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A NOVEL ROBUST MULTIVARIATE REGRESSION APPROACH TO OPTIMIZE MULTIPLE SURFACES

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Abstract. Response surface methodology involves relationships between different variables, specifically experimental inputs as controllable factors, and a response or responses by incorporating uncontrollable factors named nuisance. In order to optimize these response surfaces, we should have accurate response models. A common approach to estimate a response surface is the ordinary least squares (OLS) method. Since OLS is very sensitive to outliers, some robust approaches have been discussed in the literature. Most problems face with more than one response which are mostly correlated, that are called multi-response problem. This paper presents a new approach which takes the benefits of robust multivariate regression to cope with the mentioned difficulties. After estimating accurate response surfaces, optimization phase should be applied in order to have proper combination of variables and optimum solutions. Global criterion method of multi-objective optimization has also been used to reach a compromise solution which improves all response variables simultaneously. Finally, the proposed approach is described analytically by a numerical example.

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1. Introduction

In quality engineering techniques, finding out the relations between input parameters and output quality characteristics is very important. Based on this relation, the optimization of the outputs can be done. One common problem in product or process design is to determine optimal level of control variables where there are different outputs. This problem is called multi-response optimization (MRO) problem.

Several studies have presented approaches surveying multiple quality characteristics, but few papers have focused on the existence of correlation. Correlation can also meaningfully affect the analysis of MRO problem. Nuisances in experiments may be classified into the following three categories [18]. First of all, controllable factors which are known and controllable, but their effect is not of interest as a factor. Unknown and uncontrollable variables, that is, the existence of the factor is unknown and it may even be changing levels while the experiments are conducted. Final categories are "known and uncontrollable variables".

Keywords and phrases: Multi-response, simultaneous equation systems, multivariate robust regression, global criterion (GC) method.

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Modeling and optimization of correlated response surfaces have been dedicated by many researchers. Chiao and Hamada [7] considered experiments with correlated multiple responses. Analysis of these experiments consists of modeling distributional parameters in terms of the experimental factors and finding factor settings which maximize the probability of being in a specification region, *i.e.*, all responses are simultaneously meeting their respective specifications. It is assumed that the multi-response set has a multivariate normal distribution and also that each response variables is desired to be within a predefined specification region. Kazemzadeh *et al.* [14] applied multi-objective goal programming model to provide a general framework for multi-response optimization problems. Shah *et al.* [23] used the seemingly unrelated regressions (SUR) method for estimating the regression parameters where there are correlated dependent variables. The method can be useful in MRS problem with correlated responses and leads to a more precise estimate of the optimum variable setting. A useful approach in solving multi-response optimization problem is multivariate regression method that is very useful when response variables are correlated.

A common method of explaining and analyzing the results of experiments is response surface modeling. After gathering experimental data, a relationship between the factors (input data) and the response or responses (output results) should be well-defined to complete the analysis procedure. If a suitable model could be concluded to define the precise relation between the input variables and the response or the responses' consequents, then the interpretations will not be reliable. After determining an experimental design and performing experiments, the next steps include the statistical analysis and the selection of the optimal input variables. One of the most common approaches of regression coefficient estimation is the OLS method. A solution given by OLS determines the coefficient values that minimize the sum of the squares of the residuals.

Huber [12] introduced the concept of robustness in the regression. One common robust estimation approach is the M-estimator that is based on the maximum likelihood estimation (MLE) method. The main idea behind the M-estimators that works iteratively is to replace the sum of squared residuals of the OLS by another function. As a consequence, several authors (*e.g.*, [8]) called this estimator an iteratively reweighted least squares (IRLS) method. This method can be applied to robustly estimate the coefficients of the multivariate regression of this research.

The literature of the M-estimators and response surface problems is rich. Morgenthaler and Schumacher [19] discussed robust response surfaces in chemistry based on design of experiments. Hund *et al.* [13] presented various methods of outlier detection and evaluated their robustness using different experimental designs. Wiens and Wu [25] proposed a comparative study of M-estimators and presented a design that is more optimal compared with possible regression models. Maronna *et al.* [17] explained the most recent robust regression algorithms.

Recently, Bashiri and Moslemi [3] proposed a moving average iterative weighting method (MAIW) to estimate the coefficients of regression models based on M-estimators. The aim of their research was to decrease the effect of faulty points by considering the previous data to detect the outliers or the probable trends in residuals. Furthermore, Bashiri and Moslemi [4, 5] proposed an iterative weighting method to modify both the outliers that follow abnormal trends and the residuals that have non-equal variation, so that they have less effects on the coefficient estimation.

Many robust estimators of multivariate regression models were investigated by several researchers such as Maronna and Morgenthaler [16] who proposed a robust covariance estimator. Koenker and Portnoy [15] proposed multivariate regression method of M-type. Another efficient and useful robust multivariate regression estimator is based on the minimum covariance determinant (MCD) that was developed by Rousseeuw *et al.* [21]. Based on this procedure, a robust multivariate regression approach was developed by Rousseeuw *et al.* [21]. These estimators are categorized as a high breakdown point robust algorithms, whereas M-type of estimator is not categorized as a high breakdown point algorithm. Though the breakdown points of these methods are in high level, the efficiency of M-type is significant against other method. Bashiri and Moslemi [4, 5] proposed iteratively robust regression procedure based in M-estimators.

This paper proposes a novel procedure for robust estimation of the coefficients in regression equations that relate control factors to the response variables in multi-response optimization. More specifically, as OLS and relate method based on maximum likelihood approaches are very sensitive to outliers, to the non-normality of the error terms in the regression equations, as well as to correlated responses, a robust coefficient estimation

TABLE 1. A summary of the relevant works.

Authors	Methods			
	Correlation analysis	Optimization approach	OLS	Robust approach
Su & Tong (1997)	PCA	Factor effects of new components	✓	
Antony [2]	Coefficient as responses	Signal to noise maximization	✓	
Chiao & Hamada [7]	Consider correlation	Joint probability maximization	✓	
Shah *et al.* [23]	SUR method	Desirability function	✓	
Tong *et al.* (2005)	PCA	Variation mode chart of PCA	✓	
Wang [24]	PCA	Variation mode chart of PCA	✓	
Kazemzadeh *et al.* [14]	Considers correlation	Goal programming/desirability function	✓	
Ribeiro *et al.* [20]	PCA	Response surface fitting on first component	✓	
Hejazi *et al.* [11]	Considered	Goal programming	✓	
Salmasnia *et al.* [22]	PCA	Desirability function	✓	
Rousseeuw *et al.* [21]	Not considered	–		✓
Bashiri & Moslemi [4, 5]	Not considered	–		✓
Bashiri & Moslemi [4, 5]	Not considered	–		✓
Proposed paper	Considers correlation	GC method		✓

method is proposed for multi-response surfaces in multi-response optimization problem based on M-estimators. As it is clear by now, accurate estimates of the response surfaces are needed in order to achieve optimum design parameters in multi-response problems. Global criterion (GC) method of vector optimization is also applied since there are several output characteristics to be optimized. This paper proposes a methodology that can analyze correlated multiple response surfaces fitted on control factors.

As the comparative study among the major works mentioned above (shown in Tab. 1) reveals, little attention has been paid to the use of a robust approach and in the analysis of MRSO problems. Consequently, one can conclude that an attention to the robust optimization in response surface methodology and applying the method in multi-response system can be a good contribution that had not been investigated yet.

The rest of this paper is organized as follows. In Section 2, a robust approach for the model construction procedure and also GC optimization method is presented. A numerical example is given in Section 3 to illustrate the application of the proposed methodology. Finally, Section 4 contains the concluding remarks.

2. Materials and methods

When the problems involve several equations with common variables, it is recommended to estimate the parameters through a system of equations simultaneously. Various methods such as ordinary least squares (OLS), cross-equation weighting method, SUR, two-stage least squares (2SLS), weighted two-stage least squares (WTSLS), three-stage least squares (3SLS), full information maximum likelihood (FIML), and the generalized method of moments (GMM) have been proposed to solve such problems.

Besides classical estimations, robust regression methods have been surveyed by authors. Similar to a single response, robust estimation of the regression coefficients in a multi-response problem is an important issue. Besides, due to correlations between multiple responses, treating each response separately and applying robust single response procedures may lead into incorrect interpretations of the results. Thus, considering all responses simultaneously and estimating their variance–covariance matrix seems necessary. Another difference between a single response surface and a multi-response surface is the distance measure involved. In a single response problem, Euclidean distance of residuals is used while in a multi-response problem the Mahalanobis distance that takes into account the correlation between responses is considered. In the proposed RMRS approach, lower weights are assigned to residuals with larger distance measure. In each iteration, the proposed weighting function down-weights the residuals by considering all responses simultaneously (Tab. 2).

TABLE 2. Characteristics of the major methods of system estimation.

Estimation method	Limiting assumptions			
	Normality	[1]	[2]	Outlier modifying
OLS	–	*	*	–
Cross equation weighting	–	*	*	–
SUR [26]	–	–	–	–
2SLS [6]	–	–	*	–
WTSLS	–	–	*	–
3SLS [27]	–	–	–	–
FIML [1]	*	–	–	–
GMM [10]	–	*	–	–
Robust multivariate regression	–	*	*	*

[1] Independency between predictors and error.
[2] Independent error terms.

2.1. Model representation

A general multi-response problem can be expressed as:

$$\text{Min}\, R(x) = \begin{pmatrix} \hat{R}_1(x) \\ \hat{R}_2(x) \\ \vdots \\ \hat{R}_p(x) \end{pmatrix}$$
$$\text{Subject to}: L < x < u, \tag{2.1}$$

where $\hat{R}_i(x)$ represents response surface for ith quality characteristic; x is vector of control factors.

Furthermore, it is assumed that the process is statistically under control.

2.2. Model building

By introducing indices i and j to represent replicates and responses, respectively, we define variable $r_{ij};\ i = 1, 2, \ldots, l, j = 1, 2, \ldots, p$ to be the residual associated with the ith replicate of the jth. The residuals for each response Y_j are first obtained using initial estimates of the responses $\hat{Y}_j$ as $r_{ij} = Y_{ij} - \hat{Y}_{ij}$. Then, the scaled residuals for each response, denoted by sr_{ij}, are obtained by subtracting their values from their sample mean $(\overline{r_j})$ and then dividing the result by their variation measure (sample standard deviation s_{r_j}). In other words,

$$sr_{ij} = \frac{r_{ij} - \overline{r_j}}{s_{r_j}}. \tag{2.2}$$

Since the controllable factors are assumed constant (not random), the correlations between the responses are next estimated using scaled residuals. These estimates are used to obtain the covariance matrix $\hat{\Sigma}$. Note that since the covariance matrix can also be under influence of outliers, it should be robustly estimated using M-estimator. Assuming p responses and denoting $r(i) = [sr_{i1}, sr_{i2}, \ldots, sr_{ip}];\ i = 1, 2, \ldots, l$ the scaled residual matrix of the responses in ith replicate, the Mahalanobis distance is computed and consequently the weighting scheme is obtained based on this distance. The Mahalanobis distance of each estimated response in a replicate is obtained as:

$$d(r(i)) = \sqrt{(r(i))^T \hat{\Sigma}^{-1} r(i)}. \tag{2.3}$$

The distribution of the squared Mahalanobis distance is approximately a chi-square with p degrees of freedom [18]. The critical point of this distribution at α confidence level ($x^2_{p,\alpha}$) is used to assign the weights. In other words, if the squared Mahalanobis distance is less than $x^2_{p,\alpha}$, then the weight assigned takes the value of 1. Otherwise, the weight is obtained proportional to sum of the distances using equation (2.4).

$$w_i = \begin{cases} 1; & \text{if } d\left(r(i)\right) < \chi^2_{p,\alpha} \\ \frac{\chi^2_{p,\alpha}}{\sum_{j=1}^{l} d(r(i))}; & \text{otherwise} \end{cases}. \tag{2.4}$$

In the next section, the performance of the proposed RMRS approach in terms of sum of squared error of estimates (SSE) is investigated using a numerical illustration. The error involved to estimate regression coefficient θ using $\hat{\theta}$ is defined as

$$\text{Error} = (\theta - \hat{\theta}). \tag{2.5}$$

In the numerical example, it is assumed that some outliers are present in the experiments.

2.3. Optimization method (global criterion)

This method allows one to transform a multi-objective optimization problem into a single-objective problem. The function traditionally used in this method is distance. The multi-objective method can be written as follows:

$$\begin{aligned} &\text{Optimize } F\left(x\right) = \begin{array}{c} \left(\sum_i \left|\frac{T_i - \hat{R}_i(x)}{d_i}\right|^r\right)^{\frac{1}{r}} \\ [\text{minimize/maximize}] \end{array} \\ &\text{Subject to: the same constraints,} \end{aligned} \tag{2.6}$$

where T_i is the optimum value of problem objective function when only ith objective was considered, w_i the value representing importance of each objective, and d_i is the range of ith response within the observed experimental runs [9]. In this study, GC method was applied to convert problem into single objective form.

Consecutive steps of the proposed approach are as follows:

Step 1: identify input and output variables.
Step 2: select a proper design and run the experiments.
A proper design is selected for conducting the experiments regarding the number of variables and their levels.
Step 3: develop a system of equations.
3a: perform an initial response surfaces to get an insight about the more effective factors on each response considering the test of significance of regression coefficients.
3b: define an equation for relations between each response and other variables based on OLS method.
Step 4: estimate parameters of the system by proposed robust estimation approach.
Step 5: construct multi-objective optimization model including the following objective functions.
5a: Response surfaces related to quality characteristics.
Step 6: apply global criterion (GC) method to solve the multi-objective optimization model.

In Section 4, these steps are discussed in details.

3. Results and discussion

This section is organized to demonstrate the computational steps of the proposed approach. For this purpose, a numerical example from the literature is considered with some modifications [18].

Step 1: a chemical experiment with three controllable variables and two covariates is designed to be analyzed by the proposed method. The outputs are conversion (Y_1) and activity (Y_2) levels.

Step 2: a CCD design is selected and the experiments are conducted accordingly. Table 3 shows the results of experiments gathered by a central composite design (CCD).

In Table 3, some responses seems to be as contaminations. We illustrate these runs in bold. Figure 1 shows that some data deviate markedly from other observations in the sample.

Three approaches have been applied for coefficients estimation such as pure OLS (in which we have no outliers and contamination), OLS-based methodology such as FIML approach, and finally robust multivariate approach.

Step 3: understanding the strong effects helps us to fit better surfaces of response variables. The results showed that the following terms would be considered to construct the system of equations.

So considering the computations, the most effective variables are as follows:

$$Y_1 \propto \; x_1,\; x_2, x_3, x_1x_3, x_2x_3, x_2^2,\; x_3^2,$$
$$Y_2 \propto \; x_1,\; x_3, c_1, c_2, x_1c_2,\; x_3^2.$$

In this case, the problem is analyzed by proposed robust approach and FIML. The response surfaces regressed by the mentioned methods are given below in Table 4 (Minitab statistical package has been used to estimate the parameters in system).

Table 3. Results of designed experiments for numerical example.

Time (X_1)	Heat (X_2)	Catalyst (X_3)	Humidity (C_1)	Temp. (C_2)	Conversion (R_1)	Activity (R_2)
−1	−1	−1	41%	16.7	74	**53.2**
1	−1	−1	55%	17.3	**51**	62.9
−1	1	−1	67%	19.3	88	**53.4**
1	1	−1	55%	12.3	70	62.6
−1	−1	1	12%	11.5	71	57.3
1	−1	1	95%	18.5	90	67.9
−1	1	1	65%	19.2	66	59.8
1	1	1	96%	16.5	**97**	67.8
0	0	0	30%	13.2	81	59.2
0	0	0	59%	14	75	60.4
0	0	0	46%	16.4	76	59.1
0	0	0	57%	16.4	83	60.6
−1.682	0	0	59%	13.5	76	59.1
1.682	0	0	33%	13.9	79	65.6
0	−1.682	0	48%	15	85	60
0	1.682	0	38%	13.1	**97**	60.7
0	0	−1.682	29%	12.7	**55**	57.4
0	0	1.682	20%	15.8	81	63.2
0	0	0	25%	11.5	80	60.8
0	0	0	75%	19.1	91	58.9

Estimated Regression Coefficients for Conversion (R1)

Term	Coef	SE Coef	P
Constant	79.6003	3.355	0.000
Time (X1)	1.0288	3.534	0.660
Heat (X2)	3.925	3.636	0.016
Catalyst (X3)	6.2042	3.645	0.022
Humidity (C1)	−48.9311	4.589	0.508
Temp (C2)	1.8812	3.316	0.595
Time (X1) *Time (X1)	−5.2099	4.695	0.318
Heat (X2) *Heat (X2)	3.0210	4.871	0.016
Catalyst (X3) *Catalyst (X3)	−5.0190	5.101	0.041
Humidity (C1)*Humidity (C1)	1.309	0.966	0.511
Temp (C2) *Temp (C2)	0.9257	5.242	0.867
Time (X1) *Heat (X2)	8.7783	8.027	0.324
Time (X1) *Catalyst (X3)	11.4810	7.374	0.010
Time (X1) *Temp (C2)	−0.5302	6.538	0.939
Heat (X2) *Catalyst (X3)	−4.0070	7.820	0.028
Heat (X2) *Temp (C2)	−2.6728	6.950	0.716
Catalyst (X3)*Humidity (C1)	-3.952	26.899	0.907
Heat (X2)*Humidity (C1)	19.588	76.793	0.841
Time (X1)*Humidity (C1)	−6.698	31.223	0.279
Catalyst (X3) *Temp (C2)	−2.6715	5.605	0.654

S = 6.243   R-Sq = 93.0%   R-Sq(adj) = 73.3%

The analysis was done using coded units.

Estimated Regression Coefficients for Activity (R2)

Term	Coef	SE Coef	P
Constant	23.3310	0.7242	0.000
Time (X1)	0.8892	0.7630	0.050
Heat (X2)	1.1442	0.7850	0.205
Catalyst (X3)	2.1748	0.7869	0.012
Humidity (C1)	2.5911	4.589	0.008
Temp (C2)	10.8894	3.316	0.034
Time (X1) *Time (X1)	−5.2099	4.695	0.318
Heat (X2) *Heat (X2)	3.0210	4.871	0.016
Catalyst (X3) *Catalyst (X3)	1.2874	4.201	0.041
Humidity (C1) *Humidity (C1)	1.3415	0.966	0.511
Temp (C2) *Temp (C2)	0.9257	5.242	0.867
Time (X1) *Heat (X2)	8.7783	8.027	0.324
Time (X1) *Catalyst (X3)	11.4810	7.374	0.410
Time (X1) *Temp (C2)	−5.8114	6.538	0.039
Heat (X2) *Catalyst (X3)	−4.407	7.820	0.728
Heat (X2) *Temp (C2)	−18.7728	6.950	0.716
Catalyst (X3) *Humidity (C1)	−7.952	26.899	0.907
Heat (X2) *Humidity (C1)	21.588	76.793	0.841
Time (X1) *Humidity (C1)	78.698	31.223	0.279
Catalyst (X3) *Temp (C2)	−2.6715	5.605	0.654

S = 1.348   R-Sq = 96.8%   R-Sq(adj) = 87.9%

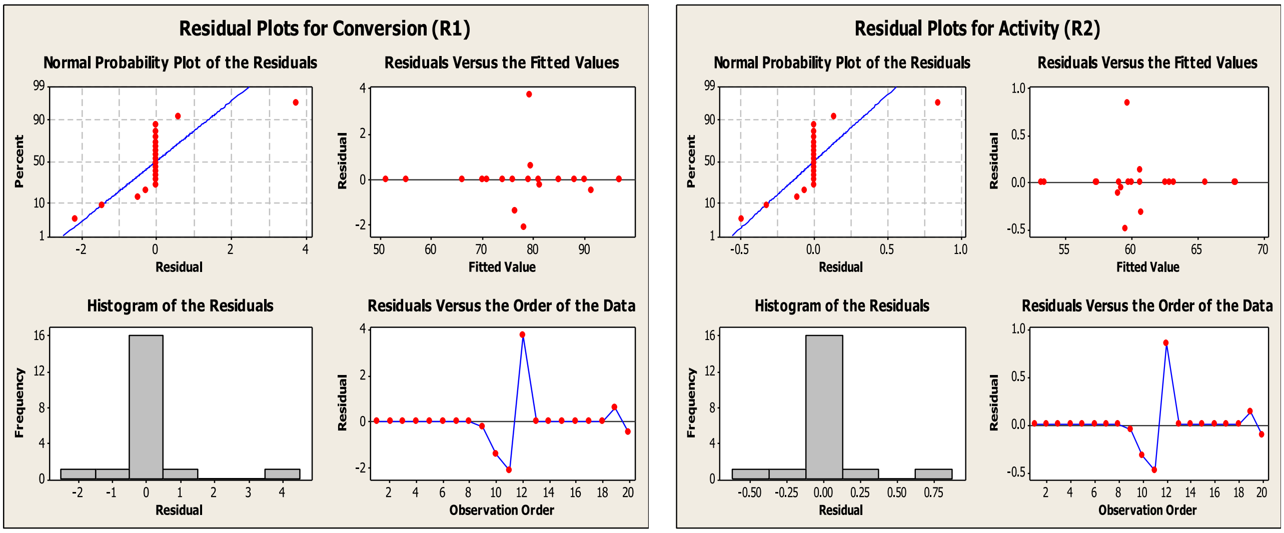

FIGURE 1. The normal probability plot and residual behavior considering contaminated data.

TABLE 4. Estimated equations in the system using FIML and robust multivariate method.

Method	Estimated responses
Robust approach	$R_1\,(X,C) = 78.9 + 1.04x_1 + 3.62x_2 + 6.04x_3 + 11.01x_1x_3 - 4.13x_2x_3 + 3.61x_2^2 - 4.81x_3^2$ $R_2\,(X,C) = 43.23 + 0.909x_1 + 2.57x_3 + 1.125c_1 + 10.059c_2 - 5.921x_1c_2 + 0.89x_3^2$
FIML	$R_1\,(X,C) = 79.6 + 1.028x_1 + 3.925x_2 + 6.204x_3 + 11.481x_1x_3 - 4.007x_2x_3 + 3.021x_2^2 - 5.01x_3^2$ $R_2\,(X,C) = 23.33 + 0.889x_1 + 2.17x_3 + 2.595c_1 + 10.859c_2 - 5.811x_1c_2 + 1.287x_3^2$

TABLE 5. Total squared errors of the estimated coefficients for the response surfaces.

	Y_1		Y_2	
	Robust multivariate	FIML	Robust multivariate	FIML
Total SE	0.89	3.88	30.365	211.8192

In order to find the efficiency of proposed robust approach, we can estimate the response surfaces by pure OLS approach. Considering SE (sum of square) of estimation errors criteria, the efficiency of the proposed approach can be presented. The model is presented in equation (3.1).

$$R_1(X,C) = 78.1 + 1.01x_1 + 3.58x_2 + 6.02x_3 + 10.99x_1x_3 - 4.27x_2x_3 + 3.981x_2^2 - 4.51x_3^2,$$
$$R_2(X,C) = 28.73 + 1.129x_1 + 2.69x_3 + 1.89c_1 + 10.99c_2 - 6.121x_1c_2 + 0.77x_3^2. \qquad (3.1)$$

The comparison between classical OLS-based model and also proposed robust multivariate regression methods based on pure model is presented in Table 5.

The results in Table 5 show that in comparison with the FIML the robust multivariate regression procedure has the smallest SE to estimate the coefficients of all responses.

Step 4: construct the multi-objective optimization model. Two response surfaces and two probability functions are to be considered as objective functions with respect to input variables constrained by their specification

TABLE 6. Trade off matrix and required parameters of GC method.

	Methods of estimation	Z_1	Z_2	R_1	R_2
Target	FIML	0	0	100	78.796
	Robust	0	0	100	71.653
Best observed		0.106	0.003	97	67.9
Worst observed		1.541	2.272	51	53.2
Range		1.435	2.269	46	14.7

TABLE 7. Optimal results of the numerical example.

Method	X	C	R_1	R_2	GC
FIML	$\begin{pmatrix} 1.224 \\ 0.464 \\ 1.38 \end{pmatrix}$	$\begin{pmatrix} 0.501 \\ 14.996 \end{pmatrix}$	100	78	0.0522
Robust	$\begin{pmatrix} 1.209 \\ 0.418 \\ 1.68 \end{pmatrix}$	$\begin{pmatrix} 0.501 \\ 15.422 \end{pmatrix}$	100	70.037	0.0011

limits. Therefore, the multi-objective mathematical program for this problem is developed in which the decision variables consist of three factors and two interdependent covariates. Table 6 gives a summary of optimal solutions obtained by solving the above model for each objective functions separately.

$$\begin{aligned} &\text{Max } F = \begin{pmatrix} R_1(X,C) \\ R_2(X,C) \end{pmatrix} \\ &\text{Subject to: } \begin{pmatrix} -1.68 \\ -1.68 \\ -1.68 \end{pmatrix} \le \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \le \begin{pmatrix} 1.68 \\ 1.68 \\ 1.68 \end{pmatrix}. \end{aligned} \tag{3.2}$$

Considering Table 6, the final multi-objective mathematical model using global criterion can be constructed by replacing the objective functions of the above multi-objective program as equation (3.2). In order to finalize the optimizing approach, considering equation (3.3), GC method's main objective function would be applied in this example.

$$\text{Min GC} = \left(\left(\frac{R_1\,(X,C) - 100}{46} \right)^2 + \left(\frac{R_2\,(X,C) - 100}{14.7} \right)^2 \right)^{1/2}. \tag{3.3}$$

In this example, we consider the same important degrees for all objective functions. Table 7 shows the optimal solution and the related objective values for this example.

As it is clear in Table 7, proposed robust method would perform better in optimizing the responses considering GC criteria. Multiple response surfaces have been analyzed by the methodology. Several objective functions and performance indices of a quality engineering problem can be optimized simultaneously by using GC method, the desired direction for optimization of responses does not change after modeling and optimization.

4. Conclusion

This study proposes a new multivariate robust approach on multi-response optimization in which multivariate robust regression method is used to predict the correlated responses. Current study tries to model the multi-response problem in a simultaneous system of equations and uses the estimated equations to construct an optimization program.

We showed that outliers affect the parameter estimates of the regression coefficients and that the usual assumptions involved in a regression model are violated when the OLS method is used for estimation. However, when the proposed methodology is used, the coefficients are estimated robustly, *i.e.*, they are not affected by the outliers and that the usual assumptions are not violated. Based on the M-estimator, the proposed approach iteratively weighs the residuals by considering the correlations between the responses measured using Mahalanobis distances. In order to optimize robust response surfaces GC as a simple multi-objective approach had been applied.

This work can be extended to be applied for tolerance design. Furthermore, the use of other robust approaches such as MM-estimates in order to estimate regression coefficients can be considered in future studies. Also for further studies, the mixed set of categorical and numerical responses is suggested. In this work, only the variances of observed values were considered. Therefore, the variances of predicted responses can be another future research on this subject.

References

[1] T. Amemiya, The maximum likelihood and the nonlinear three-stage least squares estimator in the general nonlinear simultaneous equation model. *Econometrica* **45** (1977) 955–968.

[2] J. Antony, Multi-response optimization in industrial experiments using Taguchi's quality loss function and principal component analysis. *Qual. Reliab. Eng. Int.* **16** (2000) 3–8.

[3] M. Bashiri and A. Moslemi, A robust moving average iterative weighting method to analyze the effect of outliers on the response surface design. *Int. J. Ind. Eng. Comput.* **2** (2011) 851–862.

[4] M. Bashiri and A. Moslemi, Simultaneous robust estimation of multi-response surfaces in the presence of outliers. *J. Ind. Eng. Int.* **9** (2013) 1–12.

[5] M. Bashiri and A. Moslemi, The analysis of residuals variation and outliers to obtain robust response surface. *J. Ind. Eng. Int.* **9** (2013) 1–10.

[6] R.L. Basmann, A generalized classical method of linear estimation of coefficients in a structural equation. *Econometrica* **25** (1957) 77–83.

[7] C.H. Chiao and M. Hamada, Analyzing experiments with correlated multiple responses. *J. Q. Technol.* **33** (2001) 451.

[8] D.J. Cummins and C.W. Andrews, Iteratively reweighted partial least squares: a performance analysis by Monte Carlo simulation. *J. Chem.* **9** (1995) 489–507.

[9] Y. Donoso and R. Fabregat, Multi-Objective Optimization in Computer Networks Using Metaheuristics. Auerbach Publications, Boca Raton (2007).

[10] L.P. Hansen, Large sample properties of generalized method of moment's estimators. *Econometrica* **50** (1982) 1029–1054.

[11] T.H. Hejazi, M. Bashiri, J.A. Díaz-García and K. Noghondarian, Optimization of probabilistic multiple response surfaces. *Appl. Math. Model.* **36** (2012) 1275–1285.

[12] P.J. Huber, Robust Statistics. John Wiley & Sons, New York (1981).

[13] E. Hund, D.L. Massart and J. Smeyers-Verbeke, Robust regression and outlier detection in the evaluation of robustness tests with different experimental designs. *Anal. Chim. Acta* **463** (2002) 53–73.

[14] R.B. Kazemzadeh, M. Bashiri, A.C. Atkinson and R. Noorossana, A general framework for multiresponse optimization problems based on goal programming. *Eur. J. Oper. Res.* **189** (2008) 421–429.

[15] R. Koenker and S. Portnoy, L-estimation for linear models. *J. Am. Stat. Assoc.* **82** (1987) 851–857.

[16] R. Maronna and S. Morgenthaler, Robust regression through robust covariances. *Commun. Stat. Theory Methods* **15** (1986) 1347–1365.

[17] R.A.R.D. Maronna, D. Martin and V. Yohai, Robust Statistics. John Wiley & Sons, Chichester (2006) 978.

[18] D.C. Montgomery, Design and Analysis of Experiments, 6th edn. John Wiley & Sons, Hoboken (2005).

[19] S. Morgenthaler and M.M. Schumacher, Robust analysis of a response surface design. *Chem. Intel. Lab. Syst.* **47** (1999) 127–141.

[20] J.S. Ribeiro, R.F. Teófilo, F. Augusto and M.M.C. Ferreira, Simultaneous optimization of the microextraction of coffee volatiles using response surface methodology and principal component analysis. *Chem. Intel. Lab. Syst.* **102** (2010) 45–52.

[21] P. Rousseeuw, S. Van Aelst, K. Van Driessen and J. Agullo, Robust multivariate regression. *Technometrics* **46** (2004) 293–305.

[22] A. Salmasnia, R.B. Kazemzadeh, M. Seyyed-Esfahani and T.H. Hejazi, Multiple response surface optimization with correlated data. *Int. J. Adv. Manuf. Technol.* **64** (2013) 841–855.

[23] H.K. Shah, D.C. Montgomery and W.M. Carlyle, Response surface modeling and optimization in multiresponse experiments using seemingly unrelated regressions. *Qual. Eng.* **16** (2004) 387–397.

[24] C.H. Wang, Dynamic multi-response optimization using principal component analysis and multiple criteria evaluation of the grey relation model. *Int. J. Adv. Manuf. Technol.* **32** (2007) 617–624.

[25] D.P. Wiens and E.K. Wu, A comparative study of robust designs for M-estimated regression models. *Comput. Stat. Data Anal.* **54** (2010) 1683–1695.

[26] A. Zellner, An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias. *J. Am. Stat. Assoc.* **57** (1962) 348–368.

[27] A. Zellner and H. Theil, Three-stage least squares: simultaneous estimation of simultaneous equations. *Econometrica* **30** (1962) 54–78.

[28] C.T. Su and L.I. Tong, Multi-response robust design by principal component analysis. *Total Quality Manage.* **8** (199) 409–416.

[29] L.I. Tong, W. Chung-Ho and C. Hung-Cheng, Optimization of multiple responses using principal component analysis and technique for order preference by similarity to ideal solution. *Int. J. Adv. Manuf. Technol.* **27** (2005) 407–414.