

**A THEORETICAL MODEL FOR TESTING NEW  
PRODUCT SALES VELOCITY AT SMALL FORMAT  
RETAIL STORES**

HIROAKI SANDOH<sup>1</sup> AND ROY LARKE<sup>2</sup>

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**Abstract.** The present study proposes a theoretical model to test sales velocity for new products introduced in small format retail stores. The model is designed to distinguish fast moving products within a relatively short period. Under the proposed model, the sales of a newly introduced product are monitored for a prespecified period  $T$ , *e.g.*, one week, and if the number of items sold over  $T$  is equal to a prespecified integer  $k$  or more, the product is considered a fast moving product and is carried over to the following sales periods. A slow moving product could be quickly replaced with alternative merchandise in order to make best use of shelf space. The paper first presents definitions of fast and slow moving products, and then a proposed sales test policy based on the model is formulated, where the expected loss is to be minimized with respect to the integer  $k$ . Numerical examples based on actual data collected from a convenience store in Japan are also presented to illustrate the theoretical underpinnings of the proposed sales test model.

**Keywords:** Sales test, fast moving product, slow moving product, expected loss.

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<sup>1</sup> Department of Information & Management Science, University of Marketing & Distribution Sciences, Nishi, Kobe 651-2188, Japan; e-mail: [sandoh@umds.ac.jp](mailto:sandoh@umds.ac.jp)

<sup>2</sup> Department of Marketing & Distribution Sciences, University of Marketing & Distribution Sciences, Nishi, Kobe 651-2188, Japan; e-mail: [roy12@attglobal.net](mailto:roy12@attglobal.net)

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## 1. INTRODUCTION

In modern retailing, there is an on-going trend to ever increasing centralization of decisions, effectively taking the decision making away from the retail store. Such a method has proved very successful in larger format chains such as supermarkets and general merchandise stores [9, 10]. In Japan, the prestige of being a store manager, even within a large chain, comes with markedly greater responsibility than is now common in many other countries. There are even cases of store managers being involved in store building design and having a relatively free reign on aspects of layout and product display. Such methods are slowly disappearing at larger chains, however, as companies seek the efficiency of centralized decision making.

Where Japan differs, however, is in the significant number of small retailers. Since the 1930s, it has been an active government policy to attempt to protect smaller, independent retailers [7]. One result of this has been the long-term regulation on the opening of new large stores, and even the largest retail companies have had little option but to turn to smaller formats. In the 1970s, Japan imported the concept of convenience stores from the United States, and through the introduction of best practices in logistics, merchandising and information systems, built these chains into some of the most successful retailers in the world [7]. The largest retailer overall and the largest food retailer in Japan is now Seven-Eleven Japan [11].

There have been many studies on the product selection problem from the point of view of large format, centralized buying [1, 4–6, 8, 17–19, 21, 22]. The models proposed in these studies attempt to optimize nonlinear objective functions expressing, for example total profit, on the conditions that the values of parameters involved in the models are, *a priori*, known or are to be estimated based on a large volume of data associated with the parameter estimation. Such models have less relevance in the case of small, independently operated outlets because of much smaller merchandise volumes per store, and the need to adjust merchandise selection for a very limited commercial area around each store. The average convenience store has a commercial area with the radius of as little as 500 meters from the store. The types of merchandise and the specific product mix is, therefore, highly variable.

The largest convenience store chains in Japan are major buying concerns. The largest chains, which include Seven-Eleven and Lawson, gather a pool of merchandise numbering up to 4500 items. Each individual store in the thousands of stores in the chain will then select an appropriate 2000 or so items for their own store and needs. The majority of merchandise will be fairly uniform between stores, but between 20 and 30 per cent will vary from store to store. Even within a kilometer or so, two stores from the same chain may carry very different selections. A neighborhood convenience store would carry more top-up food items, more cosmetics and toiletries and so on than would a similar store located on a busy road nearby. The latter store would include more alcohol, more magazines and entertainment related merchandise.

Japanese convenience stores were created by Japan's biggest retailers as fast growth alternatives to larger formats. As such, these chains have been built with minute attention to detail. Japanese distribution has a history of being criticized for being archaic and anti-competitive (for example [2,16,20]), but in building convenience store chains, the largest retailers by-passed almost all aspects of Japan's traditional system to create state-of-the-art retail management systems. In the early 1980s, for example, a single 100 square meter convenience store would receive up to 70 deliveries of product a day from numerous suppliers. Today, a store will receive from three to five deliveries a day, but all from a centralized, cross-docking logistics center. Three to five deliveries a day may still seem a lot, but the reasons have now changed. Frequent deliveries no longer occur due to the number of different suppliers, but because product turnover within the store is so rapid. A typical convenience store will expect certain product lines to sellout three times a day, so deliveries are made to correspond to this very short sale time period. The systems used by convenience store chains are the epitome of just-in-time replenishment.

Convenience stores are exclusively FMCG (fast-moving consumer goods) retailers, and they clearly live up to their format. Not only is product turnaround so rapid, however, as an FMCG retailer, convenience stores face the added problem of keeping their merchandise assortments as current as possible. Most chains will consider as many as 10 000 new product introductions a year from FMCG manufacturers. The majority of these will not last a year, but the need to stock a significant proportion of these items in order to maintain current merchandise lists is vital as a marketing component.

In addition, Japan's largest FMCG manufacturers are well aware of convenience stores as the make-or-break option for some of their new products. The nationwide coverage of the largest chains means that new product introductions which find their way onto these shelves stand a far higher chance of success than those that do not.

In summary, Japanese convenience stores face the problems of all FMCG retailers, with the added complication that store level merchandise assortments need to be adjusted for a limited geographical area. The majority of convenience stores are also franchises, so individual store owners have greater responsibility to monitor and alter merchandise assortment within the store. Clearly this is a problem that is in a way simpler than the case for large supermarkets carrying 10 000 or more items, but more difficult in that convenience store shelf space needs to be optimized far quicker and on a far more fragmented scale than for supermarkets.

The model presented in this paper addresses this problem. From the myriad of new FMCG products which find their way onto the shelves of franchised convenience stores, how does a single store owner quickly and accurately access the suitability of each new product? To a certain extent, the store owner's knowledge of his local clientele and competitive environment is important, but, in most categories, he needs products that will sell very quickly indeed. How should he optimize this?

The model presented here aims to support decision making associated with product selection in a small format, FMCG retailer operating within a restricted geographical area. Under the proposed strategy, a sales test for a newly introduced FMCG product such as a new soft drink or potato chips product is conducted over periods  $((j-1)T, jT]$  ( $j = 1, 2, \dots$ ), where  $T$  is one or two weeks. The model aims to identify whether a particular product is fast moving or not. For simplicity, it is assumed that one facing is used for the objective product and that  $m$  ( $m = 1, 2, \dots$ ) items are arranged in the facing for total stock on the shelf. It is also assumed that when all the  $m$  items are sold out during the sales test, the facing is replenished with other  $m$  items. Given the rapid turnaround of merchandise within convenience stores and their rapid replenishment systems, it is a safe assumption that the facing would almost never be empty.

Under the proposed sales test strategy, if the number  $N(T)$  of items sold over  $j$ -th period,  $((j-1)T, jT]$  becomes equal to an non-negative integer  $k$  ( $k = 0, 1, 2, \dots$ ) or more, the sales test is continued over the next period  $(jT, (j+1)T]$ . Alternatively, if  $N(T)$  becomes less than  $k$ , the sales test is terminated at the end of the  $j$ -th period, and so releases the shelf space for use with other merchandise.

The definitions for a fast and a slow moving product are presented below, along with that of a standard product. The expected loss due to a miscalculation of the product's sales velocity under the proposed policy is then formulated, which is to be minimized. It is shown that there always exists a finite optimal integer  $k^*$  which minimizes the expected loss. Numerical examples based on the actual data collected from a single convenience store are also presented to illustrate the theoretical underpinnings of the proposed sales test policy formulation.

## 2. ASSUMPTIONS AND DEFINITIONS

It is assumed throughout this paper that demand for the newly introduced product occurs following a Poisson process. This is because using a Poisson process for the demand distribution of frequently purchased consumer goods has a long validated history [3,15]. Hence, let  $N(t)$ , denote the number of items of the product demanded on  $(0, t]$ , and we have

$$\Pr[N(t) = i] \equiv p_i[\lambda(\mathbf{x})] = \frac{[\lambda(\mathbf{x})t]^i}{i!} e^{-\lambda(\mathbf{x})t}, \quad i = 0, 1, 2, \dots, \quad (1)$$

where  $\mathbf{x} = (x_1, x_2, \dots, x_n)$  is a vector of marketing variables. In equation (1),  $\lambda(\mathbf{x})$  expresses the mean number of items demanded per unit of time when  $\mathbf{x}$  is fixed, and thus we call it a *demand rate*. In the following, we consider a specific set of the marketing variables  $\mathbf{x} = \mathbf{x}_0$ , and  $\lambda(\mathbf{x}_0)$  is written as  $\lambda$  for simplicity.

Under the above assumption, let  $\alpha$  and  $\beta$  respectively denote gross profit per item and the facing occupation cost per unit of time (see Eq. (4) below), where a gross profit signifies the profit obtained by subtracting all the cost associated with purchasing, transportation, except facing occupation from its selling price.

The definitions of a fast moving and a slow moving product are given below along with that of a standard product:

**Definition 1** (Classification of products). If the demand rate  $\lambda$  for a product satisfies

$$\alpha\lambda - \beta > (<)0 \quad (2)$$

such a product is called a *fast (slow) moving product*.

It is convenient to introduce the concept of a *standard product*, with a demand rate of  $\lambda = \lambda_0$  where

$$\alpha\lambda_0 - \beta = 0. \quad (3)$$

From equation (3), the facing occupation cost,  $\beta$  can be expressed, in terms of  $\alpha$  and  $\lambda_0$ , as

$$\beta = \alpha\lambda_0. \quad (4)$$

Hence, the subtraction of the facing occupation cost from the gross profit of a newly introduced product is equivalent to comparing the newly introduced product with a standard product in terms of the profit acquired from each.

Notations  $\Lambda_1$  and  $\Lambda_2$  shown below will be used in the following:

$$\Lambda_1 = \left\{ \lambda \left| \alpha - \frac{\beta}{\lambda} > 0 \right. \right\}, \quad (5)$$

$$\Lambda_2 = \left\{ \lambda \left| \alpha - \frac{\beta}{\lambda} < 0 \right. \right\}. \quad (6)$$

### 3. MODEL

In this study, we consider consumer nondurables, specifically FMCG merchandise such as grocery products, within a single merchandise category. This category will be allocated one product facing within the store's shelf space. Some of the product will be on display within this single facing at all times, with  $m(m = 1, 2, \dots)$  items in the facing itself, and that the product will be immediately replenished should the facing become empty. In effect, we are assuming that there is always product in the facing in question.

Under such a situation, a sales test policy is conducted over periods  $(0, T]$ ,  $(T, 2T]$ ,  $(2T, 3T]$ ,  $\dots$  using one facing, where  $T$  is predetermined as relatively a short time period of only one or two weeks. When the number of items demanded over  $((j-1)T, jT]$  becomes equal to an integer  $k(k = 0, 1, 2, \dots)$  or more, we regard the product as a fast moving product, otherwise it is regarded as a slow moving product. Being considered a fast moving product, the item will stay on the shelves for the next period. Products that are deemed by the model to be slow moving products will have their test sales period terminated at the end of the  $j$ -th

period to release the facing to another candidate product in the overall product list. The remaining products unsold by the time the sales test is terminated are assumed to be disposed of in some way. In reality, this is a feasible assumption as retailers will return the products to suppliers for refund, or, cases of a small number of products, simply dispose of them. If all the  $m$  items are sold out during the sales test period, the facing will be replenished with items of the same product as explained above.

Figure 1 illustrates the proposed sales test for three periods. In the first two periods  $(0, T]$  and  $(T, 2T]$  the number of items sold exceeded  $k$  and the sales test was continued for subsequent periods, but in the third period  $(2T, 3T]$  the number of items sold drops to below  $k$  and so would be considered for replacement.

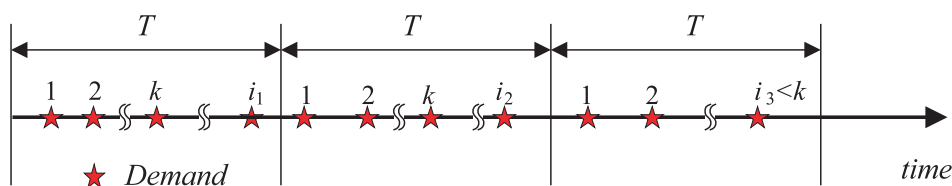


FIGURE 1. Sales test policy.

#### 4. EXPECTED LOSS

Under the proposed policy, there exists the possibility of two types of misjudgment or decision errors: (1) the misjudgment of regarding an actual fast moving product as a slow moving product where  $N(T)$  incidentally becomes less than  $k$ , and (2) the misjudgment of regarding an actual slow moving product as a fast one due to an event of  $N(T) \geq k$ . In the following, these misjudgments are called Type I and II Errors, respectively. It does not seem that the Type II Error would be a critical mistake since we have  $N(T) \geq k$  in the corresponding period, but it is not negligible because the model would suggest continuing the sales test over the next period. This would compound potential inefficiencies and therefore losses due to shelving a less than optimum merchandise selection.

The probability of making Type I and II Errors in each period is given by

$$\Pr[J_2 | \lambda = \lambda_1 \in \Lambda_1] = \sum_{i=0}^{k-1} p_i(\lambda_1), \quad (7)$$

$$\Pr[J_1 | \lambda = \lambda_2 \in \Lambda_2] = \sum_{i=k}^{\infty} p_i(\lambda_2), \quad (8)$$

$$\lambda_1 > \lambda_2$$

where  $J_1$  and  $J_2$  express to regard the objective product as a fast and a slow moving respectively, and  $\sum_{i=0}^{-1} \cdot = 0$ .

When  $\lambda = \lambda_l \in \Lambda_l$ , and if the demand rate is constant with respect to time, that is, it does not change over considerably long periods, we can derive the expected profit of the proposed policy, from a renewal reward process [13,14], as

$$A_l(k) = \sum_{i=0}^{k-1} (i\alpha - \beta T) p_i(\lambda_l) + \sum_{i=k}^{\infty} [i\alpha - \beta T + A_l(k)] p_i(\lambda_l), \quad (9)$$

$$k = 0, 1, 2, \dots, l = 1, 2.$$

In equation (9), the first term of the right-hand-side expresses the expected profit in case we terminate the sales test since the number  $N(T)$  of items sold is under  $k$ . The second term of the right-hand-side signifies the expected profit when we continue the sales test over the succeeding period due to  $N(T) \geq k$ . This expresses the expected profit until the sales test is terminated since  $N(T)$  falls to less than  $k$ . In an actual retail situation, however, it would be unwise to assume that the demand rate remains constant over a considerably long period, theoretically over an infinite period, and consequently we cannot use  $A_l(k)$  in equation (9).

In this study, we define expected profit by the sum of the expected profit over the corresponding period and over the succeeding one, assuming the demand rate will not change significantly over the subsequent periods. Then the expected profit is given by

$$A_l(k) = \sum_{i=0}^{k-1} (i\alpha - \beta T) p_i(\lambda_l) + \sum_{i=k}^{\infty} [\alpha(i + \lambda_l T) - 2bT] p_i(\lambda_l), \quad (10)$$

$$k = 0, 1, 2, \dots, l = 1, 2.$$

#### 4.1. FAST MOVING PRODUCT

Where the objective product is actually a fast moving product with demand rate  $\lambda_1 \in \Lambda_1$ , we would carry over the sales test to the next period. In such a case, the expected profit is given by

$$B_1(k) = \sum_{i=0}^{\infty} [\alpha(i + \lambda_1 T) - 2bT] p_i(\lambda_1), \quad k = 0, 1, 2, \dots, \quad (11)$$

where  $B_1(k)$  expresses the expected profit when we continue the sales test over the next period independently of the number  $i$  of sold items.

Hence, the loss incurred by the proposed policy when the product is a fast moving product can be given by

$$C_1(k) = B_1(k) - A_1(k)$$

$$= (\alpha\lambda_1 - \beta) T \sum_{i=0}^{k-1} p_i(\lambda_1), \quad k = 0, 1, 2, \dots \quad (12)$$

## 4.2. SLOW MOVING PRODUCT

If the objective product is actually a slow moving product with demand rate  $\lambda_2 \in \Lambda_2$ , we should terminate the sales test independently of the sales test results. The expected profit in such a case is given by

$$B_2(k) = \sum_{i=0}^{\infty} (i\alpha - \beta T) p_i(\lambda_2), \quad k = 0, 1, 2, \dots \quad (13)$$

Hence, the loss incurred if the product is a slow moving product can be expressed by

$$\begin{aligned} C_2(k) &\equiv B_2(k) - A_2(k) \\ &= -(\alpha\lambda_2 - \beta) T \sum_{i=k}^{\infty} p_i(\lambda_2), \quad k = 0, 1, 2, \dots \end{aligned} \quad (14)$$

## 4.3. EXPECTED LOSS

Let  $q_1 (0 < q_1 < 1)$  and  $q_2 (= 1 - q_1)$  denote respectively the prior probabilities that the objective product is a fast or a slow moving product. In this case the expected loss becomes

$$\begin{aligned} C_0(k) &\equiv C_1(k)q_1 + C_2(k)q_2 \\ &= q_1 (\alpha\lambda_1 - \beta) T \sum_{i=0}^{k-1} p_i(\lambda_1) - q_2 (\alpha\lambda_2 - \beta) T \sum_{i=k}^{\infty} p_i(\lambda_2), \quad (15) \\ &k = 0, 1, 2, \dots \end{aligned}$$

In the above, we have formulated the expected loss of the proposed policy, so if  $k = k^*$  minimizes  $C_0(k)$  in equation (15), it is optimum.

## 5. OPTIMAL POLICY

Let  $\Delta C_0(k) = C_0(k+1) - C_0(k)$ , then we have

$$\begin{aligned} \Delta C_0(k) &= q_1 (\alpha\lambda_1 - \beta) T p_k(\lambda_1) + q_2 (\alpha\lambda_2 - \beta) T p_k(\lambda_2), \quad (16) \\ &k = 0, 1, 2, \dots \end{aligned}$$

Since we have  $\alpha\lambda_1 - \beta > 0$ ,  $\Delta C_0(k) \geq 0$  which agrees with

$$\left( \frac{\lambda_1}{\lambda_2} \right)^k \geq - \frac{q_2 (\alpha\lambda_2 - \beta)}{q_1 (\alpha\lambda_1 - \beta)} e^{(\lambda_1 - \lambda_2)T}, \quad k = 0, 1, 2, \dots \quad (17)$$



Let us denote the left-hand-side and the right-hand-side of inequality (17) by  $L(k)$  and  $c$ , respectively, then we notice that  $L(k)$  is strictly increasing in  $k$  due to  $\lambda_1 > \lambda_2$  along with  $L(0) = 1$ ,  $\lim_{k \rightarrow +\infty} L(k) = +\infty$  and  $c > 0$ .

From the above observations, the optimal sales test policy can be shown as follows:

- (1) where  $c \leq L(0) = 1$ , we have  $\Delta C_0(k) \geq 0$ . It follows that  $C_0(k)$  increases with  $k$  and consequently  $k^* = 0$ . This recommends that the sales test should continue over the next period independently of the sales test result over the corresponding period;
- (2) if  $c > 1$ , the sign of  $\Delta C_0(k)$  changes from negative to positive only once. Hence, there exists an optimal positive finite integer  $k^* (> 0)$  which minimizes  $C_0(k)$ .

## 6. NUMERICAL EXAMPLES

This section presents numerical examples using sales data for bags of potato chips from a single convenience store located on the campus of the authors' university. The store is a member of *Lawson*, one of Japan's largest convenience store chains, but, as it is used for education and research purposes, sales are not completely representative of all stores in the chain. Specifically, the store is prone to heavier than usual sales during lunch periods with lighter than usual sales at other times, and suffers from significant falls in sales out of school terms. Nevertheless, the store provides appropriate and actual data for a test of the model. Being reliant on college students for much of its sales, the store reflects the importance of geographical location and the adjustments in merchandise necessary at convenience stores.

### 6.1. PARAMETER VALUES

The proposed model includes parameters,  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_0$ ,  $\alpha$ ,  $\beta$ ,  $T$ . Among these, the value of  $\alpha$  is easily determined by obtaining the mean gross profit per SKU (Stock Keeping Unit) for each of the brands in the category under study. The value of  $T$  can also be determined easily by considering its physical tractability and this is set to one week. The value of  $\beta$  can be calculated by equation (4) if the values of  $\alpha$  and  $\lambda_0$  are predetermined.

There are intuitively several possible methods for determining the values of  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_0$ . The retailer may set these values empirically or estimate the values in order to meet some specific sales or merchandising objective. In this subsection, we illustrate an intuitive method for determining the values of  $\lambda_1$ ,  $\lambda_0$  and  $\lambda_2$  based on the actual data.

Table 1 provides the actual sales data for 29 brands of potato chips, each of which occupied only a single facing. The data were collected for a ten week period between May 3, 1999 and July 5, 1999 from the single convenience store mentioned previously. In Table 1, products introduced partly through the collection period

are indicated by “\*” in the weeks prior to their introduction. Equally, brands which were removed by the store manager during the period under study are shown by “-” during the weeks when they were no longer stocked because of removal. It would be possible to omit these brands, but this would mean the data were not realistic. For this reason, we make the assumption that a week where the number of items in stock was zero due to removal (indicated by “-” in the table) is equal to a week whereby no sales were achieved for a brand, even though it was in stock. Finally, in the weeks indicated by “??”, the corresponding brands were unpredictably out of stock because of shortage, and we omit the weeks indicated by “??” in computing mean sales of the corresponding brands.

TABLE 1. Sales data.

Brand	1st <sup>week</sup>	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	Mean	Cumulative share
<i>A</i>	52	84	76	91	100	77	64	??	??	43	78.38	0.221
<i>B</i>	18	32	34	31	29	38	37	38	27	30	31.40	0.316
<i>C</i>	*	42	38	36	18	23	11	??	??	11	22.38	0.383
<i>D</i>	23	23	20	44	33	55	22	-	-	-	22.00	0.449
<i>E</i>	20	21	26	17	23	20	17	??	??	6	18.75	0.506
<i>F</i>	15	30	19	15	16	12	8	0	9	14	13.80	0.547
<i>G</i>	*	*	*	*	*	*	*	*	*	13	13.00	0.587
<i>H</i>	*	*	*	*	14	23	10	8	8	8	11.83	0.622
<i>I</i>	*	*	*	*	*	*	*	17	12	6	11.67	0.657
<i>J</i>	18	15	23	12	12	14	12	2	0	4	11.20	0.691
<i>K</i>	10	13	11	8	7	14	13	14	12	9	11.10	0.725
<i>L</i>	*	*	*	*	17	12	12	7	4	3	9.17	0.752
<i>M</i>	*	*	*	*	*	*	*	*	*	9	9.00	0.779
<i>N</i>	10	9	12	10	7	14	11	1	0	13	8.70	0.806
<i>O</i>	6	13	10	3	14	15	7	7	-	-	7.50	0.828
<i>P</i>	*	*	6	10	4	7	5	5	13	8	7.25	0.850
<i>Q</i>	7	6	4	4	5	6	9	13	6	9	6.90	0.871
<i>R</i>	*	*	13	11	7	3	9	8	5	9	6.50	0.890
<i>S</i>	15	8	8	5	4	11	3	5	3	-	6.20	0.909
<i>T</i>	*	*	*	8	5	8	4	1	5	5	5.14	0.925
<i>U</i>	*	*	*	6	6	4	3	3	5	3	4.29	0.937
<i>V</i>	6	6	9	5	8	-	-	-	-	-	3.40	0.948
<i>W</i>	12	15	6	-	-	-	-	-	-	-	3.30	0.958
<i>X</i>	*	*	*	*	*	*	3	2	2	5	3.00	0.967
<i>Y</i>	*	*	*	*	*	*	6	4	2	0	3.00	0.976
<i>Z</i>	3	8	16	-	-	-	-	-	-	-	2.70	0.984
<i>AA</i>	1	6	19	-	-	-	-	-	-	-	2.60	0.992
<i>AB</i>	16	6	-	-	-	-	-	-	-	-	2.20	0.998
<i>AC</i>	*	5	-	-	-	-	-	-	-	-	0.56	1.000

In order to determine the relevant parameters necessary for the model, a technique that is currently used by convenience store managers was employed. Brands are classified into three groups: **A**, **B** and **C** according to past sales and the store

manager’s own past experience. The three groups represent the relative level of sales accounted for by each brand. The top seven brands  $\mathcal{A}, \mathcal{B}, \dots, \mathcal{G}$  account for around 65.7 per cent of total sales. These brands are called Group **A**. The succeeding 14 brands  $\mathcal{H}, \mathcal{I}, \dots, \mathcal{W}$  account for a further 30.1 per cent of total sales. Hence, brands  $\mathcal{H}, \mathcal{I}, \dots, \mathcal{W}$  are called Group **B**. The remaining eight brands are Group **C**. This study postulates that brands in Group **A** are predetermined as meeting criteria as fast moving products, those in Group **B** are standard products, with the remainder are slow moving products.

Based on these preliminary assumptions, the demand rate  $\lambda_1$  was set to 11.67, that is the mean number of sold items per week for the fast moving product  $\mathcal{I}$  (see Tab. 1). The demand rate  $\lambda_0$  is set to 7.25, *i.e.* the median number of items sold per week for brands  $\mathcal{J}$  to  $\mathcal{W}$ . The demand rate  $\lambda_2$  is set to 3.0 or the mean number of brand  $\mathcal{X}$  sold per week (*i.e.* a slow moving product).

As noted above, where the number of items in stock fell to zero due to removal, we regarded the number of items sold to be zero. As the store had not deliberately cut the brand at this point, the item is considered to be “receiving the benefit of the doubt” and still available for sale in subsequent periods. This illustrates the importance of a manager’s personal experience as well as extraneous factors such as problems with supply or the unavailability of alternative products. Most of the brands that suffered from this problem belong to Group **C**, the slow moving group mentioned above, and so  $\lambda_2$  might be slightly underestimated as a result.

Table 2 summarizes the results. In Table 2, the gross profit  $\alpha$  is set to the mean gross profit per item of brands  $\mathcal{A}, \mathcal{B}, \dots, \mathcal{AC}$ , and the space occupation cost  $\beta$  per week is computed using equation (4).

TABLE 2. Parameters.

$\lambda_1$	$\lambda_2$	$\lambda_0$	$\alpha$ (yen)	$\beta$ (yen)	$m$	$T$ (weeks)
11.67	3.0	7.25	40	290	10	1

TABLE 3. Optimal policies.

$q_1$	0.1	0.3	0.5	0.7	0.9
$k^*$	8	7	7	6	5
$C_0(k^*)$	3.67	6.90	7.71	7.37	4.66

## 6.2. CHARACTERISTICS OF THE MODEL

Figure 2 reveals the expected loss for  $q_1 = 0.1, 0.3, 0.5, 0.7, 0.9$ , where  $q_1$  denotes the probability that the newly introduced product is a fast moving product. Table 3 shows the optimal non-negative integer  $k^*$  and its corresponding expected loss  $C_0(k^*)$  when the prior probability  $q_1$  changes. It is observed in Table 3 that  $k^*$  tends to decrease with increasing  $q_1$ . This signifies that if the objective product is,

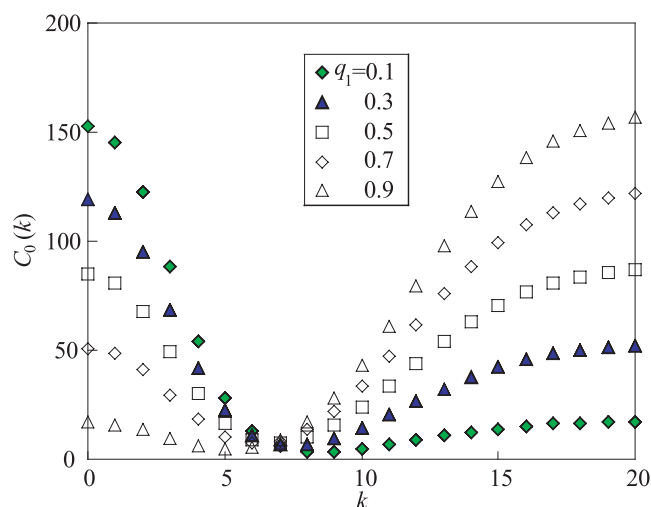


FIGURE 2. Expected loss.

*a priori*, considered to become slow moving, it will be regarded as a slow moving product unless its sales are unexpectedly high. On the contrary, when the objective product is, *a priori*, considered to be fast moving,  $k^*$  takes a small value so that the objective product is likely to be continued over numerous sales periods without significant falls in sales. It is also seen in Table 3 that the expected loss tends to take its maximum value when  $q_1 = q_2 = 0.5$ , which is equivalent to the situation where there is no information with respect to whether or not the candidate brand is a fast moving product.

This section applies the optimal integer  $k^*$  in Table 3 to another set of data associated with 16 brands of potato chips<sup>3</sup>, which were collected from the store over a ten week period between May 15, 2000 and July 17, 2000. Figure 3 shows the transition of sales per week of the top eight brands, Brand *a* to *h*, while Figure 4 reveals the results of the succeeding eight brands, Brand *i* to *p*. Table 4 shows the details of these sales data. In Table 4, marks, “\*”, “\_” and “??” have the same meanings as those in Table 1.

As discussed in 1., the authors know of no theoretical model to cope with this problem of the sales velocity of relatively small numbers of products within a small format store. Consequently, there exists no theoretical tool to verify the effectiveness of the proposed method. We can only consider the validity of the proposed method by comparing the above results with the actual decisions made by the manager of the convenience store from which we collected the above data.

Evaluation of each brand in Table 4 begins at the point where it first appears on sale in the store. Table 5 shows the week when, for each brand under evaluation,

<sup>3</sup>In 2000, the convenience store under study increased the number of categories and reduced the number of brands for each category. The number of brands of potato chips was reduced from 29 to 16.

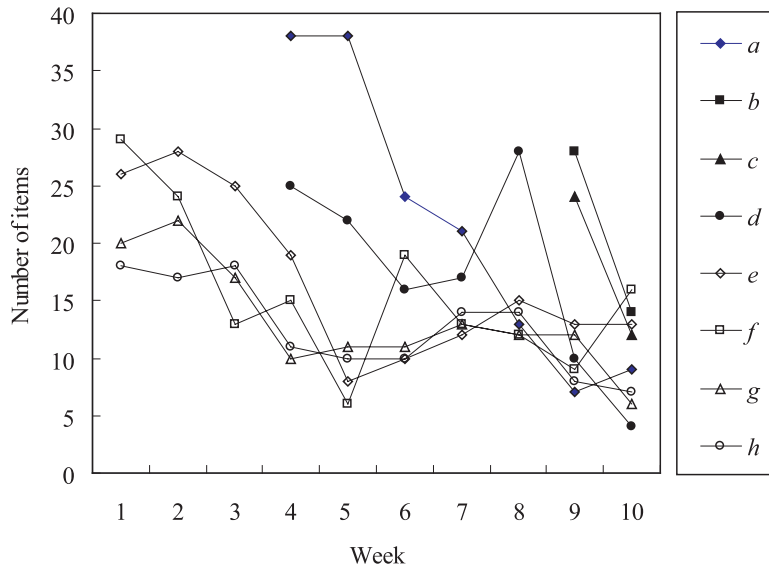


FIGURE 3. Sales data.

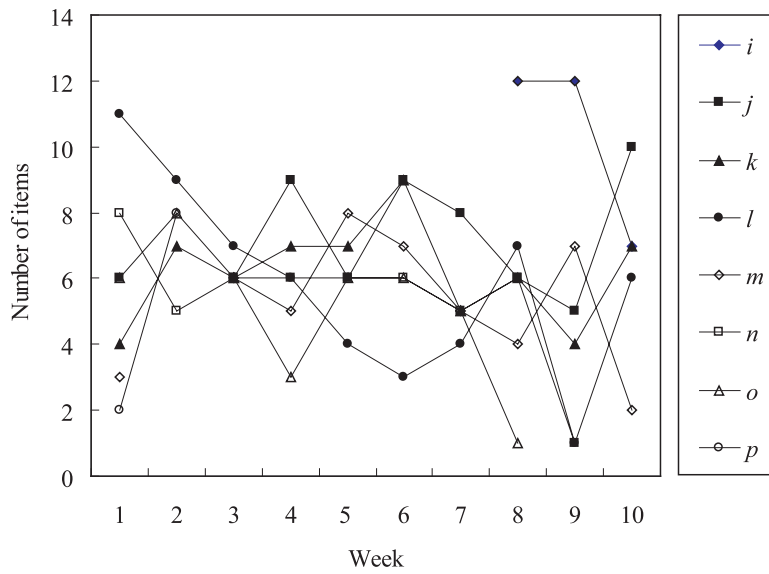


FIGURE 4. Sales data.

sales should be terminated according to the prior probabilities based upon  $k^*$  for  $q_1 = 0.1, 0.3, 0.5, 0.7, 0.9$ . In Table 5, periods marked by “+” signify that the results indicate that sales should continue beyond the data collection period. It

TABLE 4. Sales data.

Brand	1st week	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	Mean	Cumulative share
<i>a</i>	*	*	*	38	38	24	21	13	7	9	21.43	0.119
<i>b</i>	*	*	*	*	*	*	*	*	28	14	21.00	0.236
<i>c</i>	*	*	*	*	*	*	*	*	24	12	18.00	0.336
<i>d</i>	*	*	*	25	22	16	17	28	10	4	17.43	0.433
<i>e</i>	26	28	25	19	8	10	12	15	13	13	16.90	0.526
<i>f</i>	29	24	13	15	6	19	13	12	9	16	15.60	0.613
<i>g</i>	20	22	17	10	11	11	13	12	12	6	13.40	0.688
<i>h</i>	18	17	18	11	10	10	14	14	8	7	12.70	0.758
<i>i</i>	*	*	*	*	*	*	*	*	12	7	10.33	0.816
<i>j</i>	6	??	6	9	6	9	8	6	5	10	7.22	0.850
<i>k</i>	4	7	6	7	7	9	5	6	4	7	6.20	0.884
<i>l</i>	11	9	7	6	4	3	4	7	1	6	5.80	0.916
<i>m</i>	3	??	6	5	8	7	5	4	7	2	5.22	0.945
<i>n</i>	8	5	6	6	6	6	5	6	1	–	4.90	0.972
<i>o</i>	6	8	6	3	6	6	5	1	–	–	4.10	0.994
<i>p</i>	2	8	–	–	–	–	–	–	–	–	1.00	1.000

TABLE 5. Time to terminate the sales test.

Brand	$q_1 = 0.1$	$q_1 = 0.3$	$q_1 = 0.5$	$q_1 = 0.7$	$q_1 = 0.9$
<i>a</i>	9	+	+	+	+
<i>b</i>	+	+	+	+	+
<i>c</i>	+	+	+	+	+
<i>d</i>	10	10	10	10	10
<i>e</i>	+	+	+	+	+
<i>f</i>	5	5	5	+	+
<i>g</i>	10	10	10	+	+
<i>h</i>	10	+	+	+	+
<i>i</i>	10	+	+	+	+
<i>j</i>	1	1	1	9	9
<i>k</i>	1	1	1	1	1
<i>l</i>	3	4	4	5	5
<i>m</i>	1	1	1	1	1
<i>n</i>	2	2	2	2	9
<i>o</i>	1	1	1	4	4
<i>p</i>	1	1	1	1	1

should be noted here that Brand *k* was introduced in midweek of the first week and therefore its sales in the first week were smaller than those in the other weeks. If we had ignored this sales result of Brand *k* in the first week, it would have been cut off in the second week for  $q_1 = 0.1$ , in the third week for  $q_1 = 0.3, 0.5$ , and in the ninth week for  $q_1 = 0.7, 0.9$ .

If we had applied the proposed method to the brands in Table 4, Brands  $j, k, \dots, p$  would have been cut off more promptly than in Table 4 for most values of  $q_1$ . In addition, Brands  $a, b, \dots, i$  except  $d$  would have been carried over the next period for large values of  $q_1$ . From these observations, the proposed method tends to detect fast moving products without errors and slow moving products efficiently for a large value of  $q_1$ .

It should be noted in the above that the actual data shown in Tables 1 and 4, and Figures 3 and 4 do not seem to follow a Poisson process if we look at them over the all weeks at once. It should, however, be reminded that we have not assumed the demand rate  $\lambda$  remains constant over a long period. We here consider that the actual value of  $\lambda$  varies in each week like a random variable.

## 7. SUMMARY AND CONCLUDING REMARKS

This study proposed a theoretical model of for sales test policy which tests the sales velocity of newly introduced products at a retail store. Under the proposed model, a sales test is conducted for  $m$  items of a newly introduced product over period  $(0, T]$ . If the number  $N(T)$  of items of the product sold up to time  $T$  is greater than or equal to a non-negative integer  $k$ , it is regarded as a fast moving product. In contrast, if  $N(T)$  is smaller than  $k$ , we regard the newly introduced product as a slow moving product and the manager will consider cancelling the sale of the product to release shelf space to an alternative candidate product.

The expected loss incurred by the misjudgments or the decision errors by the proposed model was then formulated, which was to be minimized with respect to  $k$ . The existence of an optimum integer  $k$  was then shown for a prespecified sales test period  $T$ . Numerical examples were also presented using the actual data on potato chips collected from a single convenience store to discuss characteristics of the proposed model.

The authors are currently collecting data on a variety of products to clarify to what extent the proposed model can effectively be applied. In addition, we are investigating the model which takes account of sales interaction among brands. At present, however, the model provides a first step for managers of small, locally competitive retail stores in the FMCG sector.

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