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Winter Braids Lecture Notes

Participants & Programme

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Participants & Programme

This volume of *Winter Braids Lecture Notes* contains the lecture notes for the four mini-courses given at Winter Braids X, which took place at the Centro de Giorgi in Pisa, from February 17th to 20th, 2020. We are indepted to Fillipo Callegaro, who did a great work as local organizer for this edition of the Winter Braids school.

Participants

Albeladi	Hadeel	(University of Leeds)
Alsubhi	Basmah	(University of Leeds)
Audoux	Benjamin	(Université Aix-Marseille)
Barbensi	Agnese	(University of Oxford)
Bartulovic	Ivan	(Université de Lille)
Battista	Ludovico	(Università di Pisa)
Belletti	Giulio	(SNS, Università di Pisa)
Bellingeri	Paolo	(Université Caen Normandie)
Ben Aribi	Fathi	(Université de Louvain)
Benard	Léo	(Universitaet Göttingen)
Benvenuti	Silvia	(Università di Bologna)
Berat	Clément	(Université Grenoble Alpes)
Blanchi	Filippo	(Università di Pisa)
Balckwell	LSarah Elizabeth	(University of Georgia)
Bulgakova	Daria	(Université Aix-Marseille)
Callegaro	Filippo	(Università di Pisa)
Catabriga	Alessia	(Università di Bologna)
Cavallo	Alberto	(Max Planck Institute, Bonn)
Celoria	Daniele	(University of Oxford)
Collari	Carlo	(NY University, Abu Dhabi)
Colombari	Boris	(Université Aix-Marseille)
Dai	Irving	(MIT)
Damiani	Celeste	(University of Leeds)
Darné	Jacques	(Université de Bourgogne)
Detcherry	Renaud	(MPIM Bonn)
Di Petrillo	Gemma	(Università di Pisa)
Duran Batalla	Juan Luis	(Universidad Autonoma Barcelona)
Faes	Quentin	(Université de Bourgogne)
Fernandez	Lucas	(University of Bern)
Ferrari	Lenardo	(Università di Pisa)
Ferretti	Livio	(University of Bern)
Florens	Vincent	(Université de Pau)
Földvári	Viktória	(Eötvös Loránd University)
Fossati	Edoardo	(SNS, Università di Pisa)
Gandolfi	Guillaume	(Université Caen Normandie)

Gille	Catherine	(Université Paris Diderot)
Gobet	Thomas	(Université de Tours)
Gonzales	Martin	(MPIM Bonn)
Graff	Emmanuel	(Université Caen Normandie)
Italiano	Giovanni	(SNS, Università di Pisa)
Joseph	Jason	(MPIM Bonn)
Kallel	Sadok	(Université de Lille)
Kladov	Danil	(Univ. Novossibirsk)
Kutluay	Deniz	(Indiana Univ. Bloomington)
Kjuchukova	Alexandra	(MPIM Bonn)
Ladu	Roberto	(Imperial College)
Leturcq	David	(Université Grenoble Alpes)
Limouzineau	Maÿlis	(Univ. Köln)
Lopez	Daniel	(Univ. Paris)
Makri	Stavroula	(Université Caen Normandie)
Martel	Jules	(Université Toulouse III)
Matkovic	Irena	(University of Oxford)
McDonald	Clayton	(Boston College)
Meilhan	Jean-Baptiste	(Université Grenoble Alpes)
Merz	Alice	(Università di Pisa)
Migliorini	Matteo	(SNS, Università di Pisa)
Mihajlovic	Stefan	(Central European University)
Miller	Maggie	(Princeton)
Misev	Filip	(MPIM Bonn)
Monti	Andrea Egidio	(SNS, Università di Pisa)
Moussard	Delphine	(Université Aix-Marseille)
Nagel	Matthias	(ETH Zürich)
Nguyen	Hung	(Università di Pisa)
Pinzon Caicedo	Juanita	(Notre Dame/MPIM Bonn)
Pogtanapaisan	Puttipong	(University of Iowa)
Rasskin	Ivan	(Université de Montpellier)
Riba	Ricard	(UA Barcelona)
Riolo	Stefano	(Université de Neuchatel)
Robert	Louis-Hadrien	(Université de Genève)
Rodau	Adrien	(Université de Pau)
Ruppik	Benjamin Matthias	(MPIM Bonn)
Sakalli	Sumeyra	(MPIM Bonn)
Sané	Abdoul Karim	(Université de Grenoble)
Santoro	Diego	(Università di Pisa)
Sato	Masatoshi	(Tokyo Denki University)
Schwartz	Hannah	(MPIM Bonn)
Shaukat	Awais	(Abdus Salam SMS)
Shimizu	Tatsuro	(Kyoto University)
Silvero	Marithania	(Universidad de Huelva)
Singh	Oliver	(Durham University)
Soergel	Mireille	(Université Bourgogne)
Soulié	Arthur	(Cambridge University)
Stenhede	Eric	(Università di Pisa)
Thiel	Anne-Laure	(Université Caen Normandie)
Torzewska	Fiona	(University of Leeds)
Truöl	Paula	(ETH Zürich)
Usman	Ali	(Bahauddin Zakariya Univ. Multan)
Vera Arboleda	Anderson	(Kyoto University)
Vershinin	Vladimir	(Université de Montpellier)
Wagner	Emmanuel	(Université de Bourgogne)
Yakhin	Valeriy	(Univ. Novossibirsk)
Yoon	Seokbeom	(Seoul National University)
Zuddas	Daniele	(Universitët Bayreuth)

Abstracts of Courses

Christine Lescop (Université Grenoble Alpes)
Invariants of links and 3-manifolds that count graph configurations

We will present ways of counting configurations of uni-trivalent Feynman graphs in 3-manifolds in order to produce invariants of these 3-manifolds and of their links, following Gauss, Witten, Bar-Natan, Kontsevich and others.
We will first review the construction of the simplest invariants that can be obtained in our setting. These invariants are the linking number and the Casson invariant of integer homology 3- spheres.
Next we will see how the involved ingredients, which may be explicitly described using gradient flows of Morse functions, allow us to define a functor on the category of framed tangles in rational homology cylinders.
Finally, we will show some properties of our functor, which generalizes both a universal Vassiliev invariant for links in the ambient space and a universal finite type invariant of rational homology 3-spheres.

Carlo Petronio (Università di Pisa)
The Hurwitz existence problem for surface branched covers

To a branched cover $f : \tilde{\Sigma} \to \Sigma$ between closed surfaces one can associate a combinatorial datum given by the degree d of f the genera and orientability of $\tilde{\Sigma}$ and Σ, the number of branching points of f and the partitions of d given by the local degrees of f at the preimages of the branching points. This datum must satisfy the Riemann-Hurwitz condition plussome extra ones if either of $\tilde{\Sigma}$ or Σ or both are non-orientable. A very old question of Hurwitz asks whether a combinatorial datum satisfying the necessary conditions is actually realizable (namely, associated to some existing f) or not (in which case it is called exceptional). Or, more generally, to count the number of realizations of the datum up to a natural equivalence relation. Many partial answers have been given to the Hurwitz problem over the time, but a complete solution is still missing. I will report on ancient and recent results and techniques employed to attack the question.

Loïc Poulain d'Andecy (Université de Reims)
Fusion for the Yang-Baxter equation and the braid group

The Yang-Baxter equation is a famous equation in mathematical physics. The main goal of this mini-course is to present how its study interacts interestingly with the study of the braid group. In a few words, we will speak about the Yang–Baxter equation and the braid group, their connections, the use of quantum groups for their study, and the construction of interesting algebras such as the Hecke algebras and the BMW algebras, and some recent generalizations (going under the name of fused Hecke algebras).
In more details, after briefly introducing the Yang-Baxter equation and making connections with the braid equation via quantum groups, we will treat as motivating examples the two famous algebras for knots and links invariants, the Hecke algebra (HOMFLY-PT polynomial) and the BMW algebra (Kaufman polynomial). We will emphasize that they arise as centralizers of quantum groups representations, and that these centralizers are the interesting objects to consider for our purposes.
Then we will turn our attention towards the fusion procedure, which allows to construct new solutions of the Yang–Baxter equation starting from a given one. This old and (quite well-known in theoretical physics) procedure works originally at the level of quantum groups representations. So we will present the general idea, formulating it in a way that it can be applied to the centralizers themselves. Then we will restrict to a class of examples described recently, which allows already to produce a large family of interesting algebras containing finite-dimensional realization of the braid group. This will be the opportunity, if time allows, to present nice objects such as fused braids, fused permutations and finally fused Hecke algebras.

Rob Schneiderman (City University of New York)
An elementary introduction to Whitney towers in 4-manifolds

The general failure of the Whitney move for removing intersections among surfaces in 4 dimensions is one reason why 4-manifolds are so difficult to understand. Whitney towers are 2- complexes formed as iterated Whitney disks on immersed surfaces in 4-manifolds in an attempt to ‘measureÕ this failure, or to ‘approximate’ a successful Whitney move. Whitney towers are organized by unitrivalent trees which can represent invariants that obstruct the existence of homotopies of the underlying immersed surfaces to embeddings. For Whitney towers on immersed disks into the 4-ball these invariants are related to the Milnor invariants of the link on the boundary, and to certain higher-order Arf invariants. Analogous invariants can be formulated for immersed surfaces in arbitrary 4-manifolds, but these invariants are not generally well- understood. Many interesting questions about Whitney towers related to 3- and 4-dimensional geometric topology remain open, and the goal of these talks is to provide introductory paths to some of these questions involving link concordance and 2-spheres in 4-manifolds.

Intro to Whitney Towers I: Surfaces in 4-space
Describing and manipulating surfaces in 4-space, finger-moves and Whitney moves, Whitney tower definitions, Whitney tower goals, the 4-dimensional Jacobi identity.

Intro to Whitney towers II: Links bounding Whitney towers
Basic manipulations of Whitney towers, order n intersection invariants, the order n obstruction theory, relations with Milnor invariants, the higher-order Arf conjecture.

Intro to Whitney towers III: 2-spheres in 4-manifolds
The classical quadratic order 0 intersection form, the cubic order 1 intersection invariants, realization questions, pulling apart 2-spheres, order 2 questions.

Special Lecture in honor of Patrick Dehornoy

Mario Salvetti (Università di Pisa)
The proof of $K(\pi, 1)$ conjecture for Affine Artin groups.

We prove the $K(\pi, 1)$ conjecture for affine Artin groups: the complexified complement of an affine reflection arrangement is a classifying space.
Our proof is based on recent advancements in the theory of dual Coxeter and Artin groups, as well as on several new results and constructions.

Abstracts of Short Talks

Daniele Celoria (University of Oxford)
Double branched covers and HFK

Given a knot, one can consider its lift in the double branched cover. We will show how to extract several smooth concordance invariants from the knot Floer homology of this lift, with a focus on 2-bridge and alternating knots.

Irving Dai (MIT)
Cork involutions and Floer homology

We introduce a new set of Floer-theoretic invariants that can help detect corks. Our approach utilizes the algebraic language developed by Hendricks and Manolescu in their definition of involutive Heegaard Floer homology. We show that our invariants obstruct the extension of a given involution over any homology ball (rather than a particular contractible manifold). As an application, we construct some new families of corks and show that several known families of corks have this property. This is joint work with Matthew Hedden and Abhishek Mallick.

Guillaume Gandolfi (Univ. Caen)
Vassiliev invariants for virtual braids and Artin groups

Vassiliev invariants are a particular type of invariants of knots and braids whose connection with singular knots and singular braids has been established by Birman. On one hand, the braid groups are known to be Artin groups and on the other hand, they are also subgroups of virtual braid groups, the braided counterpart of virtual knots introduced by Kauffman. Recently, a notion of singularity has been defined for both the Artin groups by Corran and the virtual braid groups by Caprau, De la Pena and McGahan, which led naturally to the extension of the notion of Vassiliev invariants for these groups.

In this talk, I will explain how the theory of Vassiliev invariants is defined for classical braid groups, virtual braid groups and Artin groups and show that they are connected through a family of particular subgroups of virtual braid groups.

Alexandra Kjuchukova (MPIM)
Simply-connected manifolds as 3-fold covers of S^4

Let X be a four-manifold which admits a handle decomposition with no 1-handles and no 3- handles. We show that X is a 3-fold cover of S^4, branched along an embedded surface with at most 2 singularities. The theorem is proved using trisections, which allow us to extend 3-dimensional techniques of Hilden.

Daniel Lopez (Univ. Paris)
Kuperberg invariants and Reidemeister torsion

Kuperberg invariants are quantum invariants of closed 3-manifolds constructed from an arbitrary finite-dimensional Hopf algebra. In this talk we explain how these invariants, at least when the Hopf algebra is involutive, can be defined for a more general class of 3-manifolds, the so called balanced sutured 3-manifolds. Moreover, we show how for a specific Hopf algebra they reduce to Reidemeister torsion.

Maggie Miller (Princeton)
Dehn surgery on links and the Thurston norm

I will show that for L an n-component link ($n > 1$) with nonzero pairwise linking numbers (and nondegenerate Thurston norm, to be defined in the talk), a primitive norm-minimizing surface S in the link complement remains norm-minimizing after an induced Dehn surgery on L (in which the boundary of S can be capped off by disks) when $[S]$ lies outside of some codimension-1 set of rays in second homology. In particular, when $n = 2$ the surface remains norm-minimizing with finitely many exceptions. The proof involves constructing taut foliations on link complements.

Ricard Riba (UA Barcelone)
Trivial 2-cocycles and invariants of rational homology 3-spheres

In this talk we will show that all rational homology 3-spheres can be constructed from a Heegaard splitting of genus g and gluing map an element of the mod p Torelli group for a certain prime p and genus g sufficiently large. Later using the aforementioned result, we give a tool to construct invariants of rational homology 3-spheres from a family of trivial 2-cocycles on the (mod d) Torelli group, which will not give a unique invariant because of the existence of an invariant that makes to fail the unicity of the construction. Finally, if there is enough time, we will see some strategies to find suitable families of 2-cocycles in order to apply our tool and construct invariants of rational homology 3-spheres.

Stefano Riolo (Neuchatel)
Non-spin hyperbolic manifolds

Let's say that a smooth manifold is 'spin' if it admits a spin structure. Unlike higher-dimensional manifolds, every closed orientable surface or 3-manifold is spin. Let's now focus on hyperbolic manifolds. By the work of Deligne and Sullivan (1975), every finite-volume hyperbolic manifold is finitely covered by a spin manifold. On the other hand, we show that there exist non-spin closed orientable hyperbolic manifolds in all dimensions bigger than 3. The main ingredient is the construction of a hyperbolic 4-manifold with odd intersection form. Joint work with B. Martelli and L. Slavich

Abdoul Karim Sané (Univ. Grenoble Alpes)
Surgery graph on unicellular map

Unicellular maps are natural objects on surfaces and counting formulas have been proven for these objects. In this talk, we will show how to endow a geometric structure on the set of unicellular maps on a closed oriented surface: a graph coming from a topological operation.

Arthur Soulié (Univ. Glasgow)
Constructing linear representations for families of groups

The representation theory of braid groups is wild in the sense that there is no known classification schema. Hence it is useful to shape constructions of linear representations for such family of groups to understand its representation theory.

Long and Moody gave a construction on representations of braid groups which associates a representation of B_n with a representation of B_{n+1}, which complexifies the initial representation. I will present this construction and its generalizations from a functorial point of view and explain that each construction defines an endofunctor, called a Long-Moody functor. I will give generalizations of this construction to other families of groups.

I will also present joint work with Martin Palmer giving a unified functorial construction of homological representations of families of groups. For instance, this construction provides the well-known family of Lawrence-Bigelow representations for braid groups.

Seokbeom Yoon (Seoul National University)
A vanishing theorem for adjoint Reidemeister torsions

Motivated by some observations in M-theory, Gang conjectures that for a hyperbolic 3-manifold, the inverse sum of the adjoint Reidemeister torsions over certain representations vanishes. In this talk, we prove the conjecture for the figure-eight knot and present numerical verification for several knots. This is joint work with Dongmin Gang and Seonhwa Kim.