathrm{d}\Pi^{CS}}{\mathrm{d}e} = \frac{\partial \Pi^{CS}}{\partial e} = p\int_0^{q_i}\int_{q_j}^{\frac{q_i-x}{e}+q_j}(y-q_j)f_D(x,y)\mathrm{d}y\mathrm{d}x + p\int_0^{q_j}\int_{q_i}^{\frac{q_j-x}{y}+q_i}(x-q_i)f_D(x,y)\mathrm{d}x\mathrm{d}y > 0$$

Where $f_D(x,y)$ denotes the joint probability density of D_i and D_j. Therefore, Π^{CS*} increases as e increases.

A.2. Proof of Proposition 4.2

Comparing (4.1) with (4.3), we can find that $\Pi^{CT} = \Pi^{CS}$ when $e = 0$ and $v = p$. Based on Proposition 4.1, we then can get $\Pi_{\min}^{CT^*} = \Pi_{\min}^{CS^*}$. Comparing (4.1) with (4.3), we can find that $\Pi^{CT} = \Pi^{CS}$ when $e = 1$ and $v = 0$. Based on Proposition 4.1, we then can get $\Pi_{\max}^{CT^*} = \Pi_{\max}^{CS^*}$.

A.3. Proof of Proposition 5.1

With transshipment, distributor i's expected profit Π_i^T is

$$\Pi_i^T = pR_i^{T^*} + (t-v)Z_{ij}^{T^*} + (p-t)Z_{ji}^{T^*} - wq_i^{T^*} \tag{A.3}$$

From (a3), the first order derivative is

$$\frac{\mathrm{d}\Pi_i^T}{\mathrm{d}t} = \frac{\partial \Pi_i^T}{\partial t} + \frac{\partial \Pi_i^T}{\partial q_i^{T^*}}\frac{\partial q_i^{T^*}}{\partial t} + \frac{\partial \Pi_i^T}{\partial q_j^{T^*}}\frac{\partial q_j^{T^*}}{\partial t} \tag{A.4}$$

Shao *et al.* [35] proved that there exist unique Nash equilibrium solutions $(q_i^{T^*}, q_j^{T^*})$ and can obtain $(q_i^{T^*}, q_j^{T^*})$ by solving the equations and $\frac{\partial \Pi_i^T}{\partial q_i^T} = 0, \frac{\partial \Pi_j^T}{\partial q_j^T} = 0$. So we can rearrange (a4) as

$$\frac{\mathrm{d}\Pi_i^T}{\mathrm{d}t} = \frac{\partial \Pi_i^T}{\partial t} + \frac{\partial \Pi_i^T}{\partial q_j^{T^*}}\frac{\partial q_j^{T^*}}{\partial t} = Z_{ij}^{T^*} - Z_{ji}^{T*} + [(p-t)h_1^* - (t-v)h_2^*]\frac{\partial q_j^{T^*}}{\partial t} \tag{A.5}$$

And because we assume the two distributors are symmetric, $Z_{ij}^{T^*} = Z_{ji}^{T^*}$. Hence (a5) can be simplified to

$$\frac{\mathrm{d}\Pi_i^T}{\mathrm{d}t} = [(p-t)h_1^* - (t-v)h_2^*]\frac{\partial q_j^{T^*}}{\partial t} \tag{A.6}$$

From (a6), the second order derivative is

$$\frac{\mathrm{d}^2\Pi_i^T}{\mathrm{d}t^2} = [(p-t)h_1^* - (t-v)h_2^*]\frac{\partial^2 q_j^{T^*}}{\partial t^2} + \left[-h_1^* - h_2^* + (p-t)\frac{\partial h_1^*}{\partial t} - (t-v)\frac{\partial h_2^*}{\partial t}\right]\frac{\partial q_j^{T^*}}{\partial t} \tag{A.7}$$

From (6), and using the Implicit Function Theorem, we can get

$$\frac{\partial h_1^*}{\partial t} = -\frac{h_1^* + h_2^*}{t-v}, \frac{\partial h_2^*}{\partial t} = -\frac{h_1^* + h_2^*}{p-t} \tag{A.8}$$

Substituting (a8) into (a7), we can get

$$\frac{\mathrm{d}^2\Pi_i^T}{\mathrm{d}t^2} = [(p-t)h_1^* - (t-v)h_2^*]\frac{\partial^2 q_j^{T^*}}{\partial t^2} - (h_1^* + h_2^*)(1 + \frac{p-t}{t-v} + \frac{t-v}{p-t})\frac{\partial q_j^{T^*}}{\partial t} \tag{A.9}$$

We make the first derivative equal to zero, *i.e.*, $[(p-t)h_1^* - (t-v)h_2^*]\frac{\partial q_j^{T^*}}{\partial t} = 0$. Rudi *et al.* [27] proved $\frac{\partial q_j^{T^*}}{\partial t} > 0$, so we can obtain $[(p-t)h_1^* - (t-v)h_2^*] = 0$. Substituting $[(p-t)h_1^* - (t-v)h_2^*] = 0$ into (a9), we can have

$$\frac{\mathrm{d}^2\Pi_i^T}{\mathrm{d}t^2} = -(h_1^* + h_2^*)(1 + \frac{p-t}{t-v} + \frac{t-v}{p-t})\frac{\partial q_j^{T^*}}{\partial t} \tag{A.10}$$

According to Shao *et al.* [35], we can have $h_1 > 0$, $h_2 > 0$. And because $p > v$ and $t > v$, we can obtain $\frac{\mathrm{d}^2\Pi_i^T}{\mathrm{d}t^2} < 0$ when the first derivative equals zero. Therefore, we can know that Distributor i's expected profit $\Pi_i^T(t)$ is quasi-concave in t, and there exists an unique optimal transshipment price t^*, which is given by the first-order condition, *i.e.*, $t^* = \frac{vh_2^* + ph_1^*}{h_2^* + h_1^*}$, where h_2^* and h_1^* are function of t.

Proof of Proposition 5.2

We first assume that the two distributors belong to the same company, and define it as a chain store model. As a result, they are concerned with the total profit from both distributors and the transshipment price should be zero. Let $(\hat{i},\hat{j})$ denote this case when the two distributors are centralized. Under transshipment, the total expected profit of the chain store, denoted as $\Pi^T_{\hat{i}\hat{j}}$, is

$$\Pi^T_{\hat{i}\hat{j}}\left(q^T_{\hat{i}},q^T_{\hat{j}}\right)=p_iR^T_{\hat{i}}+p_jR^T_{\hat{j}}+p_iZ^T_{\hat{j}\hat{i}}-vZ^T_{\hat{i}\hat{j}}+p_jZ^T_{\hat{i}\hat{j}}-vZ^T_{\hat{j}\hat{i}}-w\left(q^T_{\hat{i}}+q^T_{\hat{j}}\right) \tag{A.11}$$

Rudi *et al.* [27] show that expected profit function is concave in $(q^T_{\hat{i}},q^T_{\hat{j}})$. So we can obtain the optimal ordering quantity as:

$$F_{\hat{i}}(q^{T^*}_{\hat{i}})=\frac{p-w}{p}+\frac{p-v}{p}(h_1-h_2) \tag{A.12}$$

Comparing (A.12) with (5.7), we find that adopting t^* lead to $q^{T^*}_{\hat{i}}=q^{T^*}_{i}$. Based on it, we can get that t^* can induce $\Pi^{T^*}_{\hat{i}\hat{j}}=2\Pi^{T^*}_{i}$. Therefore, the interests of the distributor and the chain store are consistent. Using the same method of Proof of Proposition 4.1, we can get that $\Pi^{T^*}_{\hat{i}\hat{j}}$ decreases as v increases. We then can obtain that $\Pi^{T^*}_{i}$ decreases in v.

$$\frac{\mathrm{d}\Pi^S_i}{\mathrm{d}e}=\frac{\partial\Pi^S_i}{\partial e}+\frac{\partial\Pi^S_i}{\partial q^S_i}\frac{\partial q^S_i}{\partial e}+\frac{\partial\Pi^S_i}{\partial q^S_j}\frac{\partial q^S_j}{\partial e} \tag{A.13}$$

Because we obtain the $q^{S^*}_i$ by solving the equation $\frac{\partial\Pi^S_i}{\partial q^S_i}=0$, we can rearranging (a13)

$$\frac{\mathrm{d}\Pi^S_i}{\mathrm{d}e}=\frac{\partial\Pi^S_i}{\partial e}+\frac{\partial\Pi^S_i}{\partial q^S_j}\frac{\partial q^S_j}{\partial e} \tag{A.14}$$

where

$$\frac{\partial\Pi^S_i}{\partial e}=p\int_0^{q_i}\int_{q_j}^{\frac{q_i-x}{e}+q_j}(y-q_j)f_D(x,y)\mathrm{d}y\mathrm{d}x>0$$

$$\frac{\partial\Pi^S_i}{\partial q^S_j}=g_1=\int_0^{q^S_j}\int_{\frac{q^S_j-x}{e}+q^S_i}^{+\infty}f_D(x,y)\mathrm{d}y\mathrm{d}x>0$$

From (8), we can get $q^{S^*}_{\tilde{j}}$ increases in e, so $\frac{\partial q^{S^*}_j}{\partial e}>0$ Based on above, we can obtain

$$\frac{\mathrm{d}\Pi^S_i}{\mathrm{d}e}=\frac{\partial\Pi^S_i}{\partial e}+\frac{\partial\Pi^S_i}{\partial q^S_j}\frac{\partial q^S_j}{\partial e}>0 \tag{A.15}$$

Therefore, $\Pi^{S^*}_i$ increases as e increases.

A.4. Proof of Proposition 5.3

We first compare the transshipment and no-transshipment under the chain store model. Under no-transshipment, the total expected profit of the chain store, denoted as $\Pi^S_{\hat{i}\hat{j}}$, is

$$\Pi^S_{\hat{i}\hat{j}}\left(q^{S^*}_{\hat{i}},q^{S^*}_{\hat{j}}\right)=pR^S_{\hat{i}}+pR^S_{\hat{j}}+pU^S_{\hat{j}\hat{i}}+pU^S_{\hat{i}\hat{j}}-w\left(q^S_{\hat{i}}+q^S_{\hat{j}}\right) \tag{A.16}$$

The optimal ordering quantity $(q^{S^*}_{\hat{i}},q^{S^*}_{\hat{j}})$ is characterized by

$$F_{\hat{i}}(q^{S^*}_{\hat{i}})=\frac{p-w}{p}+g_1-g_2 \tag{A.17}$$

Using the same method of Proof of Proposition 4.2, we can get $\Pi^{T^*}_{\hat{i}\hat{j}\min}=\Pi^{S^*}_{\hat{i}\hat{j}\min}$ and $\Pi^{T^*}_{\hat{i}\hat{j}\max}=\Pi^{S^*}_{\hat{i}\hat{j}\max}$.

Comparing equation (A.11) with (5.4), we can get $q_i^{S^*} = q_{\hat{i}}^{S^*}$ when $e = 0$, and $q_i^{S^*} > q_{\hat{i}}^{S^*}$ when $e = 1$. So we can get $\Pi^{S^*}_{\hat{i}\hat{j}\min} = 2\Pi^{S^*}_{i\min}$ and $\Pi^{S^*}_{\hat{i}\hat{j}\max} > 2\Pi^{S^*}_{i\min}$. In the Proof of Proposition 5.2, we have obtained $\Pi^{T^*}_{\hat{i}\hat{j}\min} = 2\Pi_i^{T^*}$, so we can get $\Pi^{T^*}_{i\max} > \Pi^{S^*}_{i\max}$ and $\Pi^{T^*}_{i\min} = \Pi^{S^*}_{i\min}$.

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