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# THE ASYMPTOTICS OF SPHERICAL FUNCTIONS AND THE CENTRAL LIMIT THEOREM ON SYMMETRIC CONES

by Genkai ZHANG

## 0. Introduction.

In multivariate statistical analysis the spaces of positive definite matrices (real or complex) are of considerable interests. One studies the spherical polynomials, central limit theorem on them. See [M]. A natural generalization of those spaces is the cone of positive elements in a formally real Jordan algebra. This class of cones includes cones of positive definite real, complex and quaternion matrices and the forward light cone. In this paper we will prove a kind of central limit theorem on any such symmetric cones.

In his paper [T2] Terras obtained a central limit theorem for rotation-invariant random variables on the space of $n \times n$ real positive definite (symmetric) matrices in the case $n = 3$. This has been recently generalized to any $n$ by Richards [R]. In this paper we will generalize this central limit theorem to the symmetric cone of positive elements in a Jordan algebra.

The main tools used in [R] are the Binet-Cauchy theorem and certain coefficients in some recurrence formulas of zonal (spherical) polynomials in [Ku]. Recently we have found some recurrence formulas on any simple and formally real Jordan algebra [Z]. To prove our central limit theorem we will

*Key words*: Spherical functions – Central limit theorem – Jordan algebra – Symmetric spaces – Heat equation – Spherical polynomial.
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use those recurrence formulas and a recent result of Faraut and Koranyi [FK] on expansion of reproducing kernels.

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## 1. Preliminaries.

In this section we recall some facts about the symmetric cone in a Jordan algebra. Most results in this section can be found, e.g. in [UU] and [KS] (on formally real Jordan algebras) and [T1] (in the special case of real symmetric matrices).

Let $X$ be a simple and formally real Jordan algebra [BK] with product $x \circ y$ and unit element $e$. Basic examples are the space $X = \mathcal{H}_r(\mathbb{K})$ of all self-adjoint $r \times r$-matrices over $\mathbb{K} = \mathbb{R}, \mathbb{C}$ or $\mathbb{H}$ (quaternions) with the anti-commutator product $x \circ y = (xy + yx)/2$, and the "spin factor" $X = \mathbb{R}^{1+n}$ of all vectors $x = (x_0, x_1, \ldots, x_n) = (x_o, x')$, with product $x \circ y = (x_0 y_0 + x'y', x_0 y' + y_0 x')$. The open set

$$\mathcal{P} = \{x^2, x \in X \text{ invertible}\}$$

is a convex cone. For the matrix algebras, $\mathcal{P}$ is the cone of positive definite matrices, whereas for the spin factor $\mathcal{P}$ is the forward light cone. The linear automorphism group

$$GL(\mathcal{P}) = \{g \in GL(X) : g(\mathcal{P}) = \mathcal{P}\}$$

is transitive on $\mathcal{P}$. Let $G$ be the identity component of $GL(\mathcal{P})$. $G$ has Lie algebra

$$\mathfrak{g} = \{M \in \mathfrak{gl}(X) : \exp(tM) \in GL(\mathcal{P}), \text{ for all } t \in \mathbb{R}\}$$

with the commutator bracket $[M, N] = MN - NM$. The Lie algebra $\mathfrak{g}$ has a Cartan decomposition $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$, where $\mathfrak{k}$ is the Lie algebra of the "orthogonal" group

$$K = O(\mathcal{P}) = \{g \in G : ge = e\}$$

and $\mathfrak{p} = \{M_x : x \in X\}$ consists of all multiplication operators

$$M_x y = x \circ y$$

on $X$. Moreover $\mathcal{P} = G/K$ is now a Riemannian symmetric space.

Let $e_1, \dots, e_r$ be a maximal orthogonal system of idempotents in $X$. Then we have

$$e = e_1 + \dots + e_r.$$

Let

$$X = \sum_{1 \le i \le j \le r} X_{ij} \tag{1.1}$$

be the Peirce decomposition induced by $\{e_1, \dots, e_r\}$. Here

$$X_{i,j} = X_{ji} = \left\{ x \in X : e_k \circ x = \frac{\delta_{jk} + \delta_{ik}}{2} x \right\}$$

are the joint eigenspaces of the multiplication operators $\{M_{e_j}\}_{j=1}^r$ on $X$. The integer $r$ is independent of the choice of the maximal orthogonal system of idempotents and is called the rank of $X$.

Let

$$\mathfrak{a} = \mathbb{R} M_{e_1} + \dots + \mathbb{R} M_{e_r}$$

be the subspace of $\mathfrak{p}$ generated by $M_{e_1}, \dots, M_{e_r}$. Then $\mathfrak{a}$ is maximal abelian in $\mathfrak{p}$. So $r$ is the rank of the Riemannian symmetric space $\mathcal{P}$. We have thus a root space decomposition of $(\mathfrak{g}, \mathfrak{a})$ [H1]. We let $\{\gamma_j\}_{j=1}^r$ be the dual basis in $\mathfrak{a}^*$ of $\{M_{e_j}\}_{j=1}^r$, i.e.

$$\gamma_j(M_{e_k}) = \delta_{jk}.$$

Then the corresponding root system consists of $\dfrac{\gamma_j - \gamma_k}{2}$, $j \neq k$ with common multiplicity $a$. The Weyl group is the symmetric group $S_r$. For example in the case of matrix algebra $\mathcal{H}_r(\mathbb{K})$ the rank is $r$ and the multiplicity is $a = 1, 2$ or $4$ according to $\mathbb{K} = \mathbb{R}, \mathbb{C}$, or $\mathbb{H}$ respectively.

We fix an ordering of $\mathfrak{a}^*$ by letting

$$\gamma_1 < \gamma_2 < \dots < \gamma_r.$$

Therefore, the positive roots are $\dfrac{\gamma_j - \gamma_k}{2}$, $j > k$. Let $\underline{\rho} = \sum_{j=1}^r \rho_j \gamma_j$ be, as usual, the half sum of positive roots

$$\underline{\rho} = \frac{a}{2} \sum_{j \ge k} \frac{\gamma_j - \gamma_k}{2} = \frac{a}{2} \sum_{j=1}^r \left( j - \frac{r+1}{2} \right) \gamma_j.$$

We will now describe the spherical functions and the heat equation on the symmetric space $\mathcal{P}$. We need the notion of Jordan algebra determinants.

Let $P(x) := 2(M_x)^2 - M_{x^2}$. The determinant of it is up to a constant a unique (integer) power of an irreducible polynomial $\Delta(x)$ of degree $r$ on $X$, normalized so that $\Delta(e) = 1$. $\Delta(x)$ is called the determinant of the Jordan algebra $X$. It can also be defined as the unique irreducible polynomial given by the "Cramer's rule":

$$x^{-1} = \frac{\text{cofactor of } x}{\Delta(x)}, \ (x \in X \text{ invertible})$$

normalized so that $\Delta(e) = 1$.

Let $X_{ij}$ be given in the Peirce decomposition (1.1) we put

$$X_j = \sum_{k,\ell \leq j} X_{k\ell}$$

and let $P_j$ be the orthogonal projection of $X$ on $X_j$; we define

$$\Delta_j(x) = \Delta_{X_j}(P_j x)$$

where $\Delta_{X_j}$ is the corresponding determinant in $X_j$.

For any $\underline{t} = (t_1, \ldots, t_r) \in \mathbb{C}^r$ we define

$$\Delta_{\underline{s}}(x) = \prod_{j=1}^{r} \Delta_j^{s_j}(x). \tag{1.2}$$

Let $\boldsymbol{\lambda} = \sum_{j=1}^{r} \lambda_j \gamma_j \in \mathfrak{a}_{\mathbb{C}}^*$, the complexification of $\mathfrak{a}^*$, we let $\underline{s} = (s_1, s_2, \ldots, s_r)$ be defined by

$$s_j = \lambda_j + \rho_j - \lambda_{j+1} - \rho_{j+1}, \tag{1.3}$$

where we put $s_r = \lambda_r + \rho_r$. The spherical function $h_{\boldsymbol{\lambda}}$ is then

$$h_{\boldsymbol{\lambda}}(x) = \int_K \Delta_{\underline{s}}(kx)dk = \int_K \prod_{j=1}^{r} \Delta_j^{s_j}(kx)dk.$$

Then $h_{\boldsymbol{\lambda}}(x)$ are $K$-invariant eigenfunctions of all the $G$-invariant differential operators on $\mathcal{P}$. The Helgason-Fourier transform of a $K$-invariant function $f$ on $\mathcal{P}$ is defined by

$$\widehat{f}(\boldsymbol{\lambda}) = \int_{\mathcal{P}} f(x) h_{\boldsymbol{\lambda}}(x) d^*x,$$

where $d^*$ is the $G$-invariant measure on $\mathcal{P}$.

For later use we also introduce the spherical polynomials, which is the analytic continuation of $h_{\boldsymbol{\lambda}}$ (in $\boldsymbol{\lambda}$). Let $\underline{\mathbf{m}} = (m_1, \ldots, m_r)$ be a $r$-tuple of integers with $m_1 \geq m_2 \geq \ldots \geq m_r \geq 0$. The functions

$$\phi_{\underline{\mathbf{m}}}(x) = \int_K \prod_{j=1}^{r} \Delta_j^{m_j - m_{j+1}}(kx)dk,$$

where $m_{r+1} = 0$, are the spherical polynomials on $\mathcal{P}$. In particular if we put

$$\underline{\mathbf{1^j}} = \sum_{k=1}^{j} \gamma_k,$$

the functions

$$\phi_{\underline{\mathbf{1^j}}}(x) = \int_K \Delta_{\underline{\mathbf{1^j}}}(kx)dk = \int_K \Delta_j(kx)dk$$

are called the fundamental spherical polynomials on $\mathcal{P}$.

Finally we recall the heat equation on $\mathcal{P}$. Let $L$ be the Laplace-Beltrami operator on $\mathcal{P}$. We normalize the $K$-invariant norm on $\mathfrak{p}$ so that norm $M_{e_1}$ has norm 1. Then $M_{e_1}, \ldots, M_{e_r}$ are orthonormal basis of $\mathfrak{a}$. This also gives a normalization of the Riemannian metric on the cone $\mathcal{P} = G/K$.

Then $\phi_{\boldsymbol{\lambda}}$ are eigenfunctions of $L$:

$$L\phi_{\boldsymbol{\lambda}} = \Big(\sum_{j=1}^{r} \lambda_j^2 - \frac{a(r^3-r)}{48}\Big)\phi_{\boldsymbol{\lambda}}.$$

The heat equation is defined

$$\begin{cases} \partial_t u(x,t) = Lu(x,t), \ (x,t) \in \mathcal{P} \times \mathbb{R}^+ \\ u(x,0) = f(x). \end{cases}$$

## 2. The main theorem and the proof.

Before stating our main theorem we introduce some notation. Let $H = \sum_{j=1}^{r} h_j M_{e_j} \in \mathfrak{a}$. We let $J(\exp H)$ be the function defined on $A$ by

$$J(\exp H) = \frac{1}{2^{\binom{n}{2}}} \prod_{i \geq j} (e^{h_i} - e^{h_i})^a \prod_{i=1}^{r} e^{-a\frac{(n-1)}{2} h_k}.$$

Our main theorem is the following central limit theorem.

THEOREM 1. — *Let $Y_m$, $m \geq 1$ be a sequence of independent, identical distributed, $K$-invariant random variables in $\mathcal{P}$, with common density function $f(Y)$. Assume that $f(Y)$ satisfies*

$$\int_{\mathfrak{a}} h_i f(\exp H) J(\exp H) dh_1 \cdots dh_r = 0$$

$$\int_{\mathfrak{a}} h_i h_j f(\exp H) J(\exp H) dh_1 \cdots dh_r = 0, \quad i \neq j$$

*and the convolution* $S_m = Y_1 \circ Y_2 \circ \cdots \circ Y_m$ *is normalized as in (3.39) of [T1] and that* $S_m$ *has density function* $f_m^\sharp$. *Then for any measurable subset* $S \subset \mathcal{P}$

$$\int_S f_m^\sharp(Y) d^*(Y) \approx \exp \frac{a^3(r^3 - r)}{96(2 + ar)} \times \int_S G_{\frac{(2+a)}{2(2+ar)}} * F_{\frac{a}{2+ar}}(Y) d^*(Y)$$

*as* $m \to \infty$.

*In the above,* $G_t$ *is the fundamental solution of the heat equation on* $\mathcal{P}$ *with Helgason-Fourier transform*

$$\widehat{G_t}(\boldsymbol{\lambda}) = \exp\left( t\Big( \sum_{i=1}^r \lambda_i^2 - \frac{a(r^3 - r)}{48} \Big) \right).$$

*Further* $F_t$ *has Helgason-Fourier transform*

$$\widehat{F_t}(\boldsymbol{\lambda}) = \exp\left( t\Big( \sum_{1 \le k < j \le r} \lambda_i \lambda_j \Big) \right).$$

Following [T1] and [R] we need to find the expansion of the spherical functions $h_{\boldsymbol{\lambda}}(\exp H)$ on $\mathfrak{a}$ around $H = 0$. Recall that spherical functions $h_{\boldsymbol{\lambda}}(\exp H)$ are Weyl group invariant in $\boldsymbol{\lambda}$ and $H \in \mathfrak{a}$. The expansion is (recalling that $H = \sum_{j=1}^r h_j M_{e_j}$)

$$\begin{aligned} h_{\boldsymbol{\lambda}}(\exp H) =& 1 + \Big( \alpha_{11} \sum_{i=1}^r \lambda_i + \alpha_{12} \Big) \Big( \sum_{i=1}^r h_i \Big) \\ &+ \Big( \alpha_{21} \sum_{i=1}^r \lambda_i^2 + \alpha_{22} \sum_{1 \le i < j \le r} \lambda_i \lambda_j + \alpha_{23} \sum_{i=1}^r \lambda_i + \alpha_{24} \Big) \Big( \sum_{i=1}^r h_i^2 \Big) \\ &+ P(\lambda_1, \ldots, \lambda_r) \Big( \sum_{1 \le i < j \le r} h_i h_j \Big) + \text{ higher terms}; \end{aligned}$$

namely we need to find the six coefficients $\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}$.

We fix $\boldsymbol{\lambda} \in \mathfrak{a}_{\mathbb{C}}^*$ and $\underline{s}$ as in (1.3). Consider the formula (1.2) in the polar coordinates

$$\Delta_{\underline{s}}(ka \cdot e) = \prod_{j=1}^r \Delta_j^{s_j}(ka \cdot e) \tag{2.1}$$

and put $a = \exp H$. Following [R] we take logarithm differentiation to obtain

$$\frac{1}{\Delta_{\underline{s}}(ka \cdot e)} \frac{\partial}{\partial h_1} \Delta_{\underline{s}}(ka \cdot e) = \sum_{j=1}^r s_j \frac{1}{\Delta_j(ka \cdot e)} \frac{\partial}{\partial h_1} \Delta_j(ka \cdot e). \tag{2.2}$$

Integrating over $K$ and putting $H=0$ we get

$$\frac{\partial}{\partial h_1} h_{\boldsymbol{\lambda}}(a\cdot e)\bigg|_{H=0} = \sum_{j=1}^{r} s_j \frac{\partial}{\partial h_1}\phi_{\underline{\mathbf{1_j}}}(a\cdot e)\bigg|_{H=0}. \tag{2.3}$$

We adopt the notation [R] to denote

$$f_j'(k) = \frac{\partial}{\partial h_1}\Delta_j(ka\cdot e)\bigg|_{H=0}$$

$$f_j''(k) = \frac{\partial^2}{\partial h_1^2}\Delta_j(ka\cdot e)\bigg|_{H=0}$$

and

$$\mathcal{E}(f) = \int_K f(k)dk$$

for a function $f$ on $K$.

We need a result of Faraut and Koranyi [FK].

LEMMA 1. — *For $a=\exp H$ we have*

$$\prod_{j=1}^{r}(1-te^{h_j}) = \sum_{j=0}^{r}(-1)^j\binom{r}{j}\phi_{\underline{\mathbf{1_j}}}(a\cdot e)t^j. \tag{2.4}$$

*Proof.* — This is a special case of Theorem 3.8 in [FK]. See also Remark 2 after the theorem. We give here a simple independent proof. We know that the polynomial $\Delta(e-z)$ is a polynomial of degree $r$ on $X$ and is $K$ invariant. Thus we have an expansion

$$\Delta(e-tz) = \sum_{j=0}^{r} c_j\phi_{\underline{\mathbf{1_j}}}(z)t^j,$$

for some constants $c_j$. Now take $z=a\cdot e$ the above equation reads

$$\prod_{j=1}^{r}(1-te^{h_j}) = \sum_{j=0}^{r} c_j\phi_{\underline{\mathbf{1_j}}}(z)t^j.$$

Evaluating the equation at $H=0$ and noting that $\phi_{\underline{\mathbf{1_j}}}(e)=1$ we find the coefficients $c_j$. □

Lemma 1 has also been proved previously by J. Arazy and Z. Yan [A].

Now we differentiate the expansion in Lemma 1 and set $H=0$,

$$-t(1-t)^{r-1} = \sum_{j=0}^{r}(-1)^j\binom{r-1}{j-1}\frac{r}{j}\frac{\partial}{\partial h_1}\phi_{\underline{\mathbf{1_j}}}(a\cdot e)\bigg|_{H=0} t^j;$$

however the LHS of the above is

$$-t(1-t)^{r-1} = \sum_{j=0}^{r}(-1)^j \binom{r-1}{j-1} t^j,$$

thus we find

$$\mathcal{E}(f_1') = \frac{\partial}{\partial h_1}\phi_{\underline{1^j}}(a\cdot e)\Big|_{H=0} = j/r.$$

Using this we see

$$\alpha_{11} = \frac{1}{r}, \quad \alpha_{12} = 0.$$

Note that if we differentiate (2.4) twice we get

$$\mathcal{E}(f_1'') = \frac{\partial^2}{\partial h_1^2}\phi_{\underline{1^j}}(a\cdot e)\Big|_{H=0} = j/r.$$

Now the coefficients of $2\sum_{i=1}^{r} h_i^2$ is

$$Q(s_1,\dots,s_r) = \frac{\partial^2}{\partial h_1^2} h_{\boldsymbol{\lambda}}(a\cdot e)\Big|_{H=0}. \tag{2.5}$$

We have

$$Q(s_1,\dots,s_r) = \mathcal{E}\Big(\Big(\sum_{j=1}^{r} s_j f_j'\Big)^2\Big) + \sum_{j=1}^{r}\big(s_j\mathcal{E}(f_j'') - s_j^2\mathcal{E}((f_j')^2)\big).$$

We need to find

$$\mathcal{E}(f_1'f_2'),\ \mathcal{E}((f_1')^2).$$

We start to calculate $\mathcal{E}(f_1'f_j')$. It follows from §1 that

$$\Delta_{\underline{1^1}+\underline{1^j}} = \Delta_1\Delta_j.$$

Differentiating at $H=0$ and integrating we find

$$\frac{\partial^2}{\partial h_1^2}\phi_{\underline{1^1}+\underline{1^j}}(ka\cdot e)\Big|_{H=0} = \mathcal{E}(f_1'') + \mathcal{E}(f_j'') + 2\mathcal{E}(f_1'')\mathcal{E}(f_j''). \tag{2.6}$$

To find out the LHS we need the a result proved in [Z] (see also [St]).

LEMMA 2. — *We have the recurrence formula of the product* $\phi_{\underline{1^1}}\phi_{\underline{1^j}}$ *of the spherical polynomials*

$$\phi_{\underline{1^1}}\phi_{\underline{1^j}} = \frac{j(1+\frac{a}{2}r)}{r(1+\frac{a}{2}j)}\phi_{\underline{1^1}+\underline{1^j}} + \frac{r-j}{r(1+\frac{a}{2}j)}\phi_{\underline{1^{j+1}}}$$

*for the product* $\phi_{\underline{1^1}}\phi_{\underline{1^j}}$.

The lemma can also be written as

$$\frac{j(1+\frac{a}{2}r)}{r(1+\frac{a}{2}j)}\phi_{\underline{1^1}+\underline{1^j}} = \phi_{\underline{1^1}}\phi_{\underline{1^j}} - \frac{r-j}{r(1+\frac{a}{2}j)}\phi_{\underline{1^{j+1}}}.$$

Thus

$$\frac{\partial^2}{\partial^2 h_1}\phi_{\underline{1^1}+\underline{1^j}}(ka\cdot e)\bigg|_{H=0} = \frac{(1+\frac{a}{2}j)(2j+r(j+1)) - (r-j)(j+1)}{(rj(1+\frac{a}{2}r)}.$$

Now we substitute this to (2.6) to get

$$\mathcal{E}(f_1'f_j') = \frac{2+aj}{2r+ar^2}.$$

In particular

$$\mathcal{E}\left((f_1')^2\right) = \frac{2+a}{2r+ar^2}.$$

Thus the coefficient of $\lambda_1\lambda_2$ in (2.5) is

$$\begin{aligned} 2\alpha_{22} &= 2(\mathcal{E}((f_1'f_2') - \mathcal{E}\left((f_1')^2\right)) \\ &= \frac{2a}{2r+ar^2} \end{aligned}$$

and the coefficient of $\lambda_1^2$ is

$$\begin{aligned} 2\alpha_{21} &= \mathcal{E}\left((f_1')^2\right) \\ &= \frac{2+a}{2r+ar^2}. \end{aligned}$$

The coefficient of $\lambda_1$ in (2.5) is

$$\begin{aligned} 2\alpha_{23} &= \mathcal{E}(f_1') - \mathcal{E}\left((f_1')^2\right) - a\sum_{j=2}^{r-1}\mathcal{E}(f_1'f_j') + \frac{a}{2}(r-1)\mathcal{E}(f_1') \\ &= 0. \end{aligned}$$

On the other hand, we have $Q(s_1,\ldots,s_r) = 0$ for $\underline{s} = 0$ or for $\lambda_j = -\frac{a}{2}(j - \frac{r+1}{2})$, that is

$$0 = \alpha_{21}\sum_{j=1}^{r}\lambda_j^2 + \alpha_{22}\sum_{1\le i<j\le r}\lambda_i\lambda_j + \alpha_{24}$$

which gives us

$$\alpha_{24} = -\frac{a(r^2-1)}{48(2+ar)}.$$

Finally we get the required expansion

$$\begin{aligned}h_{\lambda}(\exp H) =&1 + \Big(\frac{1}{r}\sum_{i=1}^{r}\lambda_i\Big)\Big(\sum_{i=1}^{r}h_i\Big)\\ &+\Big(\frac{2+a}{2(2r+ar^2)}\sum_{i=1}^{r}\lambda_i^2 + \frac{a}{2r+ar^2}\sum_{1\le i<j\le r}\lambda_i\lambda_j\\ &\qquad -\frac{a(r^2-1)}{48(2+ar)}\Big)\Big(\sum_{i=1}^{r}h_i^2\Big)\\ &+P(\lambda_1,\dots,\lambda_r)\Big(\sum_{\le i<j\le r}h_ih_j\Big) + \text{ higher terms.}\end{aligned}$$

The rest of the proof is similar to that in [T1].

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