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INVENTORY CONTROL AND PRICING FOR REGRET-AVERSE NEWSVENDOR

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Abstract. The decision maker's perception of regret affects a company's inventory control and pricing decisions. In this paper, we investigate how regret aversion behaviors affect the inventory control and pricing decisions under a newsvendor setting. To capture the regret aversion behaviors of the newsvendor, we provide a regret aversion utility function. Based on the built regret aversion utility function and the classic inventory control and pricing model, we construct utility function by integrating the profit utility and the regret aversion utility, and then analyze the conditions of optimal solution on the inventory and pricing policy under additive and multiplicative demand in details. Further, by the analysis of properties and numerical study, we show that the optimal policy for regret-averse newsvendor deviates from the one for regret-neutral newsvendor and changes with the regret aversion parameters to varying degree. We also show the impact tendency of newsvendor's regret aversion behaviors on the optimal inventory and pricing policy under the additive and multiplicative demand.

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1. Introduction

Inventory management and pricing decisions are critical for most operations management in industries, if not for all. Joint inventory control and pricing problems have been addressed intensively from the perspective of Operations Research (OR) (*e.g.*, see Whitin [35]; Lau and Lau [20]; Emmons [13]; Petruzzi and Dada [25]; Smith *et al.* [31]; You [38]; Chen and Bell [7]; Hua *et al.* [18]; Yu *et al.* [39]; Merzifonluoglu and Feng [23]; Devi *et al.*, [11], and Raza and Turiac [28]). However, most of existing analytical models and solutions are based on the strong assumption of rationality of decision makers. The research results fail to consider decision maker's behavioral factors. It leads to the limited implementation of rational solutions in the real world because the underlying assumptions of analytical OR models do not fully comply with business realities.

Behavioral economics and agent theory show that agents may also care about factors like bounded rationality, reference dependence, loss aversion, overconfidence, reciprocity, fairness, and status in addition to the direct

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economic benefits (*e.g.*, see Wu and Chen [36]; Fehr and Gächter [15]; Ho *et al.* [16]; Qiao *et al.*, [26]; Camerer and Loewenstein [5]; Wang [33]; Wang and Webster [34]; Ren and Croson [29], and Becker-Peth and Thonemann [2]). This also holds in the real world operations management processes. For example, given the demand uncertainties in reality, the newsvendor may exhibit regret aversion behaviors in the inventory control and pricing decisions. The regret may be caused by the direct missing profit or non-tangible/non-capital losses, *e.g.*, stakeholder's dissatisfaction or the lost good-will of the customers. These non-tangible losses usually play important roles in business decision making process, but they might not have been captured by traditional loss penalties in the classic profit maximization decision objectives. Therefore, it is necessary to take the impacts of regret aversion into consideration of inventory control and pricing.

In this paper, we consider the newsvendor exhibiting regret aversion in decisions, and investigate how regret aversion behaviors affect the inventory control and pricing decisions under a newsvendor setting. We first define the regret aversion utility function for surplus and stock-out regret aversion utilities. Based on the built regret aversion utility function and the classic inventory control and pricing model, we construct the newsvendor's utility function by integrating profit utility and regret aversion utility, and then analyze the conditions on the optimal value under additive and multiplicative demand. By the analysis of the properties and numerical study, we show that the regret aversion behaviors can affect the newsvendor's optimal policy to varying degree in a certain trend, and show the optimal policy deviates from the one of regret-neutral newsvendor.

The main contributions of our study are three folds: first, we analyze the behavioral utility for the joint pricing and ordering quantity decision problem with regret aversions, and propose a linear function to describe and capture the regret aversion utility of the newsvendor. Then, we formulate the inventory control and pricing model for the regret-averse newsvendor and solve the model to optimality. Thirdly, the impacts of regret aversions on the optimal policy are given for adjusting the perfectly rational solutions to better align with business practices. To our best knowledge, this is an earlier paper to study the inventory control and pricing problems with regret aversions.

The rest of the paper is organized as below. Section 2 reviews the relevant literatures. Section 3 introduces regret aversion utility into the traditional inventory control and pricing decision model in which the surplus regret aversion and stock-out regret aversion are demonstrated. Section 4 addresses the joint inventory control and pricing decisions under the additive and multiplicative demand. A numerical study is conducted in Section 5 in order to show the impacts of regret aversion on the optimal price and order quantity. Section 6 presents the managerial insights of our study. Section 7 concludes with a brief discussion of future research directions. All proofs are provided in the technical appendix.

2. Literature review

The joint inventory control and pricing decision problem has attracted considerable research interests. Whitin [35] first raises the joint inventory control and pricing decision problem based on the classical newsvendor model. Then, Petruzzi and Dada [25] extends Whitin's work, they discuss the optimal solution for the joint inventory and pricing decision problem under the additive and multiplicative demand. Since then, the joint inventory control and pricing remains a fruitful research topic. Chan *et al.* [6] provide a thorough literature review on joint inventory control and pricing decisions. Subsequently, Zhang [40] proposes a unified modeling framework for this problem, and characterizes the structure of the optimal policies on inventory control and pricing decisions. Given the limited space, we focus on the studies which are closely related to our study and the most recent developments.

Recently, various extended joint problems are addressed extensively. Dye [12] constructs a deterministic inventory model for deteriorating items with time-dependent backlogging rate. Ouyang *et al.* [24] introduce the time factor into the joint inventory control and pricing model. Chen and Bell [7] build a joint inventory control and pricing decision model considering product returns. Yang *et al.* [37] explore that if the product has greater price elasticity, the best strategy is always to price lower and order more.

However, all studies mentioned above are based on the assumption that the newsvendor is perfectly rational. In practice, the real inventory control and pricing decisions are often inconsistent with the optimal rational solutions of OR models. To mitigate the inconsistency, both analytical and experimental studies considering the newsvendor's behavioral factors are implemented. The research results show that the newsvendor's behavioral factors such as bounded rationality, fairness, and loss aversion, often affect the operations management related decisions.

Su [32] addresses a newsvendor problem with the bounded rationality. The results verify some anomalies highlighted by recent experimental findings. Wang and Webster [34] explore a scenario where a loss-averse newsvendor may order more than a risk-neutral newsvendor does when shortage cost is not negligible. Wang [33] investigates the competition problem between multiple loss-averse newsvendors and a risk-neutral supplier. He concludes that the loss aversion effect causes decreasing of the newsvendors' total inventory. Ma *et al.* [22] propose a penalty model to the loss-averse newsvendor if a target profit is not attained. Chen *et al.* [9] show that newsvendors become more rational through repeated game play, but may not converge to perfect rationality assumed by the Nash equilibrium. Cui *et al.* [10] incorporate the fairness into the supply chain coordination models. They find that the wholesale price contract can make channel coordination when considering fairness. Ho *et al.* [17] investigate how the distributional and peer-induced fairness affect the results of channel coordination.

Apart from the analytical studies above, experimental study is also prevalent in the behavioral operations management area. Schweitzer and Cachon [30] study the newsvendor problem by experiments considering multiple psychological behaviors of the newsvendor. The experiment results show that real decision-making results systematically deviate from the rational analytical solutions. Bostian *et al.* [3] discover a "pull-to-center" effect through experiments, *i.e.*, average order quantities are too low when they should be high under the optimal rationality assumptions and vice versa. Chen and Kök [8] discuss the effect of payment schemes on inventory decisions considering the role of mental accounting. Katok and Pavlov [19] implement an experimental study on the impacts of the bounded rationality, inequality aversion and incomplete information on the channel inefficiency, and show that all three factors affect human decision making behavior to varying degree.

Although behavioral factors have attracted attention of many OR scholars, the joint inventory control and pricing problem with regret aversions remains unaddressed. The newsvendor may exhibit regret aversion in reality, *i.e.*, the decision maker percepts extra (higher) loss than the over-stock or understock costs counted in the profit function. Therefore, how to integrate the regret aversion perceptions into the joint inventory control and pricing model is important both in theoretical and practical perspectives.

3. Formulations

We consider a single item inventory control and pricing problem under newsvendor settings. The newsvendor needs to make decisions on the order quantity and retail price at the beginning of the selling season. A unit ordering cost incurs for each unit of the product ordered. If there is remaining inventory at the end of the selling season, a unit salvage cost incurs; if there is unsatisfied demand during the selling season, a unit stock-out cost incurs. In addition, the newsvendor faces a price dependent stochastic demand; the demand is non-increasing in price with a random factor. In Table 1, we summarize the used symbols and show the notation.

Based on the classic newsvendor model, the profit function (Petruzzi and Dada [25]) is,

Table 1. Notation.

Symbols	Description
Decision variables:	
p	The price of the regret-averse retailer for the general demand, $p \in [\underline{p}, \bar{p}]$, $\bar{p}$ and $\underline{p}$ are the upper and lower bounds of the range of price p, $\bar{p} > \underline{p} \geq 0$. For additive demand, the price is p_a, $p_a \geq 0$; for the multiplicative demand, the price p is p_m, $p_m \geq 0$.

Q	The order quantity of the regret-averse retailer for the general demand, $Q \geq 0$. For the additive demand, the order quantity Q is Q_a, $Q_a \geq 0$; for the multiplicative demand, the order quantity Q is Q_m, $Q_m \geq 0$.
Parameters:	
c	The per-unit ordering cost, $c \geq 0$.
s	The per-unit penalty cost, $s \geq 0$.
v	The per-unit salvage value, $v \geq 0$.
ε	The random factor of demand, it is a random variable defined on the range $[A, B]$, μ is the mean of the random factor ε.
k	The regret aversion parameter, $k \geq 0$. For the surplus situation, $k = \alpha$; for the stock-out situation, $k = \beta$.
α	The surplus regret aversion parameter, $\alpha \geq 0$.
β	The stock-out regret aversion parameter, $\beta \geq 0$.
z	The safety stock of the regret-averse retailer for general demand, $z \geq 0$. For the additive demand, the safety stock z is z_a, $z_a \geq 0$; for the multiplicative demand, the safety stock z is z_m, $z_m \geq 0$.
p_x	The symbol which substitutes a long equation, it is used in the description for the optimal price under multiplicative demand.
Functions:	
$f(\cdot)$	The probability density function of the random factor ε.
$F(\cdot)$	The cumulative distribution function of the random factor ε.
$D(\cdot)$	The general demand function. The additive demand function is $D_a(\cdot)$, the multiplicative demand function is $D_m(\cdot)$.
$M(\cdot)$	The hazard rate for the additive demand.
$T(\cdot)$	The hazard rate for the multiplicative demand.
$\pi(\cdot)$	The profit function, $\pi^{\max}$ denotes the theoretical maximal profit.
$r(\cdot)$	The regret aversion utility function.
$U(\cdot)$	The utility function.
Optimal values:	
p_a^*	The optimal retail price determined by the regret-averse retailer $(\alpha, \beta > 0)$ under the additive demand.
Q_a^*	The optimal order quantity determined by regret-averse retailer $(\alpha, \beta > 0)$ under the additive demand.
z_a^*	The optimal safety stock determined by the regret-averse retailer $(\alpha, \beta > 0)$ under the additive demand.
p_m^*	The optimal retail price determined by the regret-averse retailer $(\alpha, \beta > 0)$ under the multiplicative demand.
Q_m^*	The optimal order quantity determined by regret-averse retailer $(\alpha, \beta > 0)$ under the multiplicative demand.
z_m^*	The optimal safety stock determined by the regret-averse retailer $(\alpha, \beta > 0)$ under the multiplicative demand.
p_{a-n}^*	The optimal retail price determined by the regret-neutral retailer $(\alpha = \beta = 0)$ under the additive demand.
z_{a-n}^*	The optimal safety stock determined by regret-neutral retailer $(\alpha = \beta = 0)$ under the additive demand.
p_{m-n}^*	The optimal retail price determined by the regret-neutral retailer $(\alpha = \beta = 0)$ under the multiplicative demand.
z_{m-n}^*	The optimal safety stock determined by regret-neutral retailer $(\alpha = \beta = 0)$ under the multiplicative demand.

p_a^0	The optimal retail price determined by risk-neutral retailer under the additive deterministic demand, where additive deterministic demand $D_a(\cdot) = a + bp_a + \mu$.
Q_a^0	The optimal order quantity determined by risk-neutral retailer under the additive deterministic demand, where the additive deterministic demand $D_a(\cdot) = a + bp_a + \mu$.
p_m^0	The optimal retail price determined by risk-neutral retailer under the multiplicative deterministic demand, where the multiplicative deterministic demand $D_m(\cdot) = a\,(p_m)^{-b}\mu$.
Q_m^0	The optimal order quantity determined by risk-neutral retailer under multiplicative deterministic demand, where multiplicative deterministic demand $D_m(\cdot) = a\,(p_m)^{-b}\mu$.

$$\pi = \begin{cases} (p-c)D(p,\varepsilon) - (c-v)[Q - D(p,\varepsilon)], & D(p,\varepsilon) < Q \\ (p-c)Q - s[D(p,\varepsilon) - Q], & D(p,\varepsilon) \geq Q \end{cases} \tag{3.1}$$

As we well know, the objective function max π is also to balance the over-stock and under-stock costs. Therefore, the ideal case is that the order quantity equals to the realized demand, *i.e.*, $D(p,\varepsilon) = Q$. Under the ideal case, we know that the theoretical maximal profit $\pi^{\max} = (p-c)D(p,\varepsilon)$. However, in fact, the order quantity often deviates from the realized demand and the newsvendor often experiences over-stock or under-stock situations. Although the lost sales and over-storage costs are considered in the profit maximization model, the newsvendor may also percept regrets on the decisions which are beyond the lost sales and over-storage penalties, *i.e.*, surplus regret and stock-out regret. Usually, the newsvendor exhibits regret aversion.

The surplus regret aversion utility refers to the negative utility caused by the surplus regret aversion behaviors of decision makers. The surplus regret aversion refers to the psychological behavior where the newsvendor intends to avoid that the real profit is lower than the reference profit (reference point) and the order quantity is greater than the demand. The surplus cost is the loss of unsold products when the order quantity is greater than the demand. Similarly, the stock-out regret aversion utility refers to the negative utility caused by the stock-out regret aversion behaviors of the decision makers. The stock-out regret aversion refers to the psychological behavior that the newsvendor intends to avoid when the real profit is lower than the reference profit (reference point) and the order quantity is lower than the demand. The stock-out cost is the loss of the unsatisfied demands when the order quantity is lower than the demand. Thus, we distinguish the surplus (stock-out) regret aversion utility from the surplus (stock-out) cost.

If the newsvendor orders too much such that the real profit is lower than the reference profit, the newsvendor perceives the surplus regret aversion utility; if the newsvendor orders too little such that the real profit is lower than the reference profit, then the newsvendor perceives the stock-out regret aversion utility. The utilities of the regret aversions can be measured by the difference between the realized profit and the reference profit. The reference profit may be the expected profit (it is a fixed value in most cases) or the theoretical maximum profit (it changes with price, order quantity or demand, *etc.*). In this paper, given the variability of the theoretical maximum profit and the profit-driven newsvendor, the theoretical maximum profit is considered as the reference profit.

Therefore, the regret aversion utility can be measured by the difference between the realized profit and the theoretical maximal profit and the regret sensitivity, *i.e.*, $r\,(\pi, \pi^{\max})$. For the tractability, we apply a linear regret aversion utility function commonly used in literatures (*e.g.*, see Bell [1]; Looms and Sugden [21]; Brann and Muermann [4] and Engelbrecht-Wiggans and Katok [14]), *i.e.*,

$$r\,(\pi, \pi^{\max}) = -k\,\left(\pi^{\max} - \pi\right) \tag{3.2}$$

When the inventory is higher than the realized demand, *i.e.*, $D(p,\varepsilon) < Q$, the newsvendor exhibits surplus regret aversion, and according to the regret theory, the utility of the surplus regret aversion is

$$r\left(\pi, \pi^{\max}\right)_{D<Q} = -\alpha\,\left(\pi^{\max} - \pi\right) \tag{3.3}$$

When the inventory is lower than or equal to the realized demand, *i.e.*, $D(p,\varepsilon) \geq Q$, the newsvendor exhibits stock-out regret aversion, the utility of the stock-out regret aversion is

$$r\left(\pi, \pi^{\max}\right)_{D\geq Q} = -\beta\,\left(\pi^{\max} - \pi\right) \tag{3.4}$$

Furthermore, taking the regret aversion into account, we consider the newsvendor's decision objective to be maximizing the profit and minimizing the regrets. Thus, the integrated utility function of the newsvendor can be written as

$$U(\pi, \pi^{\max}) = \begin{cases} \pi - \alpha(\pi^{\max} - \pi),\ D(p,\varepsilon) < Q \\ \pi - \beta(\pi^{\max} - \pi),\ D(p,\varepsilon) \geq Q \end{cases} \tag{3.5}$$

Substitutes equation (3.1) into equation (3.5), we have

$$U(p,Q) = \begin{cases} (p-c)D(p,\varepsilon) - (1+\alpha)(c-v)[Q - D(p,\varepsilon)],\ D(p,\varepsilon) < Q \\ (p-c)Q - [s + \beta(p-c+s)]\,[D(p,\varepsilon) - Q], \quad D(p,\varepsilon) \geq Q \end{cases} \tag{3.6}$$

The equation (3.6) is the joint inventory control and pricing decision model. In the following, we provide the solutions to the constructed model under the additive and multiplicative demand, respectively.

4. Optimal solutions with endogenous price

Based on the above analysis, we further look into the conditions of optimal solutions to synchronize order quantity and price decisions. The additive and multiplicative price dependent demand functions commonly applied in literatures are considered in this section.

4.1. Additive demand

The additive demand function can be defined as (Petruzzi and Dada [25]), *i.e.*,

$$D_a(p_a,\varepsilon) = y(p_a) + \varepsilon \tag{4.1}$$

Where $y(p_a) = a - bp_a$, $p_a \in [\underline{p}, \bar{p}]$, and parameter a $(a > 0)$ represents the market size of the product, b $(b > 0)$ is the price sensitivity.

By substituting equation (4.1) into equation (3.6), the utility function can be rewritten as

$$U(p_a,Q_a) = \begin{cases} (p_a-c)[y(p_a)+\varepsilon] - (1+\alpha)(c-v)\{Q_a - [y(p_a)+\varepsilon]\},\ y(p_a)+\varepsilon < Q_a \\ (p_a-c)Q_a - [s+\beta(p_a-c+s)]\,\{[y(p_a)+\varepsilon] - Q_a\}, \qquad y(p_a)+\varepsilon \geq Q_a \end{cases} \tag{4.2}$$

To facilitate the further analysis, let $z_a = Q_a - y(p_a)$ (Petruzzi and Dada [25]), then $Q_a = y(p_a) + z_a$, and hence, the case $D_a(p_a,\varepsilon) < Q_a$ is the equivalent of $\varepsilon < z_a$, the case $D_a(p_a,\varepsilon) \geq Q_a$ is the equivalent of $\varepsilon \geq z_a$, then equation (4.2) can be rewritten as

$$U(z_a,p_a) = \begin{cases} (p_a-c)[y(p_a)+\varepsilon] - [(1+\alpha)(c-v)](z_a - \varepsilon), \qquad \varepsilon < z_a \\ (p_a-c)[y(p_a)+z_a] - [\beta(p_a-c+s)+s](\varepsilon - z_a),\ \varepsilon \geq z_a \end{cases} \tag{4.3}$$

Given $\varepsilon \in [A, B]$, the expected utility function can be determined, *i.e.*,

$$\begin{aligned} E[U(z_a, p_a)] &= \int_A^{z_a} \{(p_a - c)[y(p_a) + \varepsilon] - [(1+\alpha)(c-v)](z_a - \varepsilon)\}\,\mathrm{d}\varepsilon \\ &\quad + \int_{z_a}^{B} \{(p_a - c)[y(p_a) + z_a] - [\beta(p_a - c + s) + s](\varepsilon - z_a)\}\,\mathrm{d}\varepsilon \\ &= (p_a - c)[y(p_a) + \mu] - (1+\alpha)(c-v)\int_A^{z_a} (z_a - \varepsilon) f(\varepsilon)\mathrm{d}\varepsilon \\ &\quad - (1+\beta)(p_a - c + s)\int_{z_a}^{B} (\varepsilon - z_a) f(\varepsilon)\mathrm{d}\varepsilon \end{aligned} \tag{4.4}$$

For ease of exposition, we rewrite equation (4.4) as

$$E[U(z_a, p_a)] = \varphi(p_a) - (1+\alpha)L_a(z_a, p_a) - (1+\beta)S_a(z_a, p_a) \tag{4.5}$$

where $L_a(z_a, p_a) = (c-v)\int_A^{z_a} (z_a - \varepsilon) f(\varepsilon)\mathrm{d}\varepsilon$ denotes the surplus loss which leads to the surplus regret under the additive demand, $S_a(z_a, p_a) = (p_a - c + s)\int_{z_a}^{B} (\varepsilon - z_a) f(\varepsilon)\mathrm{d}\varepsilon$ denotes the stock-out loss which leads to the stock-out regret under the additive demand. $\varphi_a(p_a) = (p_a - c)[y(p_a) + \mu]$ denotes the deterministic expected utility of the newsvendor under the additive demand. According to $\varphi_a(p_a)$, the optimal risk-neutral price, $p_a^0 = \frac{a+bc+\mu}{2b}$, is obtained for the deterministic situation, and the corresponding order quantity is $Q_a^0 = y(p_a^0) + \mu = a - bp_a^0 + \mu$.

Then, according to equation (4.4), the first and second order derivatives with respect to z_a and p_a can be obtained, *i.e.*,

$$\frac{\partial E[U(z_a, p_a)]}{\partial z_a} = -\left[(1+\alpha)(c-v) + (1+\beta)(p_a - c + s)\right] y(p_a)\, F(z_a) + (1+\beta)(p_a - c + s) y(p_a) \tag{4.6}$$

$$\frac{\partial^2 E[U(z_a, p_a)]}{\partial {z_a}^2} = -\left[(1+\alpha)(c-v) + (1+\beta)(p_a - c + s)\right] y(p_a)\, f(z_a) < 0 \tag{4.7}$$

$$\frac{\partial E[U(z_a, p_a)]}{\partial p_a} = 2b(p_a^0 - p_a) - (1+\beta)\int_{z_a}^{B} (\varepsilon - z_a) f(\varepsilon)\mathrm{d}\varepsilon \tag{4.8}$$

$$\frac{\partial^2 E[U(z_a, p_a)]}{\partial {p_a}^2} = -2b < 0 \tag{4.9}$$

Furthermore, to maximize the expected utility of the newsvendor, we have Lemma 4.1 and Theorem 4.2.

Lemma 4.1. *For a fixed z_a, we have*:

(a) *If $\beta \geq \frac{2b(p_a^0 - \underline{p})}{\int_{z_a}^{B} (\varepsilon - z_a) f(\varepsilon)\mathrm{d}\varepsilon} - 1$ or $0 \leq \beta \leq \frac{2b(p_a^0 - \bar{p})}{\int_{z_a}^{B} (\varepsilon - z_a) f(\varepsilon)\mathrm{d}\varepsilon} - 1$, then price p_a^* is the boundary price, i.e., $p_a^* = \underline{p}$ or $p_a^* = \bar{p}$.*

(b) *If $0 \leq \frac{2b(p_a^0 - \bar{p})}{\int_{z_a}^{B} (\varepsilon - z_a) f(\varepsilon)\mathrm{d}\varepsilon} - 1 \leq \beta \leq \frac{2b(p_a^0 - \underline{p})}{\int_{z_a}^{B} (\varepsilon - z_a) f(\varepsilon)\mathrm{d}\varepsilon} - 1$, then price p_a^* is determined uniquely as a function of z_a, i.e.,*

$$p_a^* = p(z_a) = p_a^0 - \frac{1+\beta}{2b}\int_{z_a}^{B} (\varepsilon - z_a) f(\varepsilon)\mathrm{d}\varepsilon \tag{4.10}$$

and $p_a^* < p_a^0$.

Proof. See Appendix.

It is easy to see that $p_a^* < p_a^0$ for (b) in Lemma 4.1, and that the optimal regret-neutral price is obtained when $\beta = 0$, *i.e.*, $p_{a-n}^* = p_a^0 - \frac{1}{2b}\int_{z_a}^{B}(\varepsilon - z_a)f(\varepsilon)\mathrm{d}\varepsilon$, then the regret-neutral safety stock z_{a-n}^* corresponding to p_{a-n}^* can be determined.

By substituting $p_a^* = p(z_a)$ into $\max\limits_{z_a,p_a} E[U(z_a,p_a)]$, the optimization problem becomes maximization over a single variable z_a, *i.e.*, $\max\limits_{z_a} E\{U[z_a,p(z_a)]\}$. On the basis of this, When z_a is determined, price p_a^* and order quantity Q_a^* are determined. For the convenience of the description, let $M(z_a) = f(z_a)/[1-F(z_a)]$, $M(z_a)$ is the hazard rate.

Theorem 4.2. *For the additive demand, the optimal policy is to order* $Q_a^* = y(p_a^*) + z_a^*$ *units to sell at price* p_a^*, *where* p_a^* *is specified by Lemma* 4.1 *and* z_a^* *is determined as shown below*:

(a) *If price* p_a^* *is* $\underline{p}$ *or* $\bar{p}$, *i.e.*, $\beta \geq \frac{2b(p_a^0-\underline{p})}{\int_{z_a}^{B}(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon} - 1$ *or* $0 \leq \beta \leq \frac{2b(p_a^0-\bar{p})}{\int_{z_a}^{B}(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon} - 1$, *then the optimal value of* z_a *can be determine by*

$$F(z_a) = \frac{(1+\beta)(p_a^* - c + s)}{(1+\alpha)(c-v) + (1+\beta)(p_a^* - c + s)} \tag{4.11}$$

where $p_a^* = \bar{p}$ *or* $p_a^* = \underline{p}$.

(b) *If price* $p_a^* = p^0 - \frac{1+\beta}{2b}\int_{z}^{B}(\varepsilon - z)f(\varepsilon)\mathrm{d}\varepsilon$, *i.e.*, $0 \leq \frac{2b(p_a^0-\bar{p})}{\int_{z_a}^{B}(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon} - 1 \leq \beta \leq \frac{2b(p_a^0-\underline{p})}{\int_{z_a}^{B}(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon} - 1$, *then the optimal value of* z_a *can be determined as described below*:

(i) *If* $F(\varepsilon)$ *satisfies the condition* $2M(z_a)^2 + \mathrm{d}M(z_a)/\mathrm{d}z_a \geq 0$, *then* z_a^* *is the largest* z_a *in the region* $[A,B]$ *that satisfies* $\partial E[U(z_a,p_a^*)]/\partial z_a = 0$.

(ii) *If* $2M(z_a)^2 + \mathrm{d}M(z_a)/\mathrm{d}z_a \geq 0$ *and* $2b(p_a^0 - c + s) - (1+\beta)(\mu - A) > 0$, *then* z_a^* *is the unique* z_a *in the region* $[A,B]$ *that satisfies* $\partial E[U(z_a,p_a^*)]/\partial z_a = 0$.

Proof. See Appendix.

Corollary 4.3. *The condition* $2M(z_a)^2 + \mathrm{d}M(z_a)/\mathrm{d}z_a \geq 0$ *in Theorem 4.2 holds for the uniform distribution and the exponential distribution.*

Proof. See Appendix.

The economic meaning of the condition (*i.e.*, $2M(z_a)^2 + \mathrm{d}M(z_a)/\mathrm{d}z_a \geq 0$) refers to the constraint of the requirement of the hazard rate. Here, the hazard rate is related to the probability density function and the cumulative distribution function.

According to Theorem 4.2, the optimal solution of z_a^* is determined by the following condition, *i.e.*,

$$F(z_a^*) = \frac{(1+\beta)(p_a^* - c + s)}{(1+\alpha)(c-v) + (1+\beta)(p_a^* - c + s)} \tag{4.12}$$

Furthermore, when z_a^* is determined, according to equation (4.10) and $Q_a^* = y(p_a^*) + z_a^*$, the optimal order quantity is determined, *i.e.*,

$$Q_a^* = y(p_a^*) + z_a^* = a - bp_a^* + z_a^* \tag{4.13}$$

Based on the above analysis, we analyze the impacts of the newsvendor's regret aversion parameters α and β on price p_a^* under the additive demand.

Proposition 4.4. *Given p_a^*, z_a^* decreases with parameter α, and Q_a^* decreases with parameter α, i.e., the higher the newsvendor's surplus regret aversion degree is, the smaller the order quantity is. z^* increases with parameter β, and Q_a^* increases with parameter β, i.e., the higher the newsvendor's stock-out regret aversion degree is, the higher the order quantity is.*

Proof. See Appendix.

Proposition 4.5. *Given z_a^*, p_a^* increases with parameter α, i.e., the higher the newsvendor's surplus regret aversion degree is, the higher the price is, until the price reaches the upper bound of the range $\bar{p}$. p_a^* decreases with parameter β, i.e., the higher the newsvendor's stock-out regret aversion degree is, the lower the price is, until the price reaches the lower bound of the range $\underline{p}$.*

Proof. See Appendix.

4.2. Multiplicative demand

The multiplicative demand function can be defined as

$$D_m(p_m, \varepsilon) = y(p_m)\,\varepsilon \tag{4.14}$$

where $y(p_m) = a\,(p_m)^{-b}$ denotes a non-increasing function of price p_m, $p_m \in [\underline{p}, \bar{p}]$, a ($a > 0$) represents the market size of the product, b ($b > 1$) is the price sensitivity.

According to equations (3.6) and (4.14), the newsvendor's utility function is given below, *i.e.*,

$$U(p_m, Q_m) = \begin{cases} (p_m - c)y(p_m)\,\varepsilon - (1+\alpha)(c-v)[Q_m - y(p_m)\,\varepsilon], & y(p_m)\,\varepsilon < Q_m \\ (p_m - c)Q_m - [s + \beta(p_m - c + s)]\,[y(p_m)\,\varepsilon - Q_m], & y(p_m)\,\varepsilon \geq Q_m \end{cases} \tag{4.15}$$

Let $z_m = Q_m/y(p_m)$, then $Q_m = y(p_m)z_m$, hence, the case $D_m(p_m, \varepsilon) < Q_m$ is the equivalent of $\varepsilon < z_m$, the case $D_m(p_m, \varepsilon) \geq Q_m$ is the equivalent of $\varepsilon \geq z_m$, then equation (4.15) can be rewritten as

$$U(z_m, p_m) = \begin{cases} (p_m - c)[y(p_m)\,\varepsilon] - [(1+\alpha)(c-v)]\,y(p_m)\,(z_m - \varepsilon), & \varepsilon < z_m \\ (p_m - c)[y(p_m)\,z_m] - [\beta(p_m - c + s) + s]\,y(p_m)\,(\varepsilon - z_m), & \varepsilon \geq z_m \end{cases} \tag{4.16}$$

Given $\varepsilon \in [A, B]$, according to equation (4.16), the expected utility maximization function is $\max E[U(z_m, p_m)]$, where $E[U(z_m, p_m)]$ is shown below, *i.e.*,

$$\begin{aligned} E[U(z_m, p_m)] = {} & (p_m - c)\,y(p_m)\,\mu - (1+\alpha)(c-v)\,y(p_m)\int_A^{z_m}(z_m - \varepsilon)f(\varepsilon)\mathrm{d}\varepsilon \\ & - (1+\beta)(p_m - c + s)\,y(p_m)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon \end{aligned} \tag{4.17}$$

For ease of exposition, equation (4.17) is rewritten as

$$E[U(z_m, p_m)] = \varphi_m(p_m) - (1+\alpha)L_m(z_m, p_m) - (1+\beta)S_m(z_m, p_m) \tag{4.18}$$

where $L_m(z_m, p_m) = (c-v)\,y(p_m)\int_A^{z_m}(z_m - \varepsilon)f(\varepsilon)\mathrm{d}\varepsilon$ denotes the surplus loss which leads to the surplus regret under the multiplicative demand, $S_m(z_m, p_m) = (p_m - c + s)\,y(p_m)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon$ denotes the stock-out loss which leads to the stock-out regret under the multiplicative demand, $\varphi_m(p_m) = (p_m - c)\,y(p_m)\,\mu$ denotes the deterministic expected utility function under the multiplicative demand. According to $\varphi_m(p_m)$, we can obtain the risk neutral price $p_m^0 = \frac{bc}{b-1}$, and the corresponding order quantity is $Q_m^0 = y(p_m^0)\mu = a(p_m^0)^{-b}\mu$.

According to equation (4.17), we know that

$$\frac{\partial E[U(z_m,p_m)]}{\partial p_m} = a{p_m}^{-b-1}\left[b\,c - \,p\,(b-1)\right]\left[\mu-(1+\beta)\int_{z_m}^{B}(\varepsilon-z_m)f(\varepsilon)\mathrm{d}\varepsilon\right] + a{p_m}^{-b-1}b\,(c-v)(1+\alpha)\int_{A}^{z_m}(z_m-\varepsilon)f(\varepsilon)\mathrm{d}\varepsilon + \,a{p_m}^{-b-1}b\,s\,(1+\beta)\int_{z_m}^{B}(\varepsilon-z_m)f(\varepsilon)\mathrm{d}\varepsilon \tag{4.19}$$

By analyzing equation (4.19) we have the following Lemma 4.6 and Theorem 4.7.

Lemma 4.6. *For a fixed z_m, we have:*

(a) *If $\beta \geq \frac{\mu}{\int_{z_m}^{B}(\varepsilon-z_m)f(\varepsilon)\mathrm{d}\varepsilon}-1$, then price p_m^* is the boundary price, i.e., $p_m^*=\bar{p}$ or $p_m^*=\underline{p}$.*
(b) *If $\beta < \frac{\mu}{\int_{z_m}^{B}(\varepsilon-z_m)f(\varepsilon)\mathrm{d}\varepsilon}-1$, then price p_m^* is determined uniquely as a function of z_m, i.e.,*

$$p_m^* = \frac{bc}{b-1}+\frac{b}{b-1}\left[\frac{(1+\alpha)(c-v)\int_A^{z_m}(z_m-\varepsilon)f(\varepsilon)\mathrm{d}\varepsilon+\,s\,(1+\beta)\int_{z_m}^{B}(\varepsilon-z_m)f(\varepsilon)\mathrm{d}\varepsilon}{\mu-(1+\beta)\,\int_{z_m}^{B}(\varepsilon-z_m)f(\varepsilon)\mathrm{d}\varepsilon}\right] \tag{4.20}$$

and $p_m^* \geq p_m^0$.

Proof. See Appendix.

By substituting $p_m^* = p(z_m)$ into function $\max_{z_m,p_m} E[U(z_m,p_m)]$, the problem becomes a single variable problem in z_m, *i.e.*, $\max_{z_m} E\{U[z_m,p(z_m)]\}$. When the optimal solution of z_m is determined, the optimal price and the inventory are determined indirectly. For convenience of description, let $T_m(z_m)=f(z_m)/[1-F(z_m)]$ and $p_x = \frac{bc}{b-1}+\frac{b}{b-1}[\frac{(1+\alpha)(c-v)\int_A^{z}(z-\varepsilon)f(\varepsilon)\mathrm{d}\varepsilon+\,s\,(1+\beta)\int_z^B(\varepsilon-z)f(\varepsilon)\mathrm{d}\varepsilon}{\mu-(1+\beta)\,\int_z^B(\varepsilon-z)f(\varepsilon)\mathrm{d}\varepsilon}]$, where $T(z_m)$ is the hazard rate, then we have $p_m^*=p_x$.

Theorem 4.7. *For the multiplicative demand, the optimal policy is to order $Q_m^* = y(p_m^*)z_m^*$ units to sell at price p_m^*, where p_m^* is specified by Lemma 4.6 and z_m^* is determined as shown below:*

(a) *When price p_m^* is $\underline{p}$ or $\bar{p}$, then the optimal value of z_m can be determined by*

$$F[z]=\frac{(1+\beta)[p_m^*-c+s]}{(1+\alpha)(c-v)+(1+\beta)[p_m^*-c+s]} \tag{4.21}$$

where $p_m^*=\bar{p}$ or $p_m^*=\underline{p}$.
(b) *When the price p_m^* is p_x, i.e., $\beta<\frac{\mu}{\int_{z_m}^{B}(\varepsilon-z_m)f(\varepsilon)\mathrm{d}\varepsilon}-1$, if $F(\varepsilon)$ satisfies the condition $2T(z_m)^2+\mathrm{d}T(z_m)/\mathrm{d}z>0$ for $\forall z_m \in [A,B]$, and if $b\geq 2$, then z_m^* is the unique z_m in the region $[A,B]$ that satisfies $\partial E[U(z_m,p_m^*)]/\partial z_m$=0.*

Proof. See Appendix.

Furthermore, we can also obtain the optimal regret-neutral price $p_{m-n}^* = p_m^0 + \frac{b}{b-1}[\frac{(c-v)\int_A^{z_m}(z_m-\varepsilon)f(\varepsilon)\mathrm{d}\varepsilon+s\int_{z_m}^B(\varepsilon-z_m)f(\varepsilon)\mathrm{d}\varepsilon}{\mu-\int_{z_m}^B(\varepsilon-z_m)f(\varepsilon)\mathrm{d}\varepsilon}]$, and the variable z_{m-n}^* for the regret-neutral newsvendor corresponding to p_{m-n}^* can be determined.

Corollary 4.8. *The condition $2T(z_m)^2+\mathrm{d}T(z_m)/\mathrm{d}z>0$ in Theorem 4.6 holds for the uniform distribution and the exponential distribution.*

The Proof is same to the one of Corollary 4.3 since $T(z_m)$ is similar to $M(z_a)$.

According to Theorem 4.7, the optimal solution of z_m^* is determined by the following condition, *i.e.*,

$$F(z_m^*) = \frac{(1+\beta)[p_m^* - c + s]}{(1+\alpha)(c-v) + (1+\beta)[p_m^* - c + s]} \tag{4.22}$$

Thus, according to equation (4.20) and $Q_m^* = y(p_m^*)\, z_m^*$, the optimal order quantity Q_m^* is determined, *i.e.*,

$$Q_m^* = y(p_m^*)\, z_m^* = a(p_m^*)^{-b} z_m^* \tag{4.23}$$

In the following, we present the analysis on the impacts of the newsvendor's regret aversion parameters α and β on price p^* under the multiplicative demand.

Proposition 4.9. *Given p_m^*, z_m^* decreases with parameter α, and Q_m^* decreases with parameter α, i.e., the higher the newsvendor's surplus regret aversion degree is, the smaller the order quantity is; z_m^* increases with parameter β, and Q_m^* increases with parameter β, i.e., the higher the newsvendor's stock-out regret aversion degree is, the higher the order quantity is.*

Proof. See Appendix.

Proposition 4.10. *Given z_m^*, p_m^* increases with parameter α, i.e., the higher the newsvendor's surplus regret aversion degree is, the higher the price is, until the price reaches the upper bound of the range $\bar{p}$. p_m^* increases with parameter β, i.e., the higher the newsvendor's stock-out regret aversion degree is, the higher the price is, until the price reaches the upper bound of the range $\bar{p}$.*

Proof. See Appendix.

It is necessary to point out that, when the price is exogenous, the optimal order quantity under additive demand can be determined by $Q_a^* = y(p_a^*) + z_a^* = a - bp_a^* + z_a^*$ and $F(z_a) = \frac{(1+\beta)(p_a^*-c+s)}{(1+\alpha)(c-v)+(1+\beta)(p_a^*-c+s)}$; the optimal order quantity under multiplicative demand can be determined by $Q_m^* = y(p_m^*)\, z_m^* = a(p_m^*)^{-b} z_m^*$ and $F(z_m^*) = \frac{(1+\beta)[p_m^*-c+s]}{(1+\alpha)(c-v)+(1+\beta)[p_m^*-c+s]}$. Specially, under both additive and multiplicative demand, if $\alpha = \beta$, the optimal order quantity equals to the regret-neutral one, *i.e.*, the optimal order quantity is not related to the regret aversion of retailer if $\alpha = \beta$. This is reasonable, because when the newsvendor has equal perception on over-stock regret and under-stock regret, they offset each other, and the regret neutral decision is the optimal option.

5. Numerical examples

In this section, a numerical example is given to illustrate the impacts of regret aversions on the joint inventory and pricing decision models, and to examine the robustness of research results above by considering an appropriate scale of regret aversion parameters. In the example, the uniform distribution is applied to model the stochastic demand factor, and by using the data used by Raza [27], some instances are generated to show the optimal decision making trends when regret aversion parameters are taken into account for the additive and multiplicative demand function respectively as shown in Tables 2 and 4. It is necessary to say that, based on Bell [1], we consider $\alpha \in (0,1]$ and $\beta \in (0,1]$, and three parameter values of α and β are used in the numerical study, the used parameter values of α are $\alpha = 0.1$ (it implies lower surplus regret aversion degree), $\alpha = 0.5$ (it implies medium surplus regret aversion degree) and $\alpha = 1$ (it implies higher surplus regret aversion degree), and the used parameter values of β are $\beta = 0.1$, $\beta = 0.5$ and $\beta = 1$ for lower, medium, and higher stock-out regret aversion degree.

5.1. Sensitivity analysis under the additive demand

Nine hypothetical instances are generated to show the properties of the optimal policy under the additive demand, the boundary conditions are not considered here, and the used parameters are presented in Table 2 below.

According to the Lemma 4.1 and Theorem 4.2, we can obtain the optimal price, optimal order quantity, optimal expected profit, and optimal expected utility under the additive demand as shown in Table 3. In the following, we provide the sensitivity analysis of the regret aversion parameters under the additive demand.

In Table 3, we can see from instances 1-9 that the optimal price, order quantity, expected profit, and expected utility are affected by the regret aversions exhibited by the retailer under the additive demand. Specifically, when the surplus regret aversion parameter α is fixed (for example, see instances 1, 2, and 3), the optimal price p_a^* increases sharply with the stock-out regret aversion parameter β, but the optimal order quantity Q_a^* decreases sharply with the stock-out regret aversion parameter β. Moreover, when the stock-out regret aversion parameter β is fixed (for example, see instances 1, 4, and 7), the optimal price p_a^* decreases slowly with the surplus regret aversion parameter α, and the optimal order quantity Q_a^* also decreases with the surplus regret aversion parameter α. We can also see that the profit of the retailer decreases with the surplus (stock-out) regret aversion parameter when the stock-out (surplus) regret aversion parameter is fixed; similarly, the utility changes with the regret aversion parameters α and β to varying degree. Specially, when $\alpha = 1$ and $\beta = 1$, the price and utility take the maximum, but the order quantity and profit take the minimum.

TABLE 2. Data for analysis of the regret aversions effects under additive demand (Raza [27]).

Parameters values	a	b	v	s	c	A	B	α	β
Benchmark (Raza [27])	1000	5	2	6	5	350	650	–	–
Instance 1	1000	5	2	6	5	350	650	0.1	0.1
Instance 2	1000	5	2	6	5	350	650	0.1	0.5
Instance 3	1000	5	2	6	5	350	650	0.1	1
Instance 4	1000	5	2	6	5	350	650	0.5	0.1
Instance 5	1000	5	2	6	5	350	650	0.5	0.5
Instance 6	1000	5	2	6	5	350	650	0.5	1
Instance 7	1000	5	2	6	5	350	650	1	0.1
Instance 8	1000	5	2	6	5	350	650	1	0.5
Instance 9	1000	5	2	6	5	350	650	1	1

TABLE 3. The effects of the regret aversions under additive demand.

	p_a^*	Q_a^*	$E(\pi_a^*)$	$E(U_a^*)$
Benchmark (Raza [27])	173.0776	779.5294	142675.1425	–
Instance 1	175.1395	769.2784	142660.4500	146304.6436
Instance 2	183.5159	728.8857	142312.5884	161407.8540
Instance 3	193.9858	677.5535	140808.9604	181273.4340
Instance 4	175.0130	768.1206	142462.5056	146160.8229
Instance 5	183.3934	728.2302	142175.9514	161259.3088
Instance 6	193.8688	677.2318	140720.0915	181121.6597
Instance 7	174.8573	766.6883	142219.0153	145987.6046
Instance 8	183.2419	727.4159	142006.9991	161078.2614
Instance 9	193.7235	676.8307	140609.7987	180935.2346

The main reason of the above effects under the additive demand is that decision objective of the regret-averse retailer is not only the profit but also the regret aversion utility. This is because the regret-averse retailer cares about the negative utility from the regret aversions. Actually, the higher the retailer's regret aversion degree is, the greater the effects are.

Compared with benchmark (regret-neutral policy), the order quantity of the regret-averse retailer is lower than the one of the regret-neutral retailer. It implies that the decision of the regret-averse retailer is more conservative than the one of the regret-neutral retailer. Moreover, the price is higher than the one of the regret-neutral one. It is because that the regret-averse retailer will set higher price to make up the profit loss for the conservative order quantity. By comparing, we also know that the profit of the regret-averse retailer is lower than the one of the regret-neutral retailer. It implies that the regret-averse retailer pays the attention to the regret aversion utility which causes profit loss.

5.2. Sensitivity analysis under the multiplicative demand

Nine hypothetical instances are generated to show the properties of the optimal policy under the multiplicative demand, the parameters are presented in Table 4 below.

According to the Lemma 4.6 and Theorem 4.7, we can obtain the optimal price, optimal order quantity, optimal expected profit, and optimal expected utility under the multiplicative demand as shown in Table 5. In the following, we provide the sensitivity analysis of the regret aversion parameters under the multiplicative demand.

We can see from instances 1-9 in Table 5 that the optimal policy of the regret-averse retailer can be affected by the surplus and stock-out regret aversions under the multiplicative demand. Specifically, when the stock-out regret aversion parameter β is fixed (for example, see instances 3, 6, and 9), the optimal price p_m^* increases with the surplus regret aversion parameter α, but the optimal order quantity Q_m^* decreases sharply with the surplus regret aversion parameter α. Similarly, when the surplus regret aversion parameter α is fixed (for example, see instances 4, 5, and 6), the optimal price p_m^* increases slowly with the stock-out regret aversion parameter β, and the optimal order quantity Q_m^* decreases slowly with the stock-out regret aversion parameter β. Similar to the analyzing results under the additive demand, under the multiplicative demand, the profit of the retailer also decreases with the surplus (stock-out) regret aversion parameter when the stock-out (surplus) regret aversion parameter is fixed, and the utility changes with the regret aversion parameters α and β to varying degree. Specially, when $\alpha = 1$ and $\beta = 1$, the price takes the maximum, but the order quantity, profit and utility take the minimum, it is different from the situation under the additive demand.

TABLE 4. Data for analysis of regret aversions effects under multiplicative demand (Raza [27]).

Parameters values	a	b	v	s	c	A	B	α	β
Benchmark (Raza [27])	5000	1.5	1	6	5	0.7	1.3	–	–
Instance 1	50000	1.5	1	6	5	0.7	1.3	0.1	0.1
Instance 2	50000	1.5	1	6	5	0.7	1.3	0.1	0.5
Instance 3	5000	1.5	1	6	5	0.7	1.3	0.1	1
Instance 4	50000	1.5	1	6	5	0.7	1.3	0.5	0.1
Instance 5	50000	1.5	1	6	5	0.7	1.3	0.5	0.5
Instance 6	50000	1.5	1	6	5	0.7	1.3	0.5	1
Instance 7	50000	1.5	1	6	5	0.7	1.3	1	0.1
Instance 8	50000	1.5	1	6	5	0.7	1.3	1	0.5
Instance 9	50000	1.5	1	6	5	0.7	1.3	1	1

Table 5. The effects of the regret aversions under multiplicative demand.

	p_m^*	Q_m^*	$E(\pi_m^*)$	$E(U_m^*)$
Benchmark (Raza [27])	18.9218	783.8195	7662.3706	–
Instance 1	19.3277	759.8235	7658.7678	7581.5503
Instance 2	19.3544	759.3302	7654.0904	7576.3918
Instance 3	19.3864	758.7459	7648.3765	7570.3109
Instance 4	20.8986	675.7197	7634.0061	7291.0250
Instance 5	20.9357	674.9000	7629.0347	7284.6396
Instance 6	20.9798	673.9306	7623.0158	7277.1388
Instance 7	22.8620	590.5277	7570.1864	6970.9065
Instance 8	22.9126	589.4437	7564.6014	6963.2865
Instance 9	22.9719	588.1810	7557.9669	6954.4607

The main reason of the above effects under the multiplicative demand is that the regret-averse retailer cares about the negative utility from the surplus and stock-out regret aversions, and thus the retailer adjust his/her optimal policy to reduce the regret aversion utility. Actually, the higher the retailer's regret aversion degree is, the greater the effects are.

Compared with benchmark (regret-neutral policy), the order quantity of the regret-averse retailer is lower greatly than the one of the regret-neutral retailer, but the price is higher than the one of regret-neutral retailer. It implies that, under the multiplicative demand, the retailer with regret aversions is more conservative than the regret-neutral retailer. In addition, the profit of regret-averse retailer is lower than the one of regret-neutral retailer, and the higher the regret aversion degree is, the less the regret-averse retailer's profit is.

By the above sensitivity analysis, we find that the effects of the regret aversions on the optimal price and order quantity are different for different demands. The optimal policy is more sensitive to the regret versions under the multiplicative demand than that under the additive demand. Under the additive demand, the price and order quantity are more sensitive to the surplus regret aversion, but under the multiplicative demand, the price and order quantity are more sensitive to the stock-out regret aversion. Besides, we also find that, under both demands, the regret-averse profit is lower than the regret-neutral one.

6. Managerial insights

According to the above analysis, we know that the optimal policy will be affected by the regret aversion behavior under the additive and multiplicative demand. In the following, we provide managerial insights.

(1) Regret-averse newsvendor needs to consider behavioral effects on the policy, and determines the policy of order quantity and price with respect to the different demand types.
For the newsvendor who mainly concerns about over-stock regret, the greater the over-stock regret aversion degree is, the less the order quantity should be under both additive and multiplicative demand, and the higher the price should be under multiplicative demand, but the lower the price should be under the additive demand. For the newsvendor who mainly concerns about under-stock regret, the greater the under-stock regret aversion degree is under both additive and multiplicative demand, the less the order quantity should be, and the higher the price should be under both additive and multiplicative demand too.

(2) The policy of the regret-averse newsvendor deviates from the one of the regret-neutral newsvendor, the decision on the optimal order quantity is more conservative than the one of regret-neutral newsvendor, but the decision on the optimal price is more radical than the one of regret-neutral newsvendor. In addition, the optimal policy of regret-averse newsvendor is also different from the one of risk-neutral retailer. The

regret-averse retailer may be more conservative or radical than the risk-neutral retailer in their decision under additive or multiplicative demand.

(3) The profit of the regret-averse newsvendor is less than the one of the regret-neutral newsvendor. If the newsvendor reduces the degree of the concern on the regret, the profit increases.

7. Conclusion and further research

In this paper, we studied a joint inventory control and pricing decision problem with newsvendor's regret aversion behaviors. Specifically, we extended the classic joint inventory control and pricing model under newsvendor settings to accommodate regret aversion parameters, and constructed a new utility function of the newsvendor. By analyzing the constructed utility function, we provided the conditions of the optimal order quantity and price.

We found that the regret-averse policy was different from the regret-neutral one, and that the regret aversion can affect the newsvendor's optimal order quantity and price decision to varying degree. Specially, if the price was exogenous and the degree of stock-out regret aversion was equal to the degree of surplus regret aversion, then the optimal policy was the regret-neutral one which was the optimal solution to the model of Petruzzi and Dada [25].

Compared with Raza [27] and Petruzzi and Dada [25], we constructed the joint inventory control and pricing model in behavioral perspective, provided the analysis of the impacts of the regret aversions on joint inventory control and pricing decisions, and showed the trend of the impacts under the additive and multiplicative demand. Compared with the existing research on the joint order quantity and price decision considering newsvendor's behavior factors such as overconfidence, loss aversion, and bounded rationality, we compensated them by clearly describing and modeling regret aversion effects.

In addition, we analyzed the impacts of regret aversion in joint inventory control and pricing decisions successfully, and provided the sensitivity analysis of the regret aversion effects by using the data of Raza [27]. In the future research, it is necessary to investigate how to measure the regret aversion effects. Moreover, it is also interesting to look into the impacts of newsvendor's regret aversion behaviors in supply chains.

Appendix A.

Proof of Lemma 4.1. According to the Petruzzi and Dada [25], we provide the mathematical proof for the Lemma 4.1 in the following.

For a fixed z_a, we know that the expected utility function $\max\limits_{z_a,p_a} E[U(z_a,p_a)]$ with variables z_a and p_a changes into the function $\max\limits_{p_a} E[U(z_a,p_a)]$ with only one variable p_a. According to equation (4.9), we have that $\partial^2 \max\limits_{p_a} E[U(z_a,p_a)]/\partial {p_a}^2 = -2b < 0$, it implies that $\max\limits_{p_a} E[U(z_a,p_a)]$ is a concave function with respect to p_a, *i.e.*, there is unique optimal price for the function $\max\limits_{p_a} E[U(z_a,p_a)]$, and the optimal price meets first order condition. Since the first order condition is $\partial \max\limits_{p_a} E[U(z_a,p_a)]/\partial p_a = 0$, according to equation (4.8), we have $\partial \max\limits_{p_a} E[U(z_a,p_a)]/\partial p_a = 2b(p_a^0 - p_a) - (1+\beta)\int_{z_a}^{B} (\varepsilon - z_a)f(\varepsilon)\mathrm{d}\varepsilon = 0$, then we know

$$p_a^* = p(z_a) = p_a^0 - \frac{1+\beta}{2b}\int_{z_a}^{B} (\varepsilon - z_a)f(\varepsilon)\mathrm{d}\varepsilon \tag{A.1}$$

It is necessary to point out that, according to equation (A.1), we have that, when $\beta \geq \frac{2b(p_a^0-\underline{p})}{\int_{z_a}^{B}(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon} - 1$, the optimal p_a^* is lower than the lower bound price $\underline{p}$, *i.e.*, $p_a^* \leq \underline{p}$. Since the price p_a^* is usually in a range in reality, *i.e.*, $p_a^* \in [\underline{p},\bar{p}]$, and thus, we set that $p_a^* = \underline{p}$ when $\beta \geq \frac{2b(p_a^0-\underline{p})}{\int_{z_a}^{B}(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon} - 1$. Similarly, when

$0 \leq \beta \leq \frac{2b(p_a^0-\bar{p})}{\int_{z_a}^B(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon}-1$, the optimal p_a^* is greater than the upper bound price $\bar{p}$, *i.e.*, $p_a^* \geq \bar{p}$. Since the price p_a^* is usually in a range in reality, *i.e.*, $p_a^* \in [\underline{p}, \bar{p}]$, and thus, we set that $p_a^* = \bar{p}$ when $0 \leq \beta \leq \frac{2b(p_a^0-\bar{p})}{\int_{z_a}^B(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon}-1$. Furthermore, when $0 \leq \frac{2b(p_a^0-\bar{p})}{\int_{z_a}^B(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon}-1 \leq \beta \leq \frac{2b(p_a^0-\underline{p})}{\int_{z_a}^B(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon}-1$, the optimal price p_a^* is in the range of $[\underline{p}, \bar{p}]$, and $p_a^* = p(z_a) = p_a^0 - \frac{1+\beta}{2b}\int_{z_a}^B(\varepsilon - z_a)f(\varepsilon)\mathrm{d}\varepsilon$. □

Proof of Theorem 4.2. According to the Petruzzi and Dada [25], we provide the mathematical proof for the Theorem 4.2 in the following.

Proof of (i) *in* (b). Based on the analysis of solution in Petruzzi and Dada [25], we give the specific proof. Since $p_a^* = p^0 - \frac{1+\beta}{2b}\int_{z_a}^B(\varepsilon - z_a)f(\varepsilon)\mathrm{d}\varepsilon$, by substituting $p_a^* = p(z_a)$ into $\max\limits_{z_a,p_a} E[U(z_a,p_a)]$, the problem for $\max\limits_{z_a,p_a} E[U(z_a,p_a)]$ changes into the one for $\max\limits_{z_a} E[U(z_a,p(z_a))]$. According to equation (4.6), we know the first order derivative of $E[U(z_a,p(z_a))]$ with respect to z_a, *i.e.*,

$$\frac{\mathrm{d}E[U(z_a,p(z_a))]}{\mathrm{d}z_a} = (1+\beta)[p_a^0 - c + s - \frac{1+\beta}{2b}\int_{z_a}^B(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon][1-F(z_a)] - (1+\alpha)(c-v)F(z_a) \tag{A.2}$$

In the following, we will obtain the optimal safety stock z_a^* by analyzing the function $\mathrm{d}E[U(z_a,p(z_a))]/\mathrm{d}z_a$. For the convenience of description, let $r(z_a) = \mathrm{d}E[U(z_a,p(z_a))]/\mathrm{d}z_a$, then the first order derivative of $r(z_a)$ with respect to safety stock z_a can be obtained, *i.e.*,

$$\begin{aligned}\frac{\mathrm{d}r(z_a)}{\mathrm{d}z_a} = {} & \frac{(1+\beta)^2}{2b}[1-F(z_a)]^2 - f(z_a)(1+\alpha)(c-v) \\ & - f(z_a)(1+\beta)\left[p_a^0 - c + s - \frac{(1+\beta)}{2b}\int_{z_a}^B(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon\right]\end{aligned} \tag{A.3}$$

Then, the second order derivative of $r(z_a)$ with respect to safety stock z_a can be obtained, *i.e.*,

$$\begin{aligned}\frac{\mathrm{d}^2r(z_a)}{\mathrm{d}{z_a}^2} = {} & -\frac{3(1+\beta)^2[1-F(z_a)]f(z_a)}{2b} - (1+\alpha)(c-v)\left[\frac{\mathrm{d}M(z_a)}{\mathrm{d}z_a} - M(z_a)^2\right][1-F(z_a)] \\ & - (1+\beta)\left[p_a^0 - c + s - \frac{(1+\beta)}{2b}\int_{z_a}^B(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon\right]\left[\frac{\mathrm{d}M(z_a)}{\mathrm{d}z_a} - M(z_a)^2\right][1-F(z_a)]\end{aligned} \tag{A.4}$$

where $M(z_a) = f(z_a)/[1-F(z_a)]$.

Since $\mathrm{d}f(z_a)/\mathrm{d}z_a = \left[\mathrm{d}M(z_a)/\mathrm{d}z_a - M(z_a)^2\right][1-F(z_a)]$, $\mathrm{d}^2r(z_a)/\mathrm{d}{z_a}^2$ can be further rewritten as

$$\begin{aligned}\frac{\mathrm{d}^2r(z_a)}{\mathrm{d}{z_a}^2} &= -\left\{(1+\alpha)(c-v) + (1+\beta)\left[p_a^0 - c + s - \frac{(1+\beta)}{2b}\int_{z_a}^B(\varepsilon-z_a)f(\varepsilon)\mathrm{d}\varepsilon\right]\right\}\frac{\mathrm{d}f(z_a)}{\mathrm{d}z_a} \\ &\quad - \frac{3(1+\beta)^2[1-F(z_a)]f(z_a)}{2b} \\ &= \frac{\mathrm{d}r(z_a)/\mathrm{d}z_a}{f(z_a)}\frac{\mathrm{d}f(z_a)}{\mathrm{d}z_a} - \frac{(1+\beta)^2[1-F(z_a)]f(z_a)}{2bM(z_a)^2}\left[2M(z_a)^2 + \frac{\mathrm{d}M(z_a)}{\mathrm{d}z_a}\right]\end{aligned} \tag{A.5}$$

Furthermore, the second order derivative $\mathrm{d}^2 r(z_a)/\mathrm{d}{z_a}^2$ when $\mathrm{d}r\,(z_a)/\mathrm{d}z_a = 0$ can be obtained, *i.e.*,

$$\left.\frac{\mathrm{d}^2 r\,(z_a)}{\mathrm{d}{z_a}^2}\right|_{\mathrm{d}r(z_a)/\mathrm{d}z_a=0} = -\frac{(1+\beta)^2\,[1-F\,(z_a)]\,f\,(z_a)}{2bM(z_a)^2}\left[2M(z_a)^2 + \frac{\mathrm{d}M\,(z_a)}{\mathrm{d}z_a}\right] \tag{A.6}$$

According to Petruzzi and Dada [25], we give the analysis of the optimal safety stock z_a^* based on equation (A.5). Specifically, if $F(\cdot)$ is a distribution satisfying the condition $2M(z_a)^2 + \mathrm{d}M\,(z_a)/\mathrm{d}z_a \geq 0$, then we know that $\mathrm{d}^2 r(z_a)/\mathrm{d}{z_a}^2 \leq 0$ at $\mathrm{d}r\,(z_a)/\mathrm{d}z_a = 0$, it follows that $r(z_a)$ has at most two roots. Since $r(B) = -(1+\alpha)(c-v) < 0$, if $r(z_a)$ has only one root, it indicates a change of sign for $r(z_a)$ from positive to negative, and thus it corresponds to a local maximum of $E\{U[z_a, p(z_a)]\}$; if $r(z_a)$ has two roots, the larger root corresponds to the a local maximum of $E\{U[z_a, p(z_a)]\}$, the smaller root corresponds to a local minimum of $E\{U[z_a, p(z_a)]\}$. Obviously, in either case, $E\{U[z_a, p(z_a)]\}$ has only one local maximum, for the situation of the only one root, the optimal safety stock z_a^* is identified as the unique value that satisfies $r(z_a) = \mathrm{d}E\{U[z_a, p(z_a)]\}/\mathrm{d}z_a = 0$; for the situation of two roots, the optimal safety stock z_a^* is identified as the larger one of the two values of z_a that satisfies $r(z_a) = \mathrm{d}E\{U[z_a, p(z_a)]\}/\mathrm{d}z_a = 0$.

Proof of (ii) *in* (b). According to Petruzzi and Dada [25], we give the analysis of the uniqueness. According to equation (A.2) and $r(z_a) = \mathrm{d}E[U(z_a, p(z_a))]/\mathrm{d}z_a$, we know that $r(B) = -(1+\alpha)(c-v) < 0$. Since $E\{U[z_a, p(z_a)]\}$ is unimodal if $r(z_a)$ has only one root (still assuming that $2M(z_a)^2 + \mathrm{d}M\,(z_a)/\mathrm{d}z_a \geq 0$), if $r(A) > 0$ is satisfied, *i.e.*, $2b(p_a^0 - c + s) - (1+\beta)(\mu - A) > 0$, then we know that $E\{U[z_a, p(z_a)]\}$ is unimodal. That is, if $2b(p_a^0 - c + s) - (1+\beta)(\mu - A) > 0$, then there exists the unique optimal solution. □

Proof of Corollary 4.3. Let $\varpi = 2M(z_a)^2 + \mathrm{d}M\,(z_a)/\mathrm{d}z_a$. Since $M(z_a) = f(z_a)/[1-F(z_a)]$, ϖ can be converted into

$$\varpi = \frac{1}{[1-F(z_a)]^2}\left\{3f(z_a)^2 + f(z_a)'[1-F(z_a)]\right\} \tag{A.7}$$

In the following, we provide the proof to show that both uniform distribution $U[A, B]$ and exponential distribution $E(\lambda)$ meets the condition of $2M(z_a)^2 + \mathrm{d}M\,(z_a)/\mathrm{d}z_a \geq 0$.

(1) Uniform distribution $U[A, B]$. According to the probability density function and the cumulative distribution function of the uniform distribution, *i.e.*, $f_{Uni}(z_a) = \frac{1}{B-A}$ and $F_{Uni}(z_a) = \begin{cases} 0, & z_a \leq A \\ \frac{z_a - A}{B-A}, & A < z_a \leq B \\ 1, & z_a > B \end{cases}$ we have that $f_{Uni}(z_a) > 0$ and $f_{Uni}(z_a)' = 0$, then we know that $\varpi \geq 0$. By this, we have that the uniform distribution satisfies the condition $2M(z_a)^2 + \mathrm{d}M\,(z_a)/\mathrm{d}z_a \geq 0$.

(2) Exponential distribution $E(\lambda)$. The probability density function and the cumulative distribution function of the exponential distribution are $f_{Exp}(z_a) = \begin{cases} \lambda \mathrm{e}^{-\lambda z_a}, & z_a > 0 \\ 0, & z_a \leq 0 \end{cases}$ and $F_{Exp}(z_a) = \begin{cases} 1 - \mathrm{e}^{-\lambda z_a}, & z_a > 0 \\ 0, & z_a \leq 0 \end{cases}$, respectively. Obviously, if $z_a \leq 0$, we have that $f_{Exp}(z_a) = 0$, $F_{Exp}(z_a) = 0$, and $f_{Exp}(z_a)' = 0$, then we know $\varpi = 0$; if $z_a > 0$, we have $f_{Exp}(z_a) = \lambda \mathrm{e}^{-\lambda z_a}$, $F_{Exp}(z_a) = 1 - \mathrm{e}^{-\lambda z_a}$, and $f_{Exp}(z_a)' = -\lambda^2 \mathrm{e}^{-\lambda z_a}$, then we know that $\varpi = \frac{1}{\mathrm{e}^{-2\lambda z_a}}\left\{3\lambda^2 \mathrm{e}^{-2\lambda z_a} + [-\lambda^2 \mathrm{e}^{-\lambda z_a}]\mathrm{e}^{-\lambda z_a}\right\} = 2\lambda^2 \geq 0$. Therefore, we have that the exponential distribution satisfies the condition $2M(z_a)^2 + \mathrm{d}M\,(z_a)/\mathrm{d}z_a \geq 0$. □

Proof of Proposition 4.4. Given p_a^*, according to equation (4.12) and $Q_a^* = a - bp_a^* + z_a^*$, we can determine the first order condition of optimal order quantity with respect to surplus and stock-out regret aversion parameters, *i.e.*,

$$\frac{\partial Q_a^*}{\partial \alpha} = \frac{\partial z_a^*}{\partial \alpha} = -\frac{(1+\beta)(p_a^* - c + s)(c-v)}{[(1+\alpha)(c-v) + (1+\beta)(p_a^* - c + s)]^2 f(z_a^*)} \tag{A.8}$$

$$\frac{\partial Q_a^*}{\partial \beta} = \frac{\partial z_a^*}{\partial \beta} = \frac{(1+\alpha)(c-v)(p_a^*-c+s)}{\left[(1+\alpha)(c-v)+(1+\beta)(p_a^*-c+s)\right]^2 f(z_a^*)} \tag{A.9}$$

Since $p_a^* \geq c \geq v$, $f(z_a^*) \geq 0$ and $\alpha, \beta \geq 0$, we know that $\partial Q_a^*/\partial\alpha < 0$ and $\partial Q_a^*/\partial\beta > 0$. □

Proof of Proposition 4.5. Given z_a^*, according to equations (4.20) and (4.22), we can determine the first order condition of optimal price with respect to surplus and stock-out regret aversion parameters, *i.e.*,

$$\frac{\partial p_a^*}{\partial \alpha} = \frac{(c-v)F(z_a^*)}{[1-F(z_a^*)](1+\beta)} \tag{A.10}$$

$$\frac{\partial p_a^*}{\partial \beta} = -\frac{\int_{z_a^*}^{B}(\varepsilon - z_a^*)f(\varepsilon)\mathrm{d}\varepsilon}{2b} \tag{A.11}$$

Since $\int_{z_a^*}^{B}(\varepsilon - z_a^*)f(\varepsilon)\mathrm{d}\varepsilon \geq 0$ and $c \geq v$, we know that $\partial p_a^*/\partial\alpha > 0$ and $\partial p_a^*/\partial\beta < 0$. □

Proof of Lemma 4.6. Following the solution process of Petruzzi and Dada [25], we provide the mathematical proof for the Lemma 4.6. Specifically, we conduct the proof in two cases: one is for $\beta = \frac{\mu}{\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} - 1$; the other is for $\beta \neq \frac{\mu}{\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} - 1$.

(1) Since $\beta = \frac{\mu}{\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} - 1$, we have $\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon = 0$ for $\forall z_m \in [A, B]$. On the basis, according to the equation (4.19), we know that the first order derivative of expected utility function $E[U(z_m, p_m)]$ with respect to price p_m, *i.e.*,

$$\begin{aligned}\frac{\partial E[U(z_m,p_m)]}{\partial p_m} &= a{p_m}^{-b-1}b\,(c-v)(1+\alpha)\int_{A}^{z_m}(z_m-\varepsilon)f(\varepsilon)\mathrm{d}\varepsilon \\ &\quad + a{p_m}^{-b-1}b\,s\,(1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon\end{aligned} \tag{A.12}$$

Since $\int_{A}^{z_m}(z_m-\varepsilon)f(\varepsilon)\mathrm{d}\varepsilon \geq 0$, $\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon \geq 0$, $c \geq v$, and $\alpha, \beta \geq 0$, we know that $\partial E[U(z_m,p_m)]/\partial p_m > 0$. Given $p_m \in [\underline{p}, \bar{p}]$, so we can determine that $\bar{p}$ is the optimal solution of $E[U(z_m,p_m)]$, *i.e.*, $p_m^* = \bar{p}$.

(2) Since $\beta \neq \frac{\mu}{\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} - 1$, we have $\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon \neq 0$ for $\forall z_m \in [A, B]$, we know that the first order derivative of expected utility function $E[U(z_m,p_m)]$ with respect to price p_m as shown in equation (4.19). Since $\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon \neq 0$, equation (4.19) can be rewritten as

$$\begin{aligned}\frac{\partial E[U(z_m,p_m)]}{\partial p_m} &= a{p_m}^{-b-1}(b-1)\left[\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon\right] \\ &\times \left\{\frac{bc}{b-1} + \frac{b}{b-1}\left[\frac{(1+\alpha)(c-v)\int_{A}^{z_m}(z_m-\varepsilon)f(\varepsilon)\mathrm{d}\varepsilon + s\,(1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon}{\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon}\right] - p_m\right\}\end{aligned} \tag{A.13}$$

Further, since $p_x = \frac{bc}{b-1} + \frac{b}{b-1}[\frac{(1+\alpha)(c-v)\int_{A}^{z_m}(z_m-\varepsilon)f(\varepsilon)\mathrm{d}\varepsilon + s\,(1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon}{\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon}]$, equation (A.13) can be changed into equation (A.14), *i.e.*,

$$\frac{\partial E[U(z_m,p_m)]}{\partial p_m} = a{p_m}^{-b-1}(b-1)\left[\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon\right](p_x - p_m) \tag{A.14}$$

For $\beta \neq \frac{\mu}{\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} - 1$, we consider two situations in the following analysis, *i.e.*, $\beta > \frac{\mu}{\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} - 1$ and $\beta < \frac{\mu}{\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} - 1$.

(a) If $\beta > \frac{\mu}{\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} - 1$, then we have $\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon < 0$ for $\forall z_m \in [A, B]$. Further, because $a{p_m}^{-b-1}(b-1) \geq 0$, when $p_m < p_x$, we know that $\partial E[U(z_m, p_m)]/\partial p_m < 0$, *i.e.*, the expected utility function $E[U(z_m, p_m)]$ is a decreasing function with respect to price p_m when $p_m \in [\underline{p}, p_x]$; similarly, when $p_m > p_x$, we know that $\partial E[U(z_m, p_m)]/\partial p_m > 0$, *i.e.*, the expected utility function $E[U(z_m, p_m)]$ is an increasing function with respect to price p_m when $p_m \in [p_x, \bar{p}]$. Obviously, $E[U(z_m, p_m)]$ first decreases and then increases with price p_m, and thus $E[U(z_m, p_m)]$ reaches its minimum at $p_m = p_x$. Since $p_m \in [\underline{p}, \bar{p}]$, the optimal price is one of the bound prices, if the $E[U(z_m, p_m)]$ at $p_m^* = \underline{p}$ is greater than or equal to the one at $p_m^* = \bar{p}$, then $p_m^* = \underline{p}$; if not, $p_m^* = \bar{p}$.

(b) If $\beta < \frac{\mu}{\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} - 1$, then we have $\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon > 0$ for $\forall z_m \in [A, B]$. Further, when $p_m > p_x$, we know that $\partial E[U(z_m, p_m)]/\partial p_m < 0$; when $p_m < p_x$, we know that $\partial E[U(z_m, p_m)]/\partial p_m > 0$. Obviously, $E[U(z_m, p_m)]$ first increases and then decreases with price p_m, and thus $E[U(z_m, p_m)]$ reaches its maximum at $p_m = p_x$, *i.e.*, $p_m^* = p_x$. Further, we have

$$p_m^* = p_x = p_m^0 + \frac{b}{b-1}\left[\frac{(1+\alpha)(c-v)\int_A^{z_m}(z_m - \varepsilon)f(\varepsilon)\mathrm{d}\varepsilon + s(1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon}{\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon}\right] \tag{A.15}$$

Thus, if $\beta < \frac{\mu}{\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} - 1$ for a fixed z_m, optimal price p_m^* is determined uniquely as a function of z_m. In addition, it is easy to see that $p_m^* \geq p_m^0$. □

Proof of Theorem 4.7. According to the Petruzzi and Dada [25], we provide the mathematical proof for the Theorem 4.7 in the following.

Proof of (b). Based on the analysis of solution in Petruzzi and Dada [25], we give the specific proof. According to equation (4.17), we know first order derivative of $E\{U[z_m, p(z_m)]\}$ with respect to z_m, *i.e.*,

$$\frac{\partial E\{U[z_m, p(z_m)]\}}{\partial z_m} = y(p(z_m))\,[1 - F(z_m)]\left[(1+\beta)(p(z_m) - c + s) - \frac{(1+\alpha)(c-v)F(z_m)}{1 - F(z_m)}\right] \tag{A.16}$$

For the convenience of description, we let $R(z_m) = (1+\beta)(p_m - c + s) - \frac{(1+\alpha)(c-v)F(z_m)}{1-F(z_m)}$. If $z_m \neq B$, then we know that $y(p_m)\,[1 - F(z_m)] > 0$, in this situation, if $R(z_m) > 0$, $E\{U[z_m, p(z_m)]\}$ is the increasing function of z_m; if $R(z_m) < 0$, $E\{U[z_m, p(z_m)]\}$ is the decreasing function of z_m; and thus when $R(z_m) = 0$, $E\{U[z_m, p(z_m)]\}$ has a local optimum for any z_m. Hence, the shape of $E\{U[z_m, p(z_m)]\}$ can be determined by analyzing $R(z_m)$. According to $R(z_m)$ and $z_m \in [A, B]$, we have

$$\begin{aligned} R(A) = (1+\beta)[p(A) - c + s] &= (1+\beta)\left\{\frac{bc}{b-1} + \frac{b}{b-1}\left[\frac{s(1+\beta)(\mu - A)}{\mu - (1+\beta)(\mu - A)}\right] + s - c\right\} \\ &= (1+\beta)\frac{1}{b-1}\left\{bs\left[\frac{(1+\beta)(\mu - A)}{\mu - (1+\beta)(\mu - A)} + 1\right] + c - s\right\} \\ &= (1+\beta)\frac{1}{b-1}\left\{\frac{bs\mu}{\mu - (1+\beta)(\mu - A)} + c - s\right\} \\ &= (1+\beta)\frac{1}{b-1}\left\{s\,\frac{(b-1)\mu + (1+\beta)(\mu - A)}{\mu - (1+\beta)(\mu - A)} + c\right\} \end{aligned} \tag{A.17}$$

$$R(B) = (1+\beta)[p(B) - c + s] - \frac{(1+\alpha)(c-v)}{0} \to -\infty < 0 \tag{A.18}$$

Since $\beta < \frac{\mu}{\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} - 1$, $\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon > 0$ for $\forall z_m \in [A,B]$, for $z_m = A$, we know that $\mu - (1+\beta)\int_{A}^{B}(\varepsilon - A)f(\varepsilon)\mathrm{d}\varepsilon = \mu - (1+\beta)(\mu - A) > 0$. According to equation (A.17), we know that $R(A) > 0$.

Next, we consider how $R(z_m)$ behaves in z_m. The first and second order derivative of $R(z_m)$ with respect to z_m can be determined, *i.e.*,

$$\frac{\mathrm{d}R(z_m)}{\mathrm{d}z_m} = \frac{\mathrm{d}p(z_m)}{\mathrm{d}z_m} - \frac{(1+\alpha)(c-v)T(z_m)}{1-F(z_m)} \tag{A.19}$$

$$\frac{\mathrm{d}^2R(z_m)}{\mathrm{d}{z_m}^2} = \frac{\mathrm{d}^2p(z_m)}{\mathrm{d}{z_m}^2} - (1+\alpha)(c-v)\left[\frac{\mathrm{d}T(z_m)}{1-F(z_m)} + \frac{T(z_m)^2}{1-F(z_m)}\right] \tag{A.20}$$

where, from Lemma 4.6, we have

$$\begin{aligned}\frac{\mathrm{d}p(z_m)}{\mathrm{d}z_m} &= \frac{b(1+\alpha)(c-v)\left[(1+\beta)F(z_m)z_m - F(z_m)\,\beta\,\mu - (1+\beta)\int_{A}^{z_m}(z_m-\varepsilon)f(\varepsilon)\mathrm{d}\varepsilon\right]}{(b-1)\left[\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon\right]^2} \\ &\quad - \frac{bs\,(1+\beta)\,[1-F(z_m)]\,\mu}{(b-1)\left[\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon\right]^2}\end{aligned} \tag{A.21}$$

$$\begin{aligned}\frac{\mathrm{d}^2p(z_m)}{\mathrm{d}{z_m}^2} &= \frac{b}{b-1}f(z_m)\frac{(1+\alpha)(c-v)\,[(1+\beta)z_m - \beta\mu] + s\,(1+\beta)\mu}{\left[\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon\right]^2} \\ &\quad - \frac{2\,(1+\beta)\,[1-F(z_m)]}{\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon}\frac{\mathrm{d}p(z_m)}{\mathrm{d}z_m} \\ &= \frac{b}{b-1}\frac{(1+\alpha)(c-v)T(z_m)\left\{(1+\beta)\left[\mu - \int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon\right] - \beta\mu\right\}}{\left[\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon\right]^2} \\ &\quad - \left\{\frac{2\,(1+\beta)\,[1-F(z_m)]}{\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} + T(z_m)\right\}\frac{\mathrm{d}p(z_m)}{\mathrm{d}z_m}\end{aligned} \tag{A.22}$$

Thus, by substitution, we have

$$\begin{aligned}\frac{\mathrm{d}^2R(z_m)}{\mathrm{d}{z_m}^2} &= -(1+\alpha)(c-v)\left\{\frac{b\beta\left[\mu - \int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon\right] + (b-2)\left[\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon\right]}{(b-1)\left[\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon\right]^2}\right. \\ &\quad \left. + \frac{\mathrm{d}T(z_m)/\mathrm{d}z_m + 2T(z_m)^2}{1-F(z_m)}\right\} \\ &\quad - \left\{\frac{2\,(1+\beta)\,[1-F(z_m)]}{\mu - (1+\beta)\int_{z_m}^{B}(\varepsilon - z_m)f(\varepsilon)\mathrm{d}\varepsilon} + T(z_m)\right\}\frac{\mathrm{d}p(z_m)}{\mathrm{d}z_m}\end{aligned} \tag{A.23}$$

if $2T(z_m)^2 + \mathrm{d}T(z_m)/\mathrm{d}z_m > 0$ and $b \geq 2$, then $\frac{\mathrm{d}^2R(z_m)}{\mathrm{d}{z_m}^2}|_{\mathrm{d}R(z_m)/\mathrm{d}z_m = 0} < 0$, we further know that $R(z_m)$ is unimodal in z_m, first increasing and then decreasing. Hence, given that $2T(z_m)^2 + \mathrm{d}T(z_m)/\mathrm{d}z_m > 0$ and

$b \geq 2$, $E\{U[z_m, p(z_m)]\}$ is unimodal and reaches local maximum at the value of $z_m^* \neq B$ that satisfies $\partial E\{U[z_m, p(z_m)]\}/\partial z_m = y(p(z_m))\,[1 - F(z_m)]R(z_m) = 0$. □

Proof of Proposition 4.9. Since the price is given, according to equation (4.22) and $Q_m^* = a(p_m^*)^{-b} z_m^*$, we can determine the first order condition of the optimal order quantity with respect to surplus and stock-out regret aversion parameters, *i.e.*,

$$\frac{\partial Q_m^*}{\partial \alpha} = a(p_m^*)^{-b} \frac{\partial z_m^*}{\partial \alpha} = -\frac{a(p_m^*)^{-b}(1+\beta)(p_m^* - c + s)(c - v)}{[(1+\alpha)(c-v) + (1+\beta)(p_m^* - c + s)]^2 f(z_m^*)} \tag{A.24}$$

$$\frac{\partial Q_m^*}{\partial \beta} = a(p_m^*)^{-b} \frac{\partial z_m^*}{\partial \beta} = \frac{a(p_m^*)^{-b}(1+\alpha)(c-v)(p_m^* - c + s)}{[(1+\alpha)(c-v) + (1+\beta)(p_m^* - c + s)]^2 f(z_m^*)} \tag{A.25}$$

Since $p_m^* \geq c \geq v$, $f(z_m^*) \geq 0$, and $\alpha, \beta \geq 0$, we know that $\partial Q_m^*/\partial \alpha < 0$ and $\partial Q_m^*/\partial \beta > 0$. □

Proof of Proposition 4.10. Given z_m^*, according to the equation (4.20), we can determine the first order condition of the optimal price with respect to surplus and stock-out regret aversion parameters, *i.e.*,

$$\frac{\partial p_m^*}{\partial \alpha} = \frac{b(c-v)\int_A^{z_m^*} (z_m^* - \varepsilon) f(\varepsilon)\mathrm{d}\varepsilon}{(b-1)\left[\mu - (1+\beta)\int_{z_m^*}^{B} (\varepsilon - z_m^*) f(\varepsilon)\mathrm{d}\varepsilon\right]} \tag{A.26}$$

$$\begin{aligned}\frac{\partial p_m^*}{\partial \beta} &= \frac{b}{b-1}\frac{s\int_{z_m^*}^{B} (\varepsilon - z_m^*) f(\varepsilon)\mathrm{d}\varepsilon \left[\mu - (1+\beta)\int_{z_m^*}^{B} (\varepsilon - z_m^*) f(\varepsilon)\mathrm{d}\varepsilon\right]}{\left[\mu - (1+\beta)\int_{z_m^*}^{B} (\varepsilon - z_m^*) f(\varepsilon)\mathrm{d}\varepsilon\right]^2} \\ &\quad + \frac{b}{b-1}\frac{\left[(1+\alpha)(c-v)\int_A^{z_m^*} (z_m^* - \varepsilon) f(\varepsilon)\mathrm{d}\varepsilon + s(1+\beta)\int_{z_m^*}^{B} (\varepsilon - z_m^*) f(\varepsilon)\mathrm{d}\varepsilon\right]\int_{z_m^*}^{B} (\varepsilon - z_m^*) f(\varepsilon)\mathrm{d}\varepsilon}{\left[\mu - (1+\beta)\int_{z_m^*}^{B} (\varepsilon - z_m^*) f(\varepsilon)\mathrm{d}\varepsilon\right]^2}\end{aligned} \tag{A.27}$$

According to the Lemma 4.6, we know that $\mu - (1+\beta)\int_{z_m^*}^{B} (\varepsilon - z_m^*) f(\varepsilon)\mathrm{d}\varepsilon > 0$ when the optimal price is not the bound price (*i.e.*, $p_m^* = p_x$). Since $\int_{z_m^*}^{B} (\varepsilon - z_m^*) f(\varepsilon)\mathrm{d}\varepsilon \geq 0$ and $\int_A^{z_m^*} (z_m^* - \varepsilon) f(\varepsilon)\mathrm{d}\varepsilon \geq 0$, we know that $\partial p_m^*/\partial \alpha > 0$ and $\partial p_m^*/\partial \beta > 0$. □

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