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EDOARDO BALLICO

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A SPLITTING THEOREM FOR THE KUPKA COMPONENT OF A FOLIATION OF $\mathbb{CP}^n$, $n \geq 6$. Addendum to an addendum to a paper by Calvo-Andrade and Soares

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A codimension one singular holomorphic foliation F of $\mathbb{CP}^n$ is given by $\omega \in H^0(\mathbb{CP}^n, \Omega(k))$ (for some k) with $\omega \neq 0$, ω not vanishing on a hypersurface. The Kupka subset $K(F) := \{P \in \mathbb{CP}^n : \omega(P) = 0, d\omega(P) \neq 0\}$ of the singular set $S(F) := \{P \in \mathbb{CP}^n : \omega(P) = 0\}$ of F has remarkable properties (e.g. if not empty it is a smooth submanifold of pure codimension 2 with strong stability properties with respect to deformations of F). For much more on this topic, see [GL] and [GS]. Let $K \neq \emptyset$ be a Kupka component of F, i.e. ([CS]) a connected component of $K(F)$. It was proved in [CL] that if K is a complete intersection, then F has a meromorphic first integral. Motivated by this result in [CS] it was conjectured and proved in some cases that every Kupka component, K, is a complete intersection. This conjecture was proved in [B] under the assumption that $\deg(K)$ is not a square. Here we will remove this restrictive asssumption and prove the following result.

THEOREM. — *Let F be a codimension 1 singular holomorphic foliation of $\mathbb{CP}^n$, $n \geq 6$, induced by $\omega \in H^0(\mathbb{CP}^n, \Omega^1(k))$ and such that the*

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codimension 2 component of the singular set of F contains a Kupka component K. Then K is a complete intersection and F has a meromorphic first integral.

Let N_K be the normal bundle of K in $\mathbb{CP}^n$. By [CS], Corollary 3.5, N_K is the restriction $E|K$ to K of a rank 2 vector bundle E on $\mathbb{CP}^n$. K is a complete intersection if and only if E is the direct sum of two line bundles ([OSS]). If $n \geq 6$ every line bundle on K is the restriction of a line bundle on $\mathbb{CP}^n$ (see [FL]). Hence, by a very nice result of Faltings ([F]) if $n \geq 6$ and N_K is the direct sum of two line bundles, K is a complete intersection. Now we use the notations of [GL], page 320. There are two complex numbers (not both zero) such that the linear part of the foliation near each point of K depends from these two complex numbers. Up to a normalization we may assume that one of these numbers is 1; call μ the other one. Il $\mu \neq \pm 1$, then N_K splits into the direct sum of two line bundles ([GL]), Remark 2 at p. 320). If $\mu = -1$, then there is a two-to-one unramified covering $\pi : Z \to K$ such that $\pi^*(N_K)$ is the direct sum of two isomorphic line bundles (see [GL], the two lines before eq. (2.6)). Since K is simply connected ([FL]), Cor. 6.3), N_K splits into the direct sum of two isomorphic line bundles. Now assume $\mu = 1$. This is the only difference with respect to [B]. By [GL], eq. (2.12) and the two following lines, N_K is an extension of a line bundle L_1 by itself. Since $n \geq 6$ we have $H^1(K, \mathbb{C}) \cong \mathbb{C}$ by a theorem of Barth's (see e.g. [FL], eq. (**) at p. 83). Hence by standard Hodge theory ([GH], bottom of p. 105) we have $H^1(K, \mathcal{O}_K) = 0$. Thus any such extension of L_1, by itself splits and hence $N_K \cong L_1 \oplus L_1$, concluding the proof of the theorem.

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Edoardo BALLICO,
Department of Mathematics
University of Trento
38050 Povo (NT) (Italy).
ballico@science.unitn.it