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A STOCHASTIC APPROACH FOR FAILURE MODE AND EFFECT ANALYSIS

Elif Kiliç Delice[1] and Gulin Feryal Can[2]

Abstract. This study presents a novel approach combining Failure Mode and Effect Analysis (FMEA) and Multi-Attributive Border Approximation Area Comparison (MABAC) method based on a stochastic evaluation process to prioritize potential failure modes (FMs) in an assembly line. The aim of the proposed approach is to improve the performance of FMEA by eliminating its shortcomings addressed in the study. In this context, firstly the risk factor (RF) importance weights and the performance values of the FMs for the RFs are determined by generating random numbers having uniform distribution in a range of minimum and maximum value of a limited number of expert evaluations. In this wise, the number of experts are increased to improve effectiveness of the risk evaluation process. Diverse opinions of experts are also assessed more precisely. Secondly, the priorities of the FMs are identified by implementing MABAC method. MABAC is a practical and reliable tool which provides stability for solutions. Finally, a comparative analysis is implemented to confirm the effectiveness of Stochastic FMEA-MABAC approach.

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1. Introduction

The FMEA is a powerful proactive tool that aims to provide systematic determination of the possible root causes and the failure modes (FMs) relevant to a product, system or service for the purpose of limiting or avoiding their relative risks by prioritizing them systematically. It was first developed by NASA in 1963. Since then, it has been widely used as a risk assessment tool in different industries such as aerospace, nuclear, automotive, electronics, mechanical and medical industries [1–4] etc. FMEA specifies the key service, system or product characteristics that should be controlled more carefully and it defines improvement areas for these characteristics to increase their performance [5]. FMEA offers an easy and useful approach for users to implement their view points and knowledge in a formalized manner [6].

For risk assessment in FMEA the risk priority number (RPN), which is a product of three risk factors (RFs) named as occurrence (O), severity (S), and detection (D) is computed. RPN quantifies "how dangerous" a FM is by identifying a rank of risk priorities of FM. RPN values vary between 1 and 1000. A higher RPN value implies that the related FM has a higher degree of risk and priority.

FMEA procedure has a multi-criteria decision making (MCDM) structure. MCDM is a discipline that provides a logical and scientific framework regarding different opinions of decision makers (DMs) in decision making environments of multiple criteria [7, 8]. Similarly FMEA is a group decision making process which considers

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[1] Department of Industrial Eng., Ataturk University, 25240, Erzurum, Turkey elif.kdelice@atauni.edu.tr

[2] Department of Industrial Eng., Baskent University, 06810, Ankara, Turkey. gfcan@baskent.edu.tr

more than one RF related to relative risk of FMs and makes assessment for more than one failure in a system, product or service and cannot be performed on an individual basis [9]. As a result of FMEA, preventive-corrective measures are prioritized according to the team decision. Because of suitable structure of FMEA for MCDM, various MCDM approaches can be implemented in FMEA. In this way, FMEA can overcome some deficiencies that decrease its performance.

First of all, there are only three RFs (O, S, and D) considered in risk assessment with traditional FMEA. By using MCDM approach FMs can be evaluated in terms of various factors [10]. In this way, FMs can be ranked better and distinguished more clearly from one another. In traditional FMEA, the RPN is computed with respect to different combinations of O, S, and D factors. However, these different combinations usually result in the same RPN values for different FMs [11–13]. Nonetheless, it does not mean that the risk levels of these failures are the same. By using aggregation techniques of MCDM approaches to combine O, S, D and the other RFs in a systematic way, it is almost impossible to obtain the same RPN values for the different FMs [14–17]. What is more FMEA assumes that O, S, D have the same importance weights for risk evaluation. According to this, it neglects human/expert knowledge [14–20]. However, MCDM approaches allow for different importance weights thus it is possible to make the assessment more precise and more reliable [8,13,21]. Besides, FMEA has an ordinal valued scale for three factors. These values preserve rank but the distance between the values cannot be known since a distance computation is not performed. Therefore, FMEA cannot represent the differentiation of expert evaluations [20]. Some of MCDM approaches can utilize distance computation such as MABAC, VIKOR, TOPSIS etc. for overcoming this deficiency. In addition, the mathematical way for computing RPN is questionable. It is very sensitive to variations in RF evaluations [15,16,19]. By using MCDM approaches this shortcoming is also eliminated due to the related implementing procedures. Finally; many of the RPN numbers in the range of 1-1000 cannot be obtained by multiplication of O, S, and D as FMEA has a discrete scale. Only 120 of the 1000 numbers can be generated from the product of O, S, and D.

However, MCDM approaches have many advantages to overcome these shortcomings mentioned above; these approaches have also a disadvantage. In decision making process, there are a few experts who assess the relevant subject, because the number of experts having sufficient level of knowledge and experience related to the problem is insufficient. Additionally, these experts may come from different departments and have different backgrounds, levels of knowledge and experiences. For these reasons, there are many different opinions related to FM evaluation which have an effect on risk assessment process. In this context, if the numbers of experts increase, the performance of evaluation process also increases. This situation necessitates incorporation of more experts in the risk evaluation process. Besides, in FMEA, experts must determine and evaluate FMs before the failure occurs. Consequently, expert evaluations are defected with vagueness and imprecision which eventually results in randomness. Accordingly, evaluations related to the factors and FMs can be increased by using limited number of expert opinions with the way of generating random numbers *via* uniform distribution. In addition, the randomness feature may also be incorporated in the risk evaluation process [22]. It is also possible to expand the decision matrix related to RFs and FMs without changing maximum and minimum values of scores given by experts by generating random numbers from uniform distribution in this maximum-minimum range. The reason for choosing uniform distribution in this process is the equal occurrence probability of each continuous random variable. In this way, it is provided that there is an equal occurrence probability for generated evaluations related to weights of RFs and performance values of FMs. Considering all the facts, a Stochastic FMEA-MABAC approach is proposed for increasing FMEA performance in this study especially in terms of insufficient number of DMs and randomness feature of risk evaluation process. Besides RPN values are computed not only considering occurrence, severity and detection factors but also cost, exposure duration and system safety in this study. These factors are considered risk evaluation for an assembly line. Uniform distribution is used for computing RF weightings and performance values of FMs to obtain results that are more reasonable. Consecutively, MABAC procedure is implemented to prioritize the FMs. MABAC is selected in this study for ranking FMs because it is a reliable and powerful tool for giving logical decisions [23].

The rest of this study is structured as follows; literature related to FMEA and MABAC is given in the second section, the third section is the method section which includes Traditional FMEA and the proposed Stochastic

FMEA-MABAC approach. The fourth section consists of a numerical example utilizing the proposed approach. A comparative analysis is performed in the fifth section. Finally, the sixth section presents the conclusion of the study.

2. Fmea and mabac literature

To overcome the shortcomings as mentioned in previous section and improve the performance of Traditional FMEA, researchers have suggested many different risk evaluation and FM prioritization approaches for the long time. These approaches include uncertainty theories, MCDM techniques, mathematical programming and artificial intelligence [24]. Studies related to these approaches are given as below.

Chang *et al.* [25] developed a novel approach for FMEA to find the RPNs based on fuzzy logic and grey theory. Chang *et al.* [26] used the grey theory for computing the degrees of relational through the traditional crisp scores of FMEA. Also, Pillay and Wang [13], applied Grey Relation Analysis (GRA) method for the prioritization of FMs. Braglia *et al.* [19] utilized the The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) for prioritizing FMs by using Euclidean Distance. Keskin and Ozkan [27] proposed Fuzzy Adaptive Resonance Theory (Fuzzy ART), one of the ART networks, to compute RPN. Wang *et al.* [17] defined The FRPNs as fuzzy weighted geometric means of the fuzzy ratings for O, S and D, and computed using alpha-level sets and linear programming models. For ranking purpose, they used a new centroid defuzzification formula based on alpha-level sets. Abdelgawad and Fayek [28] implemented Fuzzy Expert System and Fuzzy Analytic Hierarchy Process (FAHP) to improve the computation process of RPN. Chang and Cheng [29] developed Fuzzy Ordered Weighted Averaging (FOWA) and Decision Making Trial and Evaluation Laboratory (DEMATEL) for evaluating the risk level of FMs. The Fuzzy Evidential Reasoning is integrated with the grey theory for FMEA implementation by Liu *et al.* [10]. Geum *et al.* [30] developed a new approach to determine and evaluate potential FMs based on service-specific FMEA and GRA method. Zammori and Gabbrielli [31] introduced an integrated approach by using FMEA and Analytic Network Process (ANP) taking into consideration a decision structure for computing the RPN. Liu *et al.* [32] used an Extended Fuzzy VIKOR for FMEA. Hadi-Vencheh and Aghajani [33] proposed a new Fuzzy Group Decision Making (FGDM) model based on α-level sets. Adhikary *et al.* [18] presented the RPN estimation approach by Grey Complex Proportional Assessment (COPRAS-G) for coal-fired thermal power plants. Ilangkumaran *et al.* [34] utilized FAHP for risk factor weights computation and proposed an assessment model integrating FMEA and FAHP to evaluate the risk priority in a paper industry. Liu *et al.* [35] proposed a FMEA model based on fuzzy set theory and Multi-objective Optimization by Ratio Analysis (MULTIMOORA) method. Liu *et al.* [14] introduced a new risk assessment model for the risk prioritization based on D numbers and an enhanced GRA method, called Grey Relational Projection (GRP). Liu *et al.* [36] proposed a risk assessment approach utilizing the Intuitionistic Fuzzy Hybrid Weighted Euclidean Distance (IFHWED) operator in FMEA. Liu *et al.* [37] proposed interval 2-Tuple Hybrid Weighted Distance (ITHWD) operator for a new risk priority assessment. Helvac lu and Ozen [38] developed an approach through Fuzzy TOPSIS (FTOPSIS) for yacht system design. Song *et al.* [16] proposed a new FMEA approach integrating rough set theory and group TOPSIS. Sharma and Sharma [39], Tsai and Yeh [40], Panchal and Kumar [41], Zhou and Thai [42] implemented GRA in FMEA. Liu *et al.* [43] developed a new modified TOPSIS method, called as Intuitionistic Fuzzy Hybrid TOPSIS, to identify the risk priorities of the FMs. Liu *et al.* [15] proposed a new FMEA methodology which includes combination of interval 2-tuple linguistic variables and GRA. Emovon *et al.* [20] utilized an improved FMEA approach combining an averaging technique with the VIÅ¡ekriterijumsko KOmpromisno Rangiranje (VIKOR) for prioritizing FMs for marine machinery systems. Liu *et al.* [44] proposed a hybrid MCDM approach combining the VIKOR, DEMATEL, and AHP for risk assessment. Lolli *et al.* [45] developed a MCDM approach by using Preference Ranking Organization Method for Enrichment Evaluations) (PROMETHEE). Hajiagha *et al.* [8] used fuzzy belief structure based VIKOR method for ranking delay causes of Tehran metro system. Safari *et al.* [46] integrated Fuzzy VIKOR (FVIKOR) and FMEA to assess risks of enterprise architecture. Vahdani *et al.* [47] integrated Fuzzy Belief Structure and TOPSIS to improve the performance of Traditional FMEA. Liu *et al.* [48] applied an ELimination Et Choix Traduisant la

REalit (ELECTRE) based outranking approach within the interval 2-tuple in linguistic environment for FMEA. Liu *et al.* [49] proposed a new model for FMEA by combining Hesitant 2-Tuple Linguistic Term Sets and An Extended QUALIFLEX approach. Zhao *et al.* [50] introduced a new FMEA approach based on Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs) and MULTIMOORA method. Wang *et al.* [51] used the combined entropy and expert evaluation method for determining the weight of the risk factors, they utilized DEMATEL to prioritize the FMs for CNC machining center.

The MABAC method is a new MCDM approach proposed recently by the research center at the University of Defense in Belgrade [23]. MABAC has a simple mathematical computation and a systematic procedure that represents the rationale of human decision making. There are a few studies related to MABAC. Pamuar and irovi proposed DEMATELMABAC approach for forklift selection [23]. Interval-valued Intuitionistic Fuzzy MABAC (IVIF-MABAC) is performed for determining the best material [52]. Choquet Integral Operator for Pythagorean Fuzzy Aggregation Operators, such as Pythagorean Fuzzy Choquet Integral Average (PFCIA) Operator and Pythagorean Fuzzy Choquet Integral Geometric (PFCIG) Operator are used with MABAC method by Peng and Yang [53]. A new MABAC approach based on the likelihood of interval type-2 fuzzy numbers (IT2FNs) is proposed for hotel selection from a tourism website [54]. Roy *et al.* [55] presented a type-2 fuzzy multi-attribute decision making methodology combining trapezoidal interval type-2 fuzzy numbers and MABAC for evaluation and selection of the suitable software company. A new assessment method is proposed by integrating rough number based AHP and rough number based MABAC for selection of the most appropriate cities in India for medical tourism [56]. Boani *et al.* [57] combined FAHP and MABAC for selecting locations for the preparation of laying-up positions.

As seen from the literature fuzzy logic based approaches are widely used for FMEA in fuzzy MCDM and artificial intelligence. The reason for implementing fuzzy logic in FMEA is to handle the uncertainty in real life applications. Besides, in terms of applicability of these types of fuzzy evaluation models in real life cases, some doubts keep.

In these fuzzy logic related studies, the extension principle and the symbolic methods are utilized. In most of these studies, it is obligatory to express the results by using an approximation process, because the computation results generally do not exactly comply with any of the initial linguistic terms. This causes a loss of information and a lack of precision in the final results. In addition, all fuzzy expert systems include stages as fuzzification, fuzzy inference and defuzzification. The risk factors are fuzzified by appropriate membership functions to obtain the degree of membership in each input class in fuzzy FMEA. The obtained fuzzy inputs are assessed in fuzzy inference engine by using well-defined rule base consisting of if-then rules and fuzzy logic operations to compute the risk level of the FMs. Then, the fuzzy output is defuzzified to obtain RPN. However, the fuzzy inference method has been commonly utilized to improve FMEA procedure; it has several limitations [4,19,29]. Firstly, it is difficult to construct a suitable membership functions for the risk factors and risk priority level. In addition, any alteration to the linguistic terms, for example, utilizing seven linguistic terms to describe D instead of five, will require re-evocation of the appropriate membership functions. There is too much information losing in fuzzy inference process for complex calculations to produce "precise" riskiness level of FMs [15]. If then rules implementation suffers from the combinatorial rule eruption problem, which causes increasing number of rules related to computation model of fuzzy RPN. In the if-then rules application, if the number of rules provided by the experts increase also prediction accuracy of the fuzzy RPN model increases. Besides, the structuring of a fuzzy if-then rule base is a difficult task which requires experts to make a large number of judgments. Because of this, construction of if-then rules is not a cost and time-consuming activity. The fuzzy if-then rules which have the same consequence but different background are not able to be distinguished from one another. For this reason, the FMs defined by these fuzzy if-then rules will not be able to be prioritized or ranked. Also, to construct proper software packages to achieve the momentary communication between risk input and output is a difficult task. For this reason, priority ranking of FMs is also difficult.

Some fuzzy FMEA approaches used a reduced if-then rule base to avoid establishing a big if-then-rule base. However, this brings with some new problems [17]. Sometimes, two if-then rules with different background can be combined or reduced. In this situation, the results of these two rules must be the same. Because of these same results, the expert cannot distinguish the two different FMs from each other. As mentioned before FMEA is usually performed by a cross-functional team. The members of the team may have different levels of expertise and knowledge. As a matter of course, different experts often use different linguistic terms for expressing their evaluations for a FM according to the certain RF. These evaluations may be precise or imprecise, certain or uncertain, and complete or incomplete. With reference to this, expert judgments are generally inconsistent. Therefore, combining and reducing rules is nearly impossible. In addition, if the rules are not reduced from a whole if-then rule base, rules reduced will be incomplete. Implementing inference process from an incomplete rule base will be prejudicial or even wrong in as much as some knowledge cannot be obtained from such a missing rule base. For overcoming this, a complete if-then rule base can be established from expert knowledge. But this time, FMs should be prioritized into different priority categories. As a result, a full priority ranking is not obtained. Unfortunately, the practicality of the fuzzy rule-based methods is suspicious because developing and testing a complete set of fuzzy rules is a cost and time-consuming activity [17]. However, the fuzzy inference system has many disadvantages; Liu *et al.* [4] presented that this system can moreover overcome most of the shortcomings mentioned in the recent literature regarding the Traditional FMEA than a Case Based Reasoning system (CBR) and a Vector Machine Support method of classification (VMS).

It has highlighted the importance of group decision in the FMEA. It is also important measuring intra-personal uncertainty and inter-personal uncertainty. Interval type-2 fuzzy sets can deal with both intra and inter-personal uncertainty. However, this approach does not sort failures into groups for multiple experts. In addition, as it is based on fuzzy logic, it requires the illustration of membership functions, which is subjective and difficult [19]. Therefore, the membership function definition may vary from person to person [58]. Generally, most of the research related to fuzzy FMEA used the same membership function for all members of the expert team. Also, it is very difficult or impossible to determine membership functions suitable in risk assessment situations [15]. In addition, results of fuzzy approaches may be very sensitive to the qualitative judgment of the linguistic variables used. Therefore, validating the doubtful results is difficult [59].

In terms of approaches, combining mathematical programming (MP) and FMEA there are also some disadvantages. It is a well-known issue that MP is the use of mathematical models, especially optimizing models, to support making decisions. Its main feature is finding the best solution of a model by optimization software automatically. If the model has been well established, the best solution should transform back into the real life as a good solution for the real life problem. If the model has not been well established, analysis of why it is no good requires greater understanding of the real life problem. Besides, MP has three main components as the constraints, the objective function and the relationships constraints. These components change continuously during the model structuring. However, the procedure of MP includes finding optimum solutions; nobody is suggesting that the solution is optimum for the real life case. Linear programming (LP) is a widely used MP approach for FMEA. But in LP approaches all of the variables that need to be considered to solve a problem cannot be quantified in a linear manner. Therefore, LP assumptions are also unrealistic, due to assumption of linear relationship. It assumes that factors never really change, when in real life they do. The possible solutions that are given in the problem are limited by the limiting the range of the problem.

These difficulties mentioned above highlight the need for new models as proposed in our manuscript, which enable to obtain priority of FMs as both accurate, and as easy as possible. In this manner, this study develops a Stochastic FMEA-MABAC approach for increasing the performance of FMEA and overcoming limitations of the fuzzy, MP, artificial intelligence and MCDM approaches. In Stochastic FMEA-MABAC approach, the requirement of membership function, inference system and rule base are eliminated. The proposed approach can overcome loss of information or lack of precision. The evaluations can be varied by increasing the number of experts. Besides, due to its stability (consistency) of its solutions, The MABAC method is a powerful tool for risk evaluation process [52]. It utilizes the distance function for normalization process of the performance values of FMs. In this wise, the differences between the opinions of experts can be considered in risk evaluation. It is also an important research topic to implement MABAC for prioritizing FMs by using the stochastic values.

Table 1. Severity scale.

Effect	Severity of Effect	Numerical Value
Hazardous	Failure is hazardous and occurs without warning. It suspends operation of the system and/or involves non-compliance with government regulations.	10
Serious	Failure involves hazardous outcomes and/or non-compliance with government regulations or standards	9
Extreme	Product is inoperable with loss of primary function. The system is inoperable.	8
Major	Product performance is severely affected but functions. The system may not operate	7
Significant	Product performance is degraded. Comfort or convince functions may not operate	6
Moderate	Moderate effect on product performance. The product requires repair	5
Low	Small effect on product performance. The product does not require repair	4
Minor	Minor effect on product or system performance	3
Very Minor	Very minor effect on product or system performance	2
None	No effect	1

Table 2. Occurrence scale.

Probability of Failure	Possible Failure Rates	Numerical Value
Extremely high: failure almost inevitable	≥1 in 2	10
Very high	1 in 3	9
Repeated failures	1 in 8	8
High	1 in 20	7
Moderately high	1 in 80	6
Moderate	1 in 400	5
Relatively low	1 in 2000	4
Low	1 in 15,000	3
Remote	1 in 150,000	2
Nearly impossible	≥1 in 1,500,000	1

3. Method

3.1. The traditional FMEA procedure

FMEA provides a tool to assign limited resources to most serious FMs by prioritizing them systematically. FMEA prioritizes FMs according to their RPN values. RPN has three components; O, S, and D as mentioned in introduction section. "O" presents the frequency that a root cause is likely to occur in a qualitative way. "S" is defined as the magnitude of the end effect of a FM. "D" is the likelihood of identifying a root cause before a failure may occurs. These three RFs are individually rated using a numerical scale ranging from "1" to "10" [60]. Table 1, 2 and 3 represents these RFs and their scoring system [61].

By using these scoring system RPN is computed as in equation (3.1)

$$RPN = O \times S \times D; \quad RPN \in [1, 1000] \tag{3.1}$$

3.2. The proposed stochastic FMEA-MABAC approach

The steps of the proposed Stochastic FMEA-MABAC approach are presented in this section. The proposed approach for prioritization of FMs consists of three main stages: determining the RFs and FMs, generating the weights of the RFs and the performance values of FMs, and obtaining the rankings of the FMs. Figure 1 delineates the flowchart of the proposed Stochastic FMEA-MABAC approach.

TABLE 3. Detection scale.

Detection	Criteria: likelihood of detection by design control	Numerical value
Absolute uncertainty	Design control does not detect a potential cause of failure or subsequent failure mode; or there is no design control	10
Very remote	Very remote chance the design control will detect a potential cause of failure or subsequent failure mode	9
Remote	Remote chance the design control will detect a potential cause of failure or subsequent failure mode	8
Very low	Very low chance the design control will detect a potential cause of failure or subsequent failure mode	7
Low	Low chance the design control will detect a potential cause of failure or subsequent failure mode	6
Moderate	Moderate chance the design control will detect a potential cause of failure or subsequent failure mode	5
Moderately high	Moderately high chance the design control will detect a potential cause of failure or subsequent failure mode	4
High	High chance the design control will detect a potential cause of failure or subsequent failure mode	3
Very high	Very high chance the design control will detect a potential cause of failure or subsequent failure mode	2
Almost certain	Design control will almost certainly detect a potential cause of failure or subsequent failure mode	1

Step 1. Form the expert group, record all potential FMs and determine the relevant RFs.

The FMs are denoted as FM_i: ith failure mode; $(i = 1, \ldots, m)$ and the RFs are denoted as RF_J: jth risk factor; $(j = 1, \ldots, n)$. These RFs and FMs are determined from k experts E_k;$(k = 1 \ldots, l)$.

Step 2. Determine the weights of RFs using importance scale.

l experts evaluate n RFs $(x_j; j = 1, \ldots, n)$ with respect to their importance by using scale depicted in Table 4.

The evaluation related to certain RF is denoted as x_{kj}. x_{kj} values form the RF evaluation matrix RF as shown below.

$$RF = \begin{array}{c} E_1 \\ E_2 \\ \vdots \\ E_l \end{array} \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{l1} & x_{l1} & \cdots & x_{ln} \end{bmatrix} \tag{3.2}$$

Step 3. Construct the performance value matrix of FMs.

According to n RFs for each of the l experts, performance value matrix of FMs FM;$(FM_i;\ i = 1, \ldots, m)$ is formed as given below.

$$FM = \begin{array}{c} FM_1 \\ FM_2 \\ \vdots \\ FM_m \end{array} \begin{bmatrix} (x_{11})_k & (x_{12})_k & \cdots & (x_{1n})_k \\ (x_{21})_k & (x_{22})_k & \cdots & (x_{2n})_k \\ \vdots & \vdots & \ddots & \vdots \\ (x_{m1})_k & (x_{m2})_k & \cdots & (x_{mn})_k \end{bmatrix} \tag{3.3}$$

where; $(x_{ij})_k$= The evaluation of ith FM of kth expert for the jth RF.

Step 4. Identify the minimum and maximum importance values of the RFs.

For jth RF, minimum and maximum values of the expert evaluations are identified. These values are denoted as below.

$$\min\{x_{kj}\} = a_j \quad and \quad \max\{x_{kj}\} = b_j;\ j = 1, \ldots, n \tag{3.4}$$

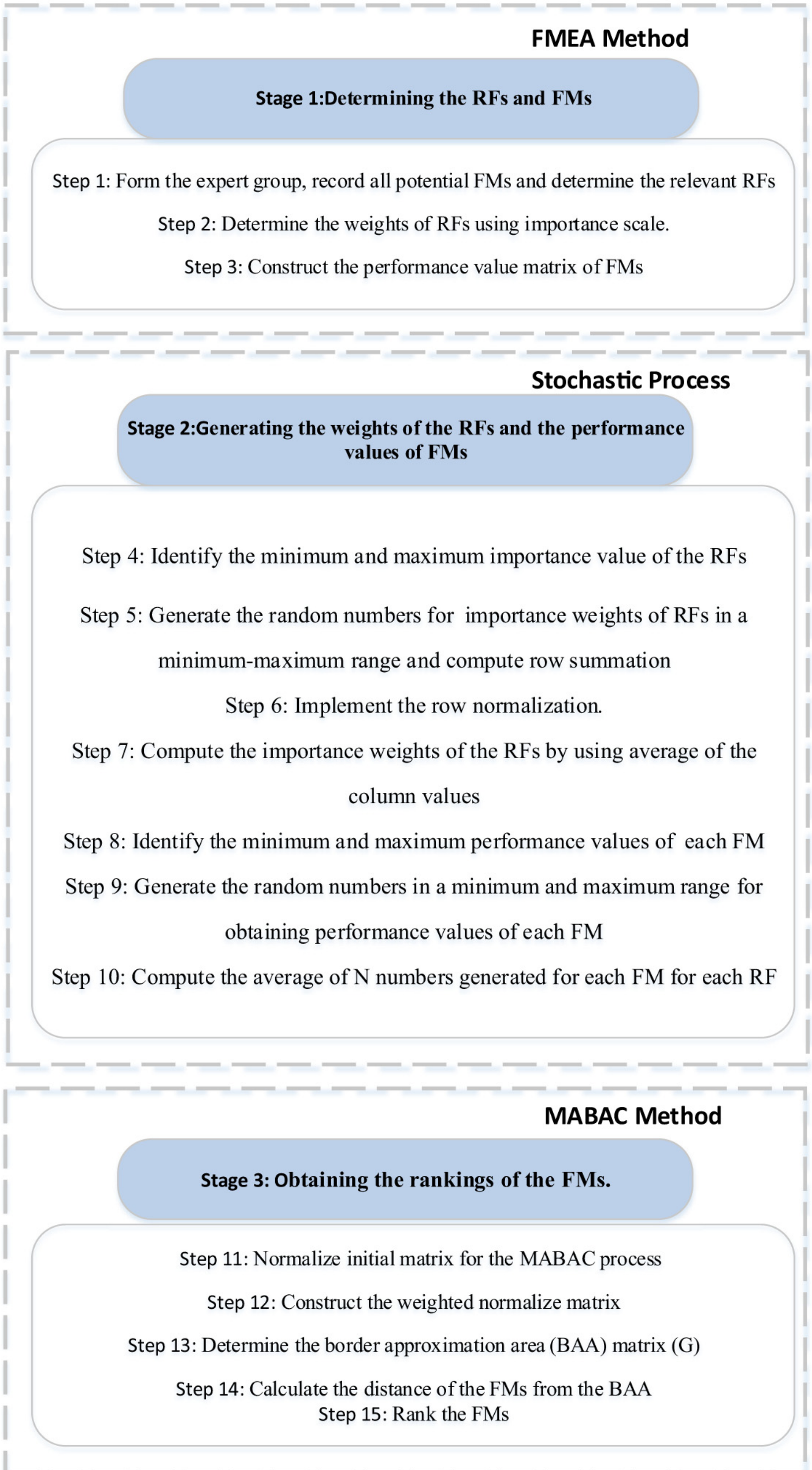

FIGURE 1. Flowchart of Stochastic FMEA-MABAC approach.

TABLE 4. Importance Scale of factors.

Score	Linguistic Terms
1	The least unimportant
2	Quite unimportant
3	Highly unimportant
4	Unimportant
5	Somewhat important
6	A little more important
7	Important
8	Highly important
9	Quite important
10	The most important

Step 5. Generate the random numbers for importance weights of RFs in a minimum-maximum range and compute the row summation.
N numbers are generated from $U(a_j, b_j)$; $j = 1, \ldots, n$ distribution for each RF. Row summation is computed as $\sum_{j=1}^{n} y_{tj}$ for each row t; $t = 1, \ldots, N; y = (x_{kj})_i;\ i = 1, \ldots, m$.

Step 6. Implement the row normalization.

Row normalization is utilized *via* using equation (3.5).

$$y_{tj}' = \frac{y_{tj}}{\sum_{j=1}^{n} y_{tj}};\ t = 1, \ldots, N,\ j = 1, \ldots, n \tag{3.5}$$

Step 7. Compute the importance weights of the RFs by using average of the column values.

For j*th* RF, the average of the column values are computed to obtain the weights of the RFs ($w_j; j = 1, \ldots, n$) as shown in equation (3.6) and the RF weight vector $W = [W_1 \ldots W_n]$ is constructed.

$$w_j = \frac{\sum_{t=1}^{N} y_{tj}'}{N},\ j = 1, \ldots, n \tag{3.6}$$

Step 8. Identify the minimum and maximum performance values of each FM.

For each expert, k numbers of minimum and maximum values of expert evaluations are determined and denoted as in equation (3.7) and (3.8).

$$\min_k \{(x_{ij})_1 \ldots (x_{ij})_l\} = a_{ij}' \tag{3.7}$$

$$\max_k \{(x_{ij})_1 \ldots (x_{ij})_l\} = b_{ij}' \tag{3.8}$$

Step 9. Generate the random numbers in a minimum and maximum range for obtaining performance values of each FM.

For m alternatives, N random numbers are generated from $U(a_{ij}', b_{ij}')$ related to each of n RF.

Step 10. Compute the average of N numbers generated for each FM for each RF.

The averages of N random numbers based on each RF for each FM x_{ij}' are computed by equation (3.9) and FM matrix is constructed.

$$x_{ij}' = \frac{\sum_{t=1}^{N} (v_{tj})_i}{N};\ t = 1, \ldots, N;\ j = 1, \ldots, n;\ v = (x_{ij})_k;\ k = 1, \ldots, l \tag{3.9}$$

$$FM' = \begin{array}{c} FM_1 \\ FM_2 \\ \vdots \\ FM_m \end{array} \begin{bmatrix} x_{11}' & x_{12}' & \cdots & x_{1n}' \\ x_{21}' & x_{22}' & \cdots & x_{2n}' \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1}' & x_{m2}' & \cdots & x_{mn}' \end{bmatrix} \tag{3.10}$$

Step 11. Normalize initial matrix for the MABAC process.

FM' matrix is accepted as the initial matrix and normalized by using equation (3.11) and (3.12).

$$n_{ij} = \frac{x_{ij}' - x_i^-}{x_i^+ - x_i^-} \text{ for benefit RFs} \tag{3.11}$$

$$n_{ij} = \frac{x_{ij}' - x_i^+}{x_i^- - x_i^+} \text{ for cost RFs} \tag{3.12}$$

Where; $x_i^+ = max(x_{ij}')$ and $x_i^- = \min(x_{ij}')$; $i = 1, \ldots, m$ and $j = 1, \ldots, n$
x_i^+ and x_i^- values are obtained from initial decision matrix. The normalized matrix N is given below.

$$N = \begin{bmatrix} n_{11} & n_{12} & \cdots & n_{1n} \\ n_{21} & n_{22} & \cdots & n_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ n_{m1} & n_{m2} & \cdots & n_{mn} \end{bmatrix} \tag{3.13}$$

Step 12. Construct the weighted normalized matrix.

The weighted normalized matrix V is formed by using equation (3.14) and (3.15), and shown as below.

$$v_{ij} = w_j(n_{ij} + 1) \tag{3.14}$$

$$V = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix} = \begin{bmatrix} w_1(n_{11}+1) & w_2(n_{12}+1) & \cdots & w_n(n_{1n}+1) \\ w_1(n_{21}+1) & w_2(n_{22}+1) & \cdots & w_n(n_{2n}+1) \\ \vdots & \vdots & \ddots & \vdots \\ w_1(n_{m1}+1) & w_2(n_{m2}+1) & \cdots & w_n(n_{mn}+1) \end{bmatrix} \tag{3.15}$$

Step 13. Determine the border approximation area (BAA) matrix.

The BAA matrix G is computed by using equation (3.16) and it is formed as an $nx1$ vector as seen in equation (3.17).

$$g_j = \left(\prod_{i=1}^{m} v_{ij} \right)^{1/m} \tag{3.16}$$

$$G = (g_1\, g_2 \ldots g_n) \tag{3.17}$$

where g_j=BAA for each RF.

Step 14. Calculate the distance of the FMs from the BAA.

The distance of the FMs from the BAA (q_{ij}) is calculated as the difference between v_{ij} in [V] and g_j in [G] as seen in equation (3.18).

$$Q = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix} - \begin{bmatrix} g_1 & g_2 & \cdots & g_n \\ g_1 & g_2 & \cdots & g_n \\ \vdots & \vdots & \ddots & \vdots \\ g_1 & g_2 & \cdots & g_n \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mn} \end{bmatrix} \tag{3.18}$$

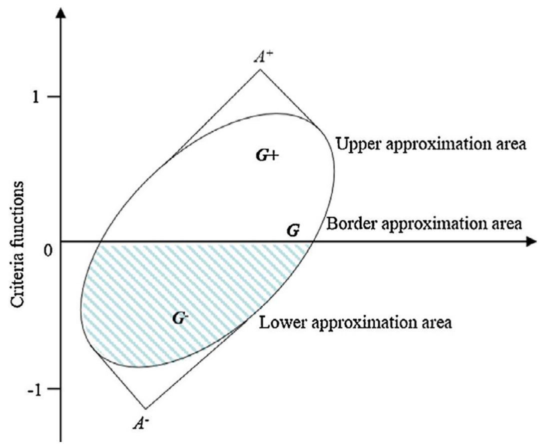

FIGURE 2. Upper, lower and border approximation areas [52].

FM_i may belong to the BAA [G], that is, $FM_i \in \{G \vee G^+ \vee G^-\}$. Here, G^+ is the upper approximation area and G^- is the lower approximation area. G^+ includes the ideal FM (FM^+), while the G^- contains the anti-ideal FM(FM^-) as shown in Figure 2.

The belonging of alternative FM_i to G, G^+ or G^- is determined by equation (3.19):

$$FM_i \in \left\{ \begin{matrix} G^+ & \text{if} & q_{ij} > 0 \\ G & \text{if} & q_{ij} = 0 \\ G^- & \text{if} & q_{ij} < 0 \end{matrix} \right\} \tag{3.19}$$

Step 15. Rank the FMs.

FMs are ranked for the descending order of S_i values. The final values of the RF functions for the FMs are obtained by calculating the sum of the row elements of [Q] as in equation (3.20).

$$S_i = \sum_{j=1}^{n} q_{ij} \; j = 1, \ldots, n; \; i = 1, \ldots, m \tag{3.20}$$

The most risky FM is the one having the highest S_i value.

4. Numerical example

The numerical example is related to the high voltage assembly line in an electro-mechanic systems manufacturing firm. The FMs of the high voltage assembly line are determined through brainstorming and the use of techniques such as root cause analysis and fault tree analysis. This assembly line was newly established and risk evaluation related to occupational health and safety had not been performed. In this context, it was decided to implement FMEA based risk evaluation to this line by using the proposed approach step by step.

Step 1. Form the expert group, record all potential FMs and determine the relevant RFs.

The FMEA team consists of three experts ($E_k; k = 1, 2, 3$). The first expert is a mechanical engineer who has a 15 years of working experience related to electro-mechanic systems. Second expert is an electronic engineer who has a 12 years of experience related to high voltage cell manufacturing. He works as a production manager in high voltage cell manufacturing workshop. The third expert is an electronic engineer who has a 10 years of working experience and he is an A class occupational health and safety expert.

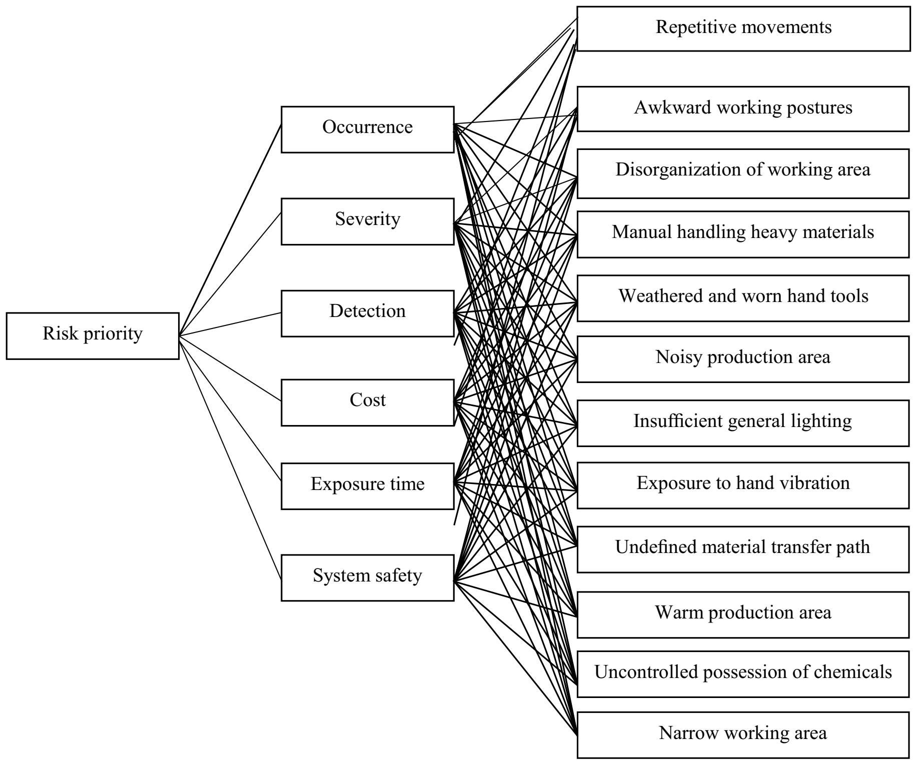

Figure 3. Hierarchical structure of risk assessment.

The twelve FMs ($FM_i; i = 1, \ldots, 12$)related to assembly line are determined by three experts. These FMs are repetitive movements (FM_1), awkward working postures (FM_2), disorganization of working area (FM_3), manual handling heavy materials (FM_4), weathered and worn hand tools (FM_5), noisy production area (FM_6), insufficient general lighting (FM_7), exposure to hand vibration (FM_8), undefined materials transfer path (FM_9), warm production area (FM_{10}), uncontrolled possession of chemicals (FM_{11}), narrow working area (FM_{12}). Additionally, six RFs ($RF_j; j = 1, \ldots, 6$) are identified by same team for evaluation of FMs. These are occurrence (RF_1), severity (RF_2), detection (RF_3), cost (RF_4), exposure time (RF_5), system safety (RF_6). The hierarchical structure related to risk prioritization is shown in Figure 3.

Step 2. Determine the weights of RFs using importance scale.

Three experts evaluated the six RFs with respect to their importance by using the scale depicted in Table 4. Importance evaluations of experts are given in Table 5.

Step 3. Construct the performance value matrix of FMs.

For each of the three experts, the performance value matrix of the twelve FMs is constructed according to the six RFs. To construct performance value matrix, scales depicted in Tables 6, 7, 8 are used for cost, exposure time and system safety factors respectively. The scales for S, O and D risk factors are given in order of Table 1, 2 and 3.

For the first expert, performance value matrix of the FMs is given in Table 9 as an example.

Table 5. Relative importance of the RFs for three experts.

RFs	Experts		
	E_1	E_2	E_3
	x_{1j}	x_{2j}	x_{3j}
RF_1	7	6	8
RF_2	5	6	7
RF_3	4	5	6
RF_4	7	8	9
RF_5	7	9	9
RF_6	7	8	10

Table 6. Scale for cost factor.

Linguistic value	Definition of system safety	Numerical Value
Very low	The cost of corrective and preventive measure is very low.	1,2
Low	The cost of corrective and preventive measure is low.	3,4
Moderate	The cost of corrective and preventive measure is moderate.	5,6,7
High	The cost of corrective and preventive measure is high.	8,9
Very high	The cost of corrective and preventive measure is very high.	10

Table 7. Scale for exposure time factor.

Linguistic value	Definition of system safety	Numerical Value
Instant exposure	There is an instant exposure to failure mode.	1
Very short term exposure	There are a few seconds exposure to failure mode.	2,3
Short term exposure	There are a few minutes exposure to failure mode.	4,5
Long term exposure	There are a few hours exposure to failure mode.	6,7
Very long term exposure	There is a half shift exposure to failure mode.	8,9
Continuous exposure	There is a shift exposure to failure mode.	10

Table 8. Scale for system safety factor.

Linguistic value	Definition of system safety	Numerical Value
Very low	The effect of corrective-preventive measure is very low to increase system safety	9,10
Low	The effect of corrective-preventive measure is low to increase system safety.	7,8
Moderate	The effect of corrective-preventive measure is moderate to increase system safety.	4,5,6
High	The effect of corrective-preventive measure is high to increase system safety.	2,3
Very high	The effect of corrective-preventive measure is very high to increase system safety.	1

Step 4. Identify the minimum and maximum importance values of the RFs.

For each of the six RFs, minimum and maximum values of the expert evaluations are found using in equation (3.4). These values are shown in Table 10.

Step 5. Generate the random numbers for importance weights of RFs in a minimum-maximum range and compute the row summation.

30 numbers having uniform distribution are generated for each RF and the row summation is computed. The generated numbers and row summation values are depicted in Table 11.

Step 6. Implement the row normalization.

The normalization is implemented by using equation (3.5) and the normalized values are depicted in Table 12.

TABLE 9. The performance values of the FMs for the first expert.

FMs	RFs					
	RF_1	RF_2	RF_3	RF_4	RF_5	RF_6
	$(x_{i1})_1$	$(x_{i2})_1$	$(x_{i3})_1$	$(x_{i4})_1$	$(x_{i5})_1$	$(x_{i6})_1$
FM_1	9	8	1	6	7	6
FM_2	7	7	1	6	10	7
FM_3	6	6	3	7	10	7
FM_4	7	6	4	8	10	9
FM_5	10	8	1	9	10	8
FM_6	5	6	3	6	6	5
FM_7	9	6	2	5	10	4
FM_8	7	7	3	8	7	4
FM_9	10	9	4	3	9	7
FM_{10}	7	6	1	8	8	9
FM_{11}	10	10	1	9	6	9
FM_{12}	6	6	1	8	9	8

TABLE 10. The minimum and maximum values of the expert evaluations for the importance weight of RFs.

RFs	Minimum Value	Maximum Value
	a_j	b_j
RF_1	6	8
RF_2	5	7
RF_3	4	6
RF_4	7	9
RF_5	7	9
RF_6	7	10

Step 7. Compute the importance weights of the RFs by using average of the column values.
For each of the six RF, the average of the column values are computed to obtain the weight of the RFs, w_j $(j = 1, \ldots, n)$ as in equation (3.6) and the RF weight vector W is constructed.

Step 8. Identify the minimum and maximum performance values of each FM.
For each E, k numbers of minimum and maximum evaluation values are determined as in equation (3.7) and (3.8). The minimum and maximum values of expert evaluations for FMs for the RFs are presented in Table 14.

Step 9. Generate the random numbers in a minimum and maximum range for obtaining performance values of each FM.
For twelve FMs, 30 random numbers in uniform distribution are generated for each of the six RFs. 30 random numbers generated for FM_1 are given in Table 15 as an example.

Step 10. Compute the average of N numbers generated for each FM for each RF.
The averages of 30 random numbers according to each of the six RFs for each of the twelve FMs are computed by equation (3.9) and FM' matrix is constructed. The average values of FMs with respect to RFs are given in Table 16.

Step 11. Normalize initial matrix for the MABAC process.
For the normalization of $[FM']$, Equation (3.11) is used for benefit RFs (RF_3, RF_6) and equation (3.12) is used for cost RFs (RF_1, RF_2, RF_4, RF_5). The normalized matrix $[N]$ is given in Table 17.

TABLE 11. Generated numbers and row summation values.

Experts	RFs						$\sum_{j=1}^{n} y_{tj}$
$(E_k; k = 1\ldots, 30)$	RF_1	RF_2	RF_3	RF_4	RF_5	RF_6	$t = 1, \ldots, 30$
E_1	6,578	5,563	4,697	7,940	7,730	9,720	42,227
E_2	6,629	6,864	5,104	8,932	8,719	7,552	43,799
E_3	7,029	5,862	5,216	7,704	8,537	9,245	43,593
E_4	6,720	6,403	4,343	7,418	8,496	7,879	41,258
E_5	6,496	5,370	4,167	7,590	7,174	9,112	39,909
E_6	6,165	5,407	4,398	7,388	7,424	9,272	40,054
E_7	6,767	6,043	4,796	8,393	8,315	8,151	42,466
E_8	7,818	6,818	5,556	7,265	7,865	8,545	43,866
E_9	7,131	5,374	4,216	7,590	7,440	7,649	39,400
E_{10}	7,090	6,221	5,776	8,752	8,193	8,291	44,322
E_{11}	6,849	5,937	5,561	7,364	7,179	7,459	40,349
E_{12}	7,838	5,719	4,187	7,670	8,550	9,523	43,487
E_{13}	6,330	5,220	4,327	7,886	7,185	8,736	39,684
E_{14}	6,650	6,921	5,748	8,063	8,717	7,141	43,241
E_{15}	6,908	6,962	5,127	8,728	7,659	8,341	43,724
E_{16}	6,135	6,775	4,218	7,779	7,471	8,318	40,695
E_{17}	7,253	6,105	4,229	8,392	7,893	7,274	41,147
E_{18}	6,516	6,411	4,068	8,469	8,513	9,139	43,116
E_{19}	7,045	6,999	5,678	8,625	8,556	8,011	44,913
E_{20}	6,888	6,177	4,887	8,126	7,473	8,222	41,773
E_{21}	6,215	6,043	4,263	8,203	8,044	7,961	40,730
E_{22}	6,187	6,695	5,512	7,279	7,214	8,868	41,755
E_{23}	6,446	5,494	5,097	8,642	7,185	8,255	41,119
E_{24}	7,284	6,577	4,168	8,169	7,988	8,069	42,254
E_{25}	7,018	6,354	5,994	8,450	8,572	9,172	45,560
E_{26}	7,699	5,304	5,053	7,781	8,196	9,199	43,231
E_{27}	6,292	6,915	5,450	8,061	8,163	7,940	42,822
E_{28}	7,928	6,834	5,297	7,664	8,653	7,226	43,603
E_{29}	6,404	5,089	4,661	7,901	7,183	9,232	40,470
E_{30}	6,086	6,164	5,501	7,957	8,302	7,103	41,114

Step 12. Construct the weighted normalized matrix.

$[V]$ is constructed by multiplying the weight of the RFs (w_j) given in Table 13 by the elements of the $[N]$ seen in Table 17 by using equation (3.14). $[V]$ is depicted in Table 18.

Step 13. Determine the border approximation area (BAA) matrix.

$[G]$ is obtained by geometrically averaging the performance values of the FMs in Table 18 with equation (3.16). $[G]$ is given in Table 19.

Step 14. Calculate the distance of the FMs from the BAA.

In this step, the distance of a FM from the BAA is determined using equation (3.18) as in Table 20.

Step 15. Rank the FMs.

The values of the RF functions for the FMs are the sum of each row of elements from matrix Q using equation (3.20) and the obtained rank of the FMs is given in Table 21. As seen from Table 21 due to the highest S_i value FM_4 is the most risky FM.

Table 12. Normalized values.

Experts	RFs					
($E_k; k = 1\ldots,30$)	RF_1	RF_2	RF_3	RF_4	RF_5	RF_6
E_1	0,156	0,132	0,111	0,188	0,183	0,230
E_2	0,151	0,157	0,117	0,204	0,199	0,172
E_3	0,161	0,134	0,120	0,177	0,196	0,212
E_4	0,163	0,155	0,105	0,180	0,206	0,191
E_5	0,163	0,135	0,104	0,190	0,180	0,228
E_6	0,154	0,135	0,110	0,184	0,185	0,231
E_7	0,159	0,142	0,113	0,198	0,196	0,192
E_8	0,178	0,155	0,127	0,166	0,179	0,195
E_9	0,181	0,136	0,107	0,193	0,189	0,194
E_{10}	0,160	0,140	0,130	0,197	0,185	0,187
E_{11}	0,170	0,147	0,138	0,183	0,178	0,185
E_{12}	0,180	0,132	0,096	0,176	0,197	0,219
E_{13}	0,160	0,132	0,109	0,199	0,181	0,220
E_{14}	0,154	0,160	0,133	0,186	0,202	0,165
E_{15}	0,158	0,159	0,117	0,200	0,175	0,191
E_{16}	0,151	0,166	0,104	0,191	0,184	0,204
E_{17}	0,176	0,148	0,103	0,204	0,192	0,177
E_{18}	0,151	0,149	0,094	0,196	0,197	0,212
E_{19}	0,157	0,156	0,126	0,192	0,190	0,178
E_{20}	0,165	0,148	0,117	0,195	0,179	0,197
E_{21}	0,153	0,148	0,105	0,201	0,198	0,195
E_{22}	0,148	0,160	0,132	0,174	0,173	0,212
E_{23}	0,157	0,134	0,124	0,210	0,175	0,201
E_{24}	0,172	0,156	0,099	0,193	0,189	0,191
E_{25}	0,154	0,139	0,132	0,185	0,188	0,201
E_{26}	0,178	0,123	0,117	0,180	0,190	0,213
E_{27}	0,147	0,161	0,127	0,188	0,191	0,185
E_{28}	0,182	0,157	0,121	0,176	0,198	0,166
E_{29}	0,158	0,126	0,115	0,195	0,177	0,228
E_{30}	0,148	0,150	0,134	0,194	0,202	0,173

Table 13. Weights of the RFs.

Weights					
w_1	w_2	w_3	w_4	w_5	w_6
0,161	0,146	0,116	0,190	0,188	0,198

5. Comparative analysis

The aim of the comparative analysis is to further demonstrate the effectiveness of the proposed Stochastic FMEA-MABAC approach. We used the example above to compare the rankings of Stochastic FMEA-MABAC, Classical FMEA-MABAC and Traditional FMEA approaches. The Classical FMEA-MABAC and Traditional FMEA approaches are computed for all FMs and the results are depicted for three approaches in Table 22.

According to the rankings of Stochastic FMEA-MABAC and Classical FMEA-MABAC approaches, manual handling of heavy materials (FM_4) is at the first rank and narrow working area (FM_{12}) is at the second rank.

TABLE 14. Minimum-maximum values of experts evaluations for FMs for the each of RF.

FMs	RFs											
	RF_1		RF_2		RF_3		RF_4		RF_5		RF_6	
	a_{i1}'	b_{i1}'	a_{i2}'	b_{i2}'	a_{i3}'	b_{i3}'	a_{i4}'	b_{i4}'	a_{i5}'	b_{i5}'	a_{i6}'	b_{i6}'
FM_1	5	9	6	8	1	3	6	7	7	10	6	9
FM_2	7	9	7	9	1	2	5	8	7	10	5	8
FM_3	6	8	4	6	1	3	7	9	9	10	4	7
FM_4	7	10	4	6	3	4	3	9	6	10	4	9
FM_5	7	10	6	8	1	2	8	9	7	10	7	8
FM_6	5	10	6	9	1	3	6	9	6	10	5	9
FM_7	6	9	6	8	1	4	5	8	9	10	4	9
FM_8	7	9	5	7	3	5	6	8	7	10	4	8
FM_9	5	10	7	9	2	4	6	9	7	10	4	7
FM_{10}	5	7	4	6	1	2	7	8	8	10	7	9
FM_{11}	8	10	6	10	1	2	6	9	6	10	7	9
FM_{12}	4	6	6	7	1	2	6	9	6	10	8	9

Exposure to hand vibration (FM_8) is at the fifth rank for both of these approaches. However, the rankings for the rest of FMs are different for these two approaches. According to Classical FMEA-MABAC, noisy assembly area (FM_6) is at the third rank while the same rank is occupied by repetitive movements (FM_1) in Stochastic FMEA-MABAC. Interviews with the workers revealed that repetitive movements were found to be more reasonable for this rank. Workers expressed their feeling of tiredness because of the same movements they have to perform all the time during a shift. They only make the same assembly operations with insignificant changes for the same product.

Disorganization of working area (FM_3) is at the final rank in the Stochastic FMEA-MABAC, which is an acceptable result for high voltage assembly line working conditions. There are more important and compelling FMs than disorganization of working area. This FM is at the tenth rank in Classical FMEA-MABAC, which is unreasonable as uncontrolled possession of chemicals (FM_{11}) occupying at eleventh rank is expected to have a higher rank. Weathered and worn hand tools (FM_5) is also expected to have a higher rank than (FM_3). As a result, the effects of damage resulting from (FM_{11}) and (FM_5) will be greater than disorganization of working area.

As seen from Table 22, there are many differences between ranking orders obtained by Traditional FMEA and the other two approaches. Since, the Traditional FMEA has the limitations mentioned in the introduction section, in determination of the FM priorities the ranking results obtained from this approach is inconsistent. For example, based on the Traditional FMEA approach undefined material transfer path (FM_9) is at the first rank and manual handling heavy materials (FM_4) is ranked behind undefined material transfer path (FM_9). However in reality, (FM_4) is more important, and the result of the proposed approach shows that FM_4 has a higher priority in comparison with FM_9. In assembly line, workers must handle high voltage cell separator (150 kg.) manually as seen in Figure 4. This part is too heavy to physically deal with, potentially posing the risk of musculoskeletal disorders and possibility of losing working ability for the workers.

Interestingly, narrow working area (FM_{12}) turned out to be the least critical FM in Traditional FMEA, while in the proposed approach and Classical FMEA-MABAC it takes the second rank. This rank obtained from Stochastic FMEA-MABAC and Classical FMEA-MABAC approaches is more reasonable result than the rank of the Traditional FMEA as the workers have to work in a small area in the cell having a narrow internal volume as shown in Figure 5. This situation limits the movement of workers.

Besides, Spearmans rank correlation coefficients (r_s) are measured to illustrate the similarity between these three different ranking approaches. Table 23 shows r_s values for the three approaches.

Table 15. 30 random numbers for FM_1.

Experts	RFs					
$(E_k; k = 1 \ldots, 30)$	RF_1	RF_2	RF_3	RF_4	RF_5	RF_6
E_1	6,759	7,488	2,061	6,634	7,824	8,351
E_2	7,554	7,066	2,092	6,765	9,506	7,676
E_3	5,189	6,080	1,349	6,255	8,907	8,729
E_4	5,276	7,483	1,650	6,136	7,079	8,064
E_5	5,769	6,332	1,612	6,141	7,496	6,326
E_6	8,525	6,623	2,143	6,120	9,766	8,867
E_7	5,148	7,076	2,163	6,935	7,394	7,685
E_8	8,377	7,418	2,290	6,436	8,794	7,981
E_9	7,574	7,763	2,341	6,085	9,875	7,802
E_{10}	7,308	6,763	2,740	6,665	7,886	7,653
E_{11}	6,535	6,825	1,274	6,175	9,308	6,628
E_{12}	7,914	6,558	2,060	6,121	7,900	8,524
E_{13}	7,210	7,467	1,504	6,881	9,338	8,933
E_{14}	7,572	7,519	1,121	6,197	9,845	8,058
E_{15}	6,284	6,974	2,213	6,600	8,047	8,912
E_{16}	5,820	7,216	2,102	6,494	7,221	6,914
E_{17}	7,810	6,737	1,180	6,807	8,947	8,517
E_{18}	7,088	7,465	2,136	6,898	9,728	6,003
E_{19}	5,936	6,710	2,175	6,867	8,880	8,837
E_{20}	7,287	6,255	1,218	6,629	8,798	6,812
E_{21}	6,856	6,100	1,630	6,839	9,972	6,394
E_{22}	7,191	6,362	1,686	6,237	8,009	8,978
E_{23}	8,917	7,494	2,512	6,699	7,445	7,064
E_{24}	8,576	7,641	1,071	6,274	7,435	7,239
E_{25}	6,831	7,794	1,831	6,958	7,823	6,761
E_{26}	7,158	7,082	1,714	6,528	8,001	8,301
E_{27}	7,298	6,991	1,269	6,169	7,275	8,814
E_{28}	6,185	6,039	2,587	6,294	7,625	8,205
E_{29}	7,467	6,109	1,224	6,671	9,648	7,757
E_{30}	8,497	7,372	2,767	6,432	9,809	6,559

Considering r_s values, it is clear that, Stochastic FMEA-MABAC approach has the similarity of %88,1 with the Classical FMEA-MABAC approach in terms of ranking. There is no correlation between the rankings of Traditional FMEA and the other two approaches. Due to the shortcomings mentioned in the introduction section, rankings provided by Traditional FMEA are not reasonable. In addition, Classical FMEA-MABAC approach has a disadvantage in terms of limited number of experts. Results obtained from Stochastic FMEA-MABAC approach are more reasonable than Classical FMEA-MABAC because of the increased numbers of experts and considered randomness. Therefore, evaluations can be made more precisely with the proposed approach.

6. Conclusions

This paper proposes a Stochastic FMEA-MABAC approach to prioritize potential FMs in an assembly line. The Stochastic FMEA method is used to determine the weights of the risk factors, and Stochastic MABAC which is a new MCDM method, is implemented to prioritize the FMs. In the proposed new approach, weights of the RFs and the performance values of FMs are determined by using random numbers generated in uniform

TABLE 16. The average values of FMs with respect to RFs.

FMs	RFs					
	RF_1	RF_2	RF_3	RF_4	RF_5	RF_6
	${x_{i1}}'$	${x_{i2}}'$	${x_{i3}}'$	${x_{i4}}'$	${x_{i5}}'$	${x_{i6}}'$
FM_1	7,064	6,960	1,857	6,498	8,519	7,778
FM_2	7,898	7,927	1,420	6,595	8,523	6,532
FM_3	7,033	5,023	2,063	7,952	9,508	5,099
FM_4	8,484	5,037	3,459	6,091	7,989	6,727
FM_5	8,912	6,938	1,479	8,598	8,550	7,411
FM_6	7,797	7,621	2,133	7,475	7,960	6,956
FM_7	7,595	6,839	2,570	6,425	9,497	6,626
FM_8	7,960	6,007	3,944	6,947	8,504	5,557
FM_9	7,502	7,922	2,818	7,235	8,486	5,767
FM_{10}	6,037	4,856	1,476	7,582	9,107	7,973
FM_{11}	9,044	8,197	1,459	7,681	8,170	8,021
FM_{12}	5,043	6,433	1,506	7,324	8,266	8,433

TABLE 17. The normalized matrix $[N]$.

FMs	RFs					
	RF_1	RF_2	RF_3	RF_4	RF_5	RF_6
	Min	Min	Max	Min	Min	Max
FM_1	0,495	0,370	0,173	0,837	0,639	0,804
FM_2	0,286	0,081	0,000	0,799	0,636	0,430
FM_3	0,503	0,950	0,255	0,258	0,000	0,000
FM_4	0,140	0,946	0,808	1,000	0,982	0,488
FM_5	0,033	0,377	0,023	0,000	0,619	0,694
FM_6	0,312	0,172	0,282	0,448	1,000	0,557
FM_7	0,362	0,407	0,456	0,867	0,007	0,458
FM_8	0,271	0,655	1,000	0,659	0,648	0,137
FM_9	0,385	0,082	0,554	0,544	0,660	0,200
FM_{10}	0,752	1,000	0,022	0,405	0,259	0,862
FM_{11}	0,000	0,000	0,015	0,366	0,865	0,877
FM_{12}	1,000	0,528	0,034	0,508	0,803	1,000

distribution, which eliminates the uncertainties in risk evaluation process to some extent. Many real life risk assessment cases feature uncertainty which results in randomness. This study demonstrates that the proposed approach is an effective and useful tool to determine the priorities of potential FMs in risk assessment problems considering randomness and uncertainty.

Compared with the other approaches integrated with FMEA, Stochastic FMEA-MABAC has the following advantages:

- By applying stochastic MCDM structure to FMEA, the ranking results of FMs obtained are more reasonable because randomness and uncertainty in real life can be reflected and modeled in risk evaluation process.
- The proposed approach has the capability of determining the objective weights of RFs *via* increasing the number of DMs through random number generation in uniform distribution.
- FMs can be evaluated according to six different RFs where FMEA uses only O, S and D.

TABLE 18. Weighted normalized matrix $[V]$.

FMs	RFs					
	RF_1	RF_2	RF_3	RF_4	RF_5	RF_6
FM_1	0,241	0,200	0,136	0,349	0,309	0,358
FM_2	0,208	0,158	0,116	0,341	0,308	0,283
FM_3	0,243	0,284	0,146	0,239	0,188	0,198
FM_4	0,184	0,284	0,210	0,380	0,373	0,295
FM_5	0,167	0,201	0,119	0,190	0,305	0,336
FM_6	0,212	0,171	0,149	0,275	0,377	0,309
FM_7	0,220	0,205	0,169	0,354	0,190	0,289
FM_8	0,205	0,241	0,232	0,315	0,311	0,225
FM_9	0,224	0,158	0,181	0,293	0,313	0,238
FM_{10}	0,283	0,292	0,119	0,267	0,237	0,369
FM_{11}	0,161	0,146	0,118	0,259	0,351	0,372
FM_{12}	0,323	0,223	0,120	0,286	0,340	0,396

TABLE 19. Border approximation area matrix $[G]$.

BAA	RFs					
	RF_1	RF_2	RF_3	RF_4	RF_5	RF_6
g_j	0,218	0,208	0,147	0,291	0,293	0,299

TABLE 20. Distances of the FMs from the BAA matrix $[Q]$.

FMs	RFs					
	RF_1	RF_2	RF_3	RF_4	RF_5	RF_6
FM_1	0,023	−0,008	−0,011	0,058	0,016	0,058
FM_2	−0,011	−0,050	−0,031	0,051	0,015	−0,016
FM_3	0,024	0,077	−0,001	−0,052	−0,105	−0,101
FM_4	−0,034	0,076	0,063	0,089	0,080	−0,004
FM_5	−0,052	−0,007	−0,028	−0,101	0,012	0,036
FM_6	−0,007	−0,037	0,002	−0,016	0,084	0,009
FM_7	0,002	−0,003	0,022	0,064	−0,103	−0,010
FM_8	−0,013	0,034	0,085	0,024	0,018	−0,074
FM_9	0,005	−0,050	0,033	0,002	0,020	−0,061
FM_{10}	0,064	0,084	−0,028	−0,024	−0,056	0,070
FM_{11}	−0,057	−0,062	−0,029	−0,031	0,058	0,073
FM_{12}	0,105	0,015	−0,027	−0,004	0,047	0,097

- With the proposed approach, priority ranking of FMs is determined by employing MABAC which is more applicable, easy to implement and provides consist ranking results.
- The proposed approach is more suitable to cope with the risk evaluation problem under the lack of sufficient number of experts. Thus, the outcome of the criticality analysis will provide more reasonable and effective information for the risk evaluation process.
- The required computations of the proposed approach are straightforward and the approach is easy-to-use in real-life risk evaluation applications.

TABLE 21. Rankings of FMs.

FMs	S_i	Rank
FM_1	0,136	3
FM_2	−0,042	8
FM_3	−0,158	12
FM_4	0,270	1
FM_5	−0,139	11
FM_6	0,036	6
FM_7	−0,029	7
FM_8	0,073	5
FM_9	−0,051	10
FM_{10}	0,110	4
FM_{11}	−0,048	9
FM_{12}	0,232	2

TABLE 22. Rankings of the FMs for three different approaches.

FMs	Stochastic FMEA-MABAC		Classical FMEA-MABAC		Traditional FMEA	
	S_i	Ranking	S_i	Ranking	RPN	Ranking
FM_1	0,136	3	0,152	4	36 902	9
FM_2	−0,042	8	−0,078	9	33 778	10
FM_3	−0,158	12	−0,119	10	38 174	8
FM_4	0,270	1	0,195	1	62 578	2
FM_5	−0,139	11	−0,128	12	51 333	6
FM_6	0,036	6	0,175	3	51 202	7
FM_7	−0,029	7	−0,004	8	53 331	4
FM_8	0,073	5	0,097	5	52 752	5
FM_9	−0,051	10	0,009	7	68 210	1
FM_{10}	0,110	4	0,052	6	30 884	11
FM_{11}	−0,048	9	−0,125	11	59 718	3
FM_{12}	0,232	2	0,190	2	24 937	12

TABLE 23. r_s values for rankings of three different approaches.

Approaches	Stochastic FMEA based MABAC	Classical FMEA based MABAC	Traditional RPN
Stochastic FMEA based MABAC	–	0,881[1]	
Classical FMEA based MABAC	0,881[1]	–	
Traditional FMEA			–

[1]Correlation is significant at the 0,01 level (2-tailed).

For the future studies, the proposed approach can be used to prioritize FMs in other sectors. Results obtained by the proposed approach in this study can be compared with other MCDM methods such as VIKOR, TOPSIS or PROMETHEE etc. Another possible future research may be related to RFs relationship. By considering the relationships between the RFs, FMEA can be implemented in a stochastic manner. Finally, different RFs may be considered for risk evaluation in FMEA.

Figure 4. Manual handling of high voltage cell separator.

Figure 5. Narrow working area.

References

[1] J. Yang, H.-Zh. Huang, L.-P. He, Sh.-P. Zhu and D. Wen, Risk evaluation in failure mode and effects analysis of aircraft turbine rotor blades using dempster–shafer evidence theory under uncertainty. *Eng. Failure Anal.* **18** (2011) 2084–2092.

[2] S. Vinodh and D. Santhosh, Application of FMEA to an automotive leaf spring manufacturing organization. *TQM J.* **24** (2012) 260–274.

[3] C. Kahraman, İ. Kaya and Ö. Şenvar, Healthcare failure mode and effects analysis under fuzziness. *Hum. Ecol. Risk Assess. Internat. J.* **19** (2013) 538–552.

[4] H.-Ch. Liu, L. Liu and N. Liu, Risk evaluation approaches in failure mode and effects analysis: A literature review. *Expert Syst. Appl.* **40** (2013) 828–838.

[5] W.G. Ireson, C.F. Coombs and R.Y. Moss, Handbook of Reliability Engineering and Management. McGraw Hill (1996).

[6] N.R. Shankar, B.S. Prabhu, D.O. Thompson, D.E. Chimenti and L. Poore, Application of fuzzy logic to matrix FMECA. In *AIP Conf. Proc.* AIP **557** (2001) 1987–1994 .

[7] M. Gul and A.F. Guneri, A fuzzy multi criteria risk assessment based on decision matrix technique: A case study for aluminum industry. *J. Loss Prev. Process Industries* **40** (2016) 89–100.

[8] S.H.R. Hajiagha, Sh.S. Hashemi, Y. Mohammadi and E.K. Zavadskas, Fuzzy belief structure based VIKOR method: an application for ranking delay causes of Tehran metro system by FMEA criteria. *Transport* **31** (2016) 108–118.

[9] K.-S. Chin, Y.-M. Wang, G.K.K. Poon and J.-B. Yang, Failure mode and effects analysis by data envelopment analysis. *Decis. Support Syst.* **48** (2009) 246–256.

[10] H.-Ch. Liu, L. Liu, Q.-H. Bian, Q.-L. Lin, N. Dong and P.-Ch. Xu, Failure mode and effects analysis using fuzzy evidential reasoning approach and grey theory. *Expert Syst. Appl.* **38** (2011) 4403–4415.

[11] J.B. Bowles and C.E. Peláez, Fuzzy logic prioritization of failures in a system failure mode, effects and criticality analysis. *Reliab. Eng. Syst. Safety* **50** (1995) 203–213.

[12] M. Ben-Daya and A. Raouf, A revised failure mode and effects analysis model. *Inter. J. Quality Reliab. Manag.* **13** (1996) 43–47.

[13] A. Pillay and J. Wang, Modified failure mode and effects analysis using approximate reasoning. *Reliab. Eng. Syst. Safety* **79** (2003) 69–85.

[14] H.-Ch. Liu, J.-X. You, X.-J. Fan and Q.-L. Lin, Failure mode and effects analysis using D numbers and grey relational projection method. *Expert Syst.h Appl.* **41** (2014) 4670–4679.

[15] H.-Ch. Liu, P. Li, J.-X. You and Y.-Z. Chen, A novel approach for FMEA: Combination of interval 2-tuple linguistic variables and gray relational analysis. *Quality Reliab. Eng. Inter.* **31** (2015) 761–772.

[16] W. Song, X. Ming, Zh. Wu and B. Zhu, A rough TOPSIS approach for failure mode and effects analysis in uncertain environments. *Quality Reliab. Eng. Inter.* **30** (2014) 473–486.

[17] Y.-M. Wang, K.-S. Chin, G. Ka Kwai Poon and J.-B. Yang, Risk evaluation in failure mode and effects analysis using fuzzy weighted geometric mean. *Expert Syst. Appl.* **36** (2009) 1195–1207.

[18] D.D. Adhikary, G.K. Bose, D. Bose and S. Mitra, Multi criteria FMECA for coal-fired thermal power plants using copras-g. *Inter. J. Quality Reliab. Manag.* **31** (2014) 601–614.

[19] M. Braglia, M. Frosolini and R. Montanari, Fuzzy criticality assessment model for failure modes and effects analysis. *Int. J. Quality Reliab. Manag.* **20** (2003) 503–524.

[20] I. Emovon, R.A. Norman, J.M. Alan and K. Pazouki, An integrated multicriteria decision making methodology using compromise solution methods for prioritising risk of marine machinery systems. *Ocean Eng.* **105** (2015) 92–103.

[21] R.K. Sharma, D. Kumar and P. Kumar, Systematic failure mode effect analysis (FMEA) using fuzzy linguistic modelling. *Inter. J. Quality Reliab. Manag.* **22** (2005) 986–1004.

[22] Y. Ayrım, K. Atalay and F.G. Can, Kargo firması seçiminde stokastik copras yaklaşımı. In Ulusal Yoneylem Araştırması ve Endustri Mhendisliği Kongresi, Izmir. *Temmuz* **36** (2016) 13–15.

[23] D. Pamučar and G. Ćirović, The selection of transport and handling resources in logistics centers using multi-attributive border approximation area comparison (MABAC). *Expert Syst. Appl.* **42** (2015) 3016–3028.

[24] H.-Ch. Liu, FMEA using uncertainty theories and MCDM methods. In *FMEA Using Uncertainty Theories and MCDM Methods*. Springer (2016) 13–27.

[25] Ch.-L. Chang, Ch.-Ch. Wei and Y.-H. Lee, Failure mode and effects analysis using fuzzy method and grey theory. *Kybernetes* **28** (1999) 1072–1080.

[26] Ch.-L. Chang, P.-H. Liu and Ch.-Ch. Wei, Failure mode and effects analysis using grey theory. *Integr. Manuf. Syst.* **12** (2001) 211–216.

[27] G.A. Keskin and C. Özkan, An alternative evaluation of fmea: fuzzy art algorithm. *Quality Reliab. Eng. Inter.* **25** (2009) 647–661.

[28] M. Abdelgawad and A.R. Fayek, Risk management in the construction industry using combined fuzzy FMEA and fuzzy AHP. *J. Constr. Eng. Manag.* **136** (2010) 1028–1036.

[29] K.-H. Chang and Ch.H. Cheng, Evaluating the risk of failure using the fuzzy OWA and DEMATEL method. *J. Intel. Manuf.* **22** (2011) 113–129.

[30] Y. Park Y. Geum and Y. Cho, A systematic approach for diagnosing service failure: *Service-specific FMEA and Grey Relational Analysis Approach* (2011).

[31] F. Zammori and R. Gabbrielli, ANP/RPN: A multi criteria evaluation of the risk priority number. *Quality Reliab. Eng. Inter.* **28** (2012) 85–104.

[32] H.-Ch. Liu, L. Liu, N. Liu and L.-X. Mao, Risk evaluation in failure mode and effects analysis with extended VIKOR method under fuzzy environment. *Expert Syst. Appl.* **39** (2012) 12926–12934.

[33] A. Hadi−Vencheh and M Aghajani, Failure mode and effects analysis: A fuzzy group MCDM approach. *J. Soft Comput. Appl.* (2013) (2013) 1–14.

[34] M. Ilangkumaran, P. Shanmugam, G. Sakthivel and K. Visagavel, Failure mode and effect analysis using fuzzy analytic hierarchy process. *Inter. J. Prod. Quality Manag.* **14** (2014) 296–313.

[35] H.-Ch. Liu, X.-J. Fan, P. Li and Y.-Z. Chen, Evaluating the risk of failure modes with extended MULTIMOORA method under fuzzy environment. *Eng. Appl. Artificial Intel.* **34** (2014) 168–177.

[36] H.-Ch. Liu, L. Liu and P. Li, Failure mode and effects analysis using intuitionistic fuzzy hybrid weighted euclidean distance operator. *Inter. J. Syst. Sci.* **45** (2014) 2012–2030.

[37] H.-Ch. Liu, J.-X. You and X.-Y. You, Evaluating the risk of healthcare failure modes using interval 2-tuple hybrid weighted distance measure. *Comput. Ind. Eng.* **78** (2014) 249–258.

[38] S. Helvacioglu and E. Ozen, Fuzzy based failure modes and effect analysis for yacht system design. *Ocean Eng.* **79** (2014) 131–141.

[39] R. K. Sharma and P. Sharma, Qualitative and quantitative approaches to analyse reliability of a mechatronic system: a case. *J. Indus. Eng. Inter.* **11** (2015) 253–268.

[40] T.-N. Tsai and J.-H. Yeh, Identification and risk assessment of soldering failure sources using a hybrid failure mode and effect analysis model and a fuzzy inference system. *J. Intell. Fuzzy Syst.* **28** (2015) 2771–2784.

[41] D. Panchal and D. Kumar, Integrated framework for behaviour analysis in a process plant. *J. Loss Prevention Process Industries* **40** (2016) 147–161.

[42] Q. Zhou and V.V. Thai, Fuzzy and grey theories in failure mode and effect analysis for tanker equipment failure prediction. *Safety Sci.* **83** (2016) 74–79.

[43] H.-Ch. Liu, J.-X. You, M.-M. Shan and L.-N. Shao, Failure mode and effects analysis using intuitionistic fuzzy hybrid TOPSIS approach. *Soft Comput.* **19** (2015) 1085–1098.

[44] H.-Ch. Liu, J.-Xin You, X.-F. Ding and Q. Su, Improving risk evaluation in fmea with a hybrid multiple criteria decision making method. *Inter. J. Quality Reliab. Manag.* **32** (2015) 763–782.

[45] F. Lolli, A. Ishizaka, R. Gamberini, Bi. Rimini, A. Maria, Ferrari, S. Marinelli and R. Savazza, Waste treatment: an environmental, economic and social analysis with a new group fuzzy PROMETHEE approach. *Clean Techn. Environ. Policy* **18** (2016) 1317–1332.

[46] H. Safari, Z. Faraji and S. Majidian, Identifying and evaluating enterprise architecture risks using FMEA and fuzzy VIKOR. *J. Intel. Manuf.* **27** (2016) 475–486.

[47] B. Vahdani, M. Salimi and M. Charkhchian, A new fmea method by integrating fuzzy belief structure and TOPSIS to improve risk evaluation process. *Inter. J. Adv. Manuf. Techn.* **77** (2015) 357–368.

[48] H.-Ch. Liu, J.-X. You, Sh. Chen and Y.-Z. Chen, An integrated failure mode and effect analysis approach for accurate risk assessment under uncertainty. *IIE Trans.* **48** (2016) 1027–1042.

[49] H.-Ch. Liu, J.-X. You, P. Li and Q. Su, Failure mode and effect analysis under uncertainty: an integrated multiple criteria decision making approach. *IEEE Trans. Reliab.* **65** (2016) 1380–1392.

[50] H. Zhao, J.-X. You and H.-Ch. Liu, Failure mode and effect analysis using multimoora method with continuous weighted entropy under interval-valued intuitionistic fuzzy environment. *Soft Comput.* (2016) 1–13.

[51] X. Wang, Y. Zhang and G. Shen, An improved FMECA for feed system of CNC machining center based on ICR and DEMATEL method. *Int. J. Adv. Manuf. Techn.* **83** (2016) 43–54.

[52] Yi.-Xi. Xue, J.-X. You, Xiao-Dong Lai and H.-Ch. Liu, An interval-valued intuitionistic fuzzy MABAC approach for material selection with incomplete weight information. *Appl. Soft Comput.* **38** (2016) 703–713.

[53] X. Peng and Y. Yang, Pythagorean fuzzy choquet integral based MABAC method for multiple attribute group decision making. *Int. J. Intell. Syst.* **31** (2016) 989–1020.

[54] Su-min Yu, J. Wang and J.-q. Wang, An interval type-2 fuzzy likelihood-based MABAC approach and its application in selecting hotels on a tourism website. *Int. J. Fuzzy Syst.* (2016) 1–15.

[55] J. Roy, A. Ranjan and A. Debnath, An extended MABAC for multi-attribute decision making using trapezoidal interval type-2 fuzzy numbers. Preprint arXiv:1607.01254 (2016).

[56] J. Roy, K. Chatterjee, A. Bandhopadhyay and S. Kar, Evaluation and selection of medical tourism sites: A rough AHP based MABAC approach. Preprint arXiv:1606.08962 (2016).

[57] D.I. Božanić, D.S. Pamučar and S.M. Karović, Use of the fuzzy AHP-MABAC hybrid model in ranking potential locations for preparing laying-up positions. *Vojnotehnički Glasnik* **64** (2016) 705–729.

[58] A. Ishizaka and N.H. Nguyen, Calibrated fuzzy AHP for current bank account selection. *Expert Syst. Appl.* **40** (2013) 3775–3783.

[59] E.P. Zafiropoulos and E.N. Dialynas, Reliability prediction and failure mode effects and criticality analysis (FMECA) of electronic devices using fuzzy logic. *Int. J. Quality Reliab. Manag.* **22** (2005) 183–200.

[60] H. Arabian-Hoseynabadi, H. Oraee and P.J. Tavner, Failure modes and effects analysis (fmea) for wind turbines. *Inter. J. Electrical Power Energy Syst.* **32** (2010) 817–824.

[61] F.M. Company, Potential failure mode and effects analysis in design (Design FMECA) and for manufacturing and assembly process (Process FMECA) instruction manual (1988).