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Section A :
Physique théorique.

Infinite mass renormalization and the Z = 0 condition (*)

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ABSTRACT. — The consequences of the condition $\delta\nu^2 = \infty$ are examined. The results are compared with those obtained from the $Z = 0$ condition which we have discussed in a previous work. It is concluded that these two conditions ($\delta\nu^2 = \infty$ and $Z = 0$) are sufficient conditions to have polynomial relations between fields, but neither of them is necessary. It is also argued that the $\delta\nu^2 = \infty$ condition is not related to compositeness since it does not permit the elimination of a field from the theory, as is the case with the $Z = 0$ condition.

INTRODUCTION

In a previous paper [1] (to be referred as I from now on) we have studied the relation between Jouvet's $Z = 0$ condition [2] for compositeness and the HNZ (Haag-Nishijima-Zimmermann) construction. We shall discuss here Nishijima's $\delta\nu = \infty$ condition [3] which lead to similar results. Let us recall the field theory models, which we shall also use now, and the results of I. We use the same notation.

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The Yukaxa model was defined by the Lagrangian

$$(1)\quad \mathcal{L}_{0Y} = - \sum_{i=a,b} \frac{1}{2} Z_{Yi} \left(\partial_\mu \varphi_i(x)\, \partial_\mu \varphi_i(x) + m_i^2\, \varphi_i(x)^2\right) - \frac{1}{2} Z_c \left(\partial_\mu \varphi_c(x)\, \partial_\mu \varphi_c(x) + \mu^2 \varphi_c(x)^2\right) + Z_1\, g\, \varphi_c(x)\, \varphi_b(x)\, \varphi^a(x) - \sum_{i=a,b} \frac{1}{2} \delta\nu_i^2\, \varphi_i(x)^2 - \frac{1}{2} \delta\nu^2\, \varphi_c(x)^2$$

while the associated Fermi model was defined in terms of the boson fields $\varphi_a(x)$ and $\varphi_b(x)$ only with the coupling $\lambda_r^2\, \varphi_a^2\, \varphi_b^2$. The results in I can be summarized as follows: (*a*) When $Z_c \to 0$ [with Z_c given by Dyson's formula $Z_c = 1 + g^2\, \pi'_1(-\mu^2)$] one has that all vacuum expectation values (VEV) of T-products in the Yukawa theory (with the only exception of the *c*-particle propagator if $\delta\nu_l^2$ is finite)

$$(2)\quad \langle 0 \mid T\left(\varphi_c(x_1)\, \varphi_c(x_2) \ldots \varphi_c(x_n)\, \varphi_{i_1}(y_1)\, \varphi_{i_2}(y_2) \ldots \varphi_{i_k}(y_k)\right) \mid 0 \rangle, \quad i_j = a, b,$$

become equal to VEV in which $\varphi_c(x)$ is replaced by the expression

$$(3)\quad j_l(x) = \frac{Z_1\, g_l}{\delta\nu_l^2}\, \varphi_a(x)\, \varphi_b(x),$$

where the index l stands for the limit quantities and g_l is the assumed finite root of the equation $Z_c(g_l) = 0$. For the propagator one obtains ($\mathcal{F}$ stands for the Fourier transform)

$$(4)\quad \mathcal{F}\{\langle 0 \mid T(\varphi_c(x)\, \varphi_c(y)) \mid 0 \rangle = \frac{1}{\delta\nu_l^2} + \mathcal{F}\{\langle 0 \mid T(j_l(x)\, j_l(y)) \mid 0 \rangle\},$$

thus showing that both expressions have the same value when $\delta\nu_l^2 = \infty$. (*b*) When $\lambda_r = \frac{1}{2} \frac{(Z_1\, g_l)^2}{\delta\nu_l^2}$ the VEV of T-products in the Yukawa theory for $Z_c = 0$ containing only the operators $\varphi_a^Y(x)$ and $\varphi_b^Y(x)$ have the same value as the corresponding ones in the Fermi theory. Furthermore, one also has that the HNZ field B (x) of the Fermi model takes the form

$$(5)\quad B(x) = \frac{Z_1\, g_l}{\delta\nu_l^2}\, \varphi_a^F(x)\, \varphi_b^F(x).$$

From (*a*) and (*b*) one concludes that when the equivalence conditions $Z_c = 0$ and $\lambda_r = \frac{1}{2} \frac{(Z_1\, g_l)^2}{\delta\nu_l^2}$ are satisfied one has

$$(6)\quad \langle 0 \mid T\left(\varphi_c(x_1) \ldots \varphi_c(x_n)\, \varphi_{i_1}^Y(y_1) \ldots \varphi_{i_h}^Y(y_h)\right) \mid 0 \rangle = \langle 0 \mid T\left(B(x_1) \ldots B(x_n)\, \varphi_{i_1}^F(y_1) \ldots \varphi_{i_k}^F(y_k)\right) \mid 0 \rangle$$

with the only possible exception of the c-particle propagator if $\delta\nu_l^2$ is finite. We recall that all the above mentioned equalities hold on shell and off shell. We also remark that it follows from the discussion in I that the condition $\lambda_r = \frac{1}{2} \frac{(Z_1 g_l)^2}{\delta\nu_l^2}$ is equivalent to require that a one-particle state $|\Phi_{\mathbf{p}}\rangle$ of mass μ satisfying $\langle 0 | \varphi_a \varphi_b | \Phi_{\mathbf{p}} \rangle \neq 0$ exists in the Fermi theory.

In the first section of this work we shall give a direct proof of a result of Nishijima [3]. We shall prove that in the Yukawa theory one has that all VEV of T-products [*see* formula (2)] become equal to VEV in which $\varphi_c(x)$ is replaced by $\hat{\varphi}(x) = \frac{Z_1 g}{\delta\nu^2} \varphi_a(x) \varphi_b(x)$ (on shell and off shell) when $\delta\nu^2 = \infty$. This implies then that $Z = 0$ or $\delta\nu = \infty$ are sufficient conditions for statement (a) to hold, but of course neither of them is necessary.

In the second section we shall discuss the relations between the two conditions, their connection to polynomial field relations and to compositeness.

I. In this section we shall discuss Nishijima's condition explicitly in the Yukawa model. We can construct here an HNZ field for the c-particle because conditions (2) of I are all satisfied. Let $\hat{\varphi}(x)$ be the simplest HNZ field given by formula (3) of I

$$\hat{\varphi}(x) = \lim_{\substack{\xi \to 0 \\ \xi^2 \text{ space-like}}} \frac{T\left(\varphi_a\left(x + \frac{\xi}{2}\right) \varphi_b\left(x - \frac{\xi}{2}\right)\right)}{f_{\mathbf{k}}(\xi)} \tag{7}$$

with

$$f_{\mathbf{k}}(\xi) = \sqrt{(2\pi)^3 \, 2\,\omega_{\mathbf{k}}} \langle 0 | T\left(\varphi\left(\frac{\xi}{2}\right) \varphi_b\left(-\frac{\xi}{2}\right)\right) | \varphi_{\mathbf{k}}\rangle. \tag{8}$$

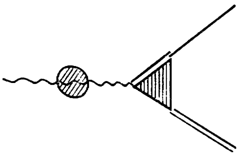
In contrast to the similar expressions in I all quantities here are Yukawa theory quantities and $|\Phi_{\mathbf{k}}\rangle$ is the one c-particle state of mass μ and momentum $\mathbf{k}$. Now, since in the Yukawa theory we originally have a field corresponding to the state $|\Phi_{\mathbf{p}}\rangle$, namely $\varphi_c(x)$, it is natural to ask for the relation between $\hat{\varphi}(x)$ and $\varphi_c(x)$. We shall look then for sufficient conditions to replace the field $\varphi_c(x)$ in any VEV of a T-product (τ-function) by the field $\hat{\varphi}(x)$. Let us recall that in I we proved that $Z_c = 0$ was a condition for so doing, except in the case of the c-propagator. We shall prove here that $\delta\nu = \infty$ also satisfies our requirements. We suppose, although we do not write it explicitly, that some cut-off Λ is introduced such that $\delta\nu_\Lambda$ is finite and $\delta\nu_\Lambda \to \infty$ when the cut-off is removed.

We want to prove then that

$$(9) \qquad \langle 0 | \mathrm{T}(\varphi_c(x)\,\mathrm{ABC}\ldots) | 0 \rangle$$

$$= \lim_{\xi \to 0} \frac{\langle 0 | \mathrm{T}\left(\varphi_a\left(x+\frac{\xi}{2}\right)\varphi_b\left(x-\frac{\xi}{2}\right)\mathrm{ABC}\ldots\right) | 0 \rangle}{f_{\mathrm{p}}(\xi)},$$

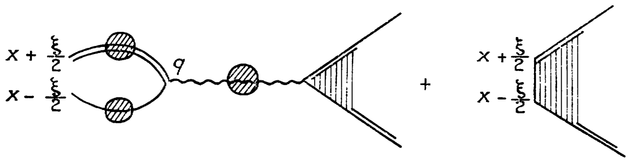
where A, B, C, ... are any fields. Let us consider first the case $\mathrm{AB} = \varphi_a\,\varphi_b$. We call $\mathrm{S}_0^1(p)$, $\mathrm{D}_0^1(p)$, $\Delta_0^1(p)$ the unrenormalized dressed propagators of φ_a, φ_b and φ_c, respectively (*see* I for notation). The left hand side of (9), graphically represented by



is given in momentum space by

$$(10) \qquad \frac{1}{(\mathrm{Z}_a\,\mathrm{Z}_b\,\mathrm{Z}_c)^{1/2}}\, g_0\,\Delta_0^1\,\Gamma_0\,\mathrm{S}_0^1\,\mathrm{D}_0^1 = g\,\Delta_{\mathrm{R}}^1\,\Gamma_{\mathrm{R}}\,\mathrm{S}_{\mathrm{R}}^1\,\mathrm{D}_{\mathrm{R}}^1,$$

where $\Gamma_{\mathrm{R}} = \mathrm{Z}_1\,\Gamma_0$ is the renormalized vertex function, and Γ_0 the unrenormalized one. We remark that we have omitted the arguments, which are quadridimensional impulsions, of the functions Δ_{R}^1, Γ_{R}, ..., since they play no role here. Unless confusion arises we shall do this in the rest of this work. On the other hand the right hand side of (9) is represented graphically [leaving aside $f_{\mathrm{p}}(\xi)$] by the sum of graphs



which have the value (once the limit $\xi \to 0$ has been taken)

$$(11) \qquad (\mathrm{Z}_1)^{-1}\,\Gamma_{\mathrm{R}}\,\mathrm{S}_{\mathrm{R}}^1\,\mathrm{D}_{\mathrm{R}}^1\,\{ g^2\,\pi_1(q^2)\,\Delta_{\mathrm{R}}^1 + 1 \}.$$

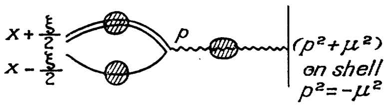
The function $\pi_1(p^2)$ has the value [*see* I, formula (27)] :

$$(12) \qquad g^2\,\pi_1(p^2) = \mathrm{Z}_1\,g^2 \int \mathrm{S}_{\mathrm{R}}^1(k)\,\mathrm{D}_{\mathrm{R}}^1(p-k)\,\Gamma_{\mathrm{R}}(p-k,k)\,d^4 k$$

and we have the relations

$$\delta\nu^2 = g^2\,\pi_1\,(-\,\mu^2), \qquad Z_c = 1 + g^2\,\pi'_1\,(-\,\mu^2), \tag{13}$$

where $\pi'_1\,(p^2) \equiv \dfrac{d\,\pi_1}{dp^2}\cdot$ The quantity $f_{\mathbf{p}}\,(\xi)$ is represented graphically by



and has the constant value

$$\lim_{\xi \to 0} f_{\mathbf{p}}\,(\xi) = \left[\frac{1}{(Z_a\,Z_b\,Z_c)^{1/2}}\,\pi\,(p^2)\,g_0\,\Delta_0^1\,.(p^2+\mu^2)\right]_{p^2=-\mu^2} = \frac{\delta\nu^2}{Z_1\,g}, \tag{14}$$

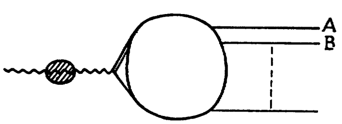
where $g_0^2\,\pi\,(p^2) = (Z_c)^{-1}\,g^2\,\pi_1\,(p^2)$. Using (11), (13) and (14) one gets for the right hand side of (9) the expression

$$\frac{g}{\delta\nu^2}\,\Gamma_R\,S_R^1\,D_R^1\,\{\,\Delta_R^1\,[\delta\nu^2 + (p^2+\mu^2)\,(Z_c - 1) + (p^2+\mu^2)^2\,g^2\,\pi_{2R}\,(p^2)] + 1\,\}, \tag{15}$$

where we have used the expansion

$$g^2\,\pi_1\,(p^2) = \delta\nu^2 + (p^2+\mu^2)\,(Z_c - 1) + (p^2+\mu^2)^2\,g^2\,\pi_{2R}\,(p^2). \tag{16}$$

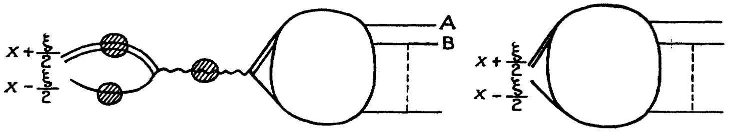
Comparing (10) and (15) we see that if $\delta\nu^2 = \infty$ then equality (9) will hold for finite p^2 when $AB = \varphi_a\,\varphi_b$. Let us generalise now this result to the case when ABC ... are any fields. We omit details since there are no essential differences with the previous calculation. The left-hand side of (9) can be graphically represented [*see* formulae (56) to (60) of I for a similar calculation] by the graph



which has the value (we do not consider here the case of the φ_c propagator)

$$(Z_c)^{-\frac{1}{2}}\,g_0\,\Delta_0^1\,C_0 = \frac{g\,Z_1}{(Z_a\,Z_b)^{1/2}}\,\Delta_R^1\,C_0, \tag{17}$$

where C_0 is defined by (17) and represents the circle of the graphical representation. Let us compute now the right-hand side of (9). Leaving $f_k(\xi)$ aside it can be represented by the sum of graphs



which have the value

$$(18) \qquad (Z_a Z_b)^{-1/2} C_0 (1 + g_0^2 \pi (p^2) \Delta_0^1).$$

Using (14) for the value of $f_p(\xi)$ we finally get for the right-hand side of (9)

$$(19) \qquad \frac{g Z_1}{(Z_a Z_b)^{1/2}} C_0 \frac{1}{\delta\nu^2} \{ \Delta_R^1 [\delta\nu^2 + (p^2 + \mu^2)(Z_c - 1) + (p^2 + \mu^2)^2 g^2 \pi_{2R}(p^2)] + 1 \}.$$

Comparison with (17) proves then equality (9) for this case when $\delta\nu^2 = \infty$. Let us remark that we are assuming that $\pi_{2R}(p^2)(\delta\nu^2)^{-1} \to 0$ when $\delta\nu^2 \to \infty$. This is true in particular if $\pi_{2R}(p^2)$ is finite, and this is the case in each order of perturbation theory. Indeed $\pi_{2R}(p^2)$ is the finite part of the self-energy loop and the renormalized c-particle propagator is expressed in terms of $\pi_{2R}(p^2)$ by

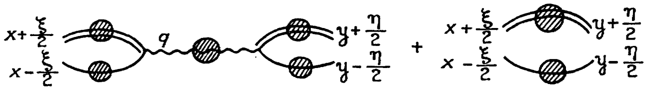
$$(20) \qquad \Delta_R^1(p^2) = \frac{1}{(p^2 + \mu^2)(1 - (p^2 + \mu^2) g^2 \pi_{2R}(p^2))}.$$

If the field $\varphi_c(x)$ appears again among the fields ABC ... in (9), we repeat the proof we have just made for each φ_c field.

Let us study now the case of the φ_c propagator. We want to prove the

$$(21) \quad \langle 0 | T(\varphi_c(x) \varphi_c(y)) | 0 \rangle = \lim_{\substack{\xi \to 0 \\ \eta \to 0}} \frac{\langle 0 | T\left(\varphi_a\left(x + \frac{\xi}{2}\right) \varphi_b\left(x - \frac{\xi}{2}\right) \varphi_a\left(y + \frac{\eta}{2}\right) \varphi_b\left(y - \frac{\eta}{2}\right)\right) | 0 \rangle}{f_p(\xi) f_k(\eta)}.$$

The left-hand side is just $\Delta_R^1(q)$. The numerator of the right-hand side is represented by the sum of graphs



which have the value (after the limits $\xi \to 0$, $\eta \to 0$)

$$(22) \quad \frac{1}{Z_a Z_b} [\pi (q^2)\, g_0^2\, \Delta_0^1\, \pi (q^2) + \pi (q^2)] = \frac{1}{(Z_1\, g)^2} [\Delta_R^1 (g^2\, \pi_1)^2 + g^3\, \pi_1 (q^2)].$$

Using (14) for the value of $f_p(\xi)$ and the expansion (16) for $g^2\, \pi_1 (p^2)$ we finally get for the right-hand side of (21)

$$(23) \quad \frac{1}{(\delta\nu^2)^2} \{ \Delta_R^1 (q) [\delta\nu^2 + (q^2 + \mu^2) (Z_c - 1) + (q^2 + \mu^2)^2\, g^2\, \pi_{2R} (q^2)]^2 + [\delta\nu^2 + (q^2 + \mu^2) (Z_c - 1) + (q^2 + \mu^2)^2\, g^2\, \pi_{2R} (q^2)] \}.$$

Inspection of this expression shows that it takes the value $\Delta_R^1 (q)$ when $\delta\nu^2 \to \infty$.

Thus we have proved that if $\delta\nu = \infty$, then $\varphi_c (x) = \hat{\varphi} (x)$ in the sense that one can replace $\varphi_c (x)$ by $\hat{\varphi} (x)$ in all τ-functions in momentum space without changing their values off-shell as well as on-shell. We recall that the important fact is that the equalities hold also off-shell, indeed if we are only interested in the on-shell S-matrix we can always replace $\varphi_c (x)$ by $\hat{\varphi} (x)$ in the τ-functions as a consequence of Borcher's theorem (*see* I, section 2). Let us give a direct proof of this statement. In the S-matrix in momentum space [*see* formula (6) of I] the τ-functions are multiplied by a factor $(p^2 + \mu^2)$ for each external φ_c leg. The value on-shell is obtained putting $p^2 = -\mu^2$. For the left-hand side of (9) we get from (17)

$$(24) \quad \frac{g\, Z_1}{(Z_a\, Z_b)^{1/2}} (p^2 + \mu^2)\, \Delta_R^1 (p)\, C_0 \Big|_{p^2 = -\mu^2} = \frac{g\, Z_1}{(Z_a\, Z_b)^{1/2}}\, C_0$$

since the residue of Δ_R^1 at the pole $p^2 = -\mu^2$ is one. The value of the right-hand side of (9) can be obtained directly from (19) multiplying by $(p^2 + \mu^2)$ and putting $p^2 = -\mu^2$. Using again $(p^2 + \mu^2)\, \Delta_R^1 (p) = 1$ on shell we get the value

$$(25) \quad \frac{g\, Z_1}{(Z_a\, Z_b)^{1/2}}\, C_0,$$

which is just (24). Let us remark that if we want to compare directly the τ-functions the proof is exactly the same, since on shell these functions have a pole and (24) and (25) prove that the residues are equal. It is clear that we can prove in the same way that expression (76) of I for the interpolating field, $\psi_c (x) = \varphi_c (x) + P (\Box_x) (\Box_x - \mu^2)\, \varphi_c (x)$, gives the same on-shell τ-functions, indeed the first term φ_c gives the original τ-function with the pole at $p^2 = -\mu^2$ while the second one is finite at $p^2 = -\mu^2$ because of the factor $(\Box_x - \mu^2)$ which in momentum space is $(p^2 + \mu^2)$ and cancels the pole.

II. In the previous section we have shown that the equality $\varphi_c(x) = \hat{\varphi}(x)$ holds if one is computing τ-functions (off-shell as well as on-shell) when $\delta\nu = \infty$. The similarity of this result with the analogous one when $Z_c = 0$ (*see* I) suggests the question of whether for $\delta\nu = \infty$ we can also relate the Yukawa model to a model with a direct Fermi coupling. Indeed the replacement of $\varphi_c(x)$ by $\hat{\varphi}(x)$ in the Yukawa Lagrangian, formula (1), does lead to a Fermi coupling, as it has been remarked by W. Zimmermann [4] who concludes then that we have a certain equivalence between both models ([4], p. 72). We shall see now that the situation when $\delta\nu = \infty$ is different from the $Z_c = 0$ one, and that no relation exists between the Fermi and the Yukawa models in this case. We shall work in the Lee model, which is sufficient for our purposes. In this case we have a Yukawa type coupling $g\,(V^+ N\,\theta + N^+ \theta^+ V)$ with V playing the role of φ_c, and a related Fermi model with the coupling $\lambda\, N^+ \theta^+ N\, \theta$. We call them the Yukawa Lee model and the Fermi Lee model respectively. For a detailed study of these two models and for a proof of their equivalence when $Z_V = 0$ (which can also be deduced from our results in I), *see* [5] and [6]. We shall use their notations here.

In the Yukawa Lee model the HNZ field for the V particle [7] written in momentum space is given by

$$\hat{b}_V(\vec{p}, t) = \frac{g}{4\,\pi^{3/2}\,\delta\nu_V} \int d^3 q\, d^3 k\, \delta(\vec{p} - \vec{q} - \vec{k}) \frac{f(\omega_k)}{\sqrt{\omega_k}}\, b_N(\vec{q}, t)\, a(\vec{k}, t), \tag{26}$$

where $\delta\nu_V = Z_V\, \delta m_V = Z_V\,(m_{V0} - m_V)$. The results of section 1, which can be explicitly verified here, show that if $\delta\nu_V = \infty$, then

$$b_V(p, t) = \hat{b}_V(p, t)$$

in the sense that we can replace b_V by $\hat{b}_V$ in the τ-functions. Now, replacement of b_V by $\hat{b}_V$ in the equations of motion of the fields $b_N(q, t)$ and $a(k, t)$ gives new equations of motion which can be formally derived from the Lagrangian

$$\begin{aligned} L_F = {} & i \int d^3 q\, b_N^+(\mathbf{q}, t)\, \dot{b}_N(\mathbf{q}, t) + i \int d^3 k\, a^+(\mathbf{k}, t)\, \dot{a}(\mathbf{k}, t) \\ & - m_N \int d^3 q\, b_N^+(\mathbf{q}, t)\, b_N(\mathbf{q}, t) - \int d^3 k\, \omega_{\mathbf{k}}\, a^+(\mathbf{k}, t)\, a(\mathbf{k}, t) \\ & + \frac{\lambda}{2\,(2\,\pi)^3} \int d^3 q_1\, d^3 q_2\, d^3 k_1\, d^3 k_2\, \delta(\mathbf{q}_1 + \mathbf{k}_1 - \mathbf{q}_2 - \mathbf{k}_2) \\ & \times \frac{f(\omega_{\mathbf{k}_1}) f(\omega_{\mathbf{k}_2})}{\sqrt{\omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2}}}\, b_N^+(\mathbf{q}_1, t)\, b_N(\mathbf{q}_2, t)\, a^+(\mathbf{k}_1, t)\, a(\mathbf{k}_2, t), \end{aligned} \tag{27}$$

with $\lambda = \frac{g^2}{\delta\nu_V}$. The Lagrangian (27) contains now a Fermi coupling $N^+ \theta^+ N \theta$, and this procedure of replacing b_V by $\hat{b}_V$ leads, as is well known, to the correct Lagrangian of the Fermi model which is equivalent to the original Yukawa model when $Z_V = 0$. Let us prove now that this is not the case when $\delta\nu_V = \infty$, $Z_V \neq 0$. Let us suppose that λ in (27) is a constant to be determined and let us see if we can fix its value so that a Fermi type theory as defined by (27) gives the same S-matrix as the former Yukawa one. We consider the $N\theta$ scattering (the only observable in the $N\theta$ sector if the V particle is stable, i. e. $m_V < m_N + \mu$), which is related to the τ-function $\langle 0 \mid T(N^+ \theta^+ N \theta) \mid 0 \rangle$. The corresponding amplitude in the Yukawa model will be proportional to the renormalized propagator $g^2 \Delta'_V(s)$ of the V-particle, i. e.

$$(28) \qquad T_Y(s) = g^2 \Delta'_V(s) = \frac{g^2}{(s + m_V)(1 - (s + m_V) g^2 B_R(s))},$$

The function $B_R(s)$ is defined by

$$(29) \quad g^2 B_0(s) = g^2 (B_0(-m_V) + (s + m_V) B'_0(-m_V) + (s + m_V)^2 B_R(s)),$$

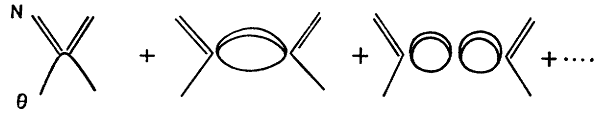
where $B_0(s)$ is the self-energy loop of the V-particle

$$(30) \qquad B_0(s) = \frac{1}{4\pi^2} \int_\mu^\infty d\omega \frac{|f(\omega)|^2 \sqrt{\omega^2 - \mu^2}}{s + m_N + \omega - i\varepsilon}.$$

One has

$$(31) \qquad \delta\nu_V = g^2 B_0(-m_V), \qquad Z_V = 1 + g^2 B'_0(-m_V).$$

On the other hand the $N\theta$ scattering amplitude in the Fermi theory will be given by the sum of graphs



which has the value

$$(32) \qquad T_F(s) = \lambda + \lambda^2 B_0(s) + \lambda^3 B_0(s)^2 + \ldots = \frac{\lambda}{1 - \lambda B_0(s)}.$$

Now, if we want to have the same function for $T_Y(s)$ and $T_F(s)$, then $T_F(s)$ must have a pole at $s = -m_V$ (this means that we impose in the Fermi theory the existence of a bound state of mass m_V). This gives

$$(33) \qquad \lambda = \frac{1}{B_0(-m_V)} = \frac{g^2}{\delta\nu_V},$$

i. e. the same value of λ obtained by direct replacement of b_V by $\hat{b}_V$ in the Lagrangian. The amplitude $T_F(s)$ takes then the form

$$T_F(s) = \frac{g^2}{\delta\nu_V - g^2 B_0(s)} \tag{34}$$

which can be written, using (31), as

$$T_F(s) = \frac{g^2}{(s + m_V)(1 - Z_V - g^2(s + m_V) B_R(s))}, \tag{35}$$

which is different from (28) when $Z_V \neq 0$. The condition $\delta\nu_V = \infty$ (which we have not used in this calculation since $\delta\nu_V$ is finally eliminated) does not relate then the two models in opposition to the $Z_V = 0$ condition which relates them. This fact can be directly verified here since $T_y(s)$ and $T_F(s)$ coincide when $Z_V = 0$. Let us remark that if we compare the V-particle propagators in both models, which are given by $(g^2)^{-1} T_y(s)$ and $(g^2)^{-1} T_F(s)$ respectively, we can see that even on shell, i. e. for $s = -m_V$, they do not coincide since the residues at the pole $s = -m_V$ are different. The reason for this is that the Yukawa one-particle state of mass m_V, $|V(\delta\nu_V \to \infty)\rangle_Y$, eigenvector of $H_Y(\delta\nu_V \to \infty)$, is different from the Fermi one-particle state with the same mass m_V, $|V(\delta\nu_V \to \infty)\rangle_Y$, eigenvector of $H_F(\delta\nu_V \to \infty)$. This implies that in general

$$\langle 0 | \hat{b}_V | V(\delta\nu_V \to \infty)\rangle_Y \neq \langle 0 | \hat{b}_V | V(\delta\nu_V \to \infty)\rangle_F$$

since the operator $\hat{b}_V$ is constructed in such a way that one has

$$\langle 0 | \hat{b}_V | V\rangle_Y = \langle 0 | b_V | V\rangle_Y.$$

The residues being determined by $\langle 0 | b_V | V\rangle_Y$ and $\langle 0 | \hat{b}_V | V\rangle_F$, we conclude that they are in general different. Indeed what happens here, when comparing with the situation which arises when $Z_V = 0$, is that statement (*a*) of the introduction is verified when $\delta\nu_V \to \infty$, while statement (*b*) is false.

We want to discuss now the independence of the conditions $\delta\nu = \infty$ and $Z_c = 0$. In the case of the Lee model one has explicit expressions for both quantities [*see* (31)] and one can verify that it is possible to realize independently both conditions. For the Yukawa model of section 1 the expressions corresponding to (31) [*see* I, formula (42), and [8]] are

$$\delta\nu^2 = g^2 \pi_1(-\mu^2) = \int_{s_0}^{\infty} \frac{F(a)\, da}{a - \mu^2} + C(a_0 - \mu^2), \tag{36}$$

$$Z_c(g^2) = 1 + g^2 \pi'_1(-\mu^2) = 1 - \int_{s_0}^{\infty} \frac{F(a)\, da}{(a - \mu^2)^2} - C. \tag{37}$$

We recall that F (a) is a positive weight function related to the Lehmann spectral function of the c-propagator by $\sigma(a) = F(a) \,|\Delta_R^1(-a)|^2$. Although we do not know the function F (a) it is clear that expressions (36) and (37) allow conditions $\delta\nu = \infty$ and $Z_c = 0$ to be realized independently.

Let us come back now to the Yukawa Lee model to consider other methods of obtaining the consequences of the $\delta\nu_V = \infty$ or the $Z_V = 0$ conditions. The equation of motion of the $b_V(p, t)$ field is

$$(38) \quad Z_V \left(i \frac{\partial}{\partial t} - m_V \right) b_V(\mathbf{p}, t) = \delta\nu_V \, b_V(\mathbf{p}, t) - \frac{g}{4 \pi^{3/2}} \int d^3 q \, d^3 k \, \delta(\mathbf{p} - \mathbf{q} - \mathbf{k}) \frac{f(\omega)_{\mathbf{k}}}{\sqrt{\omega_{\mathbf{k}}}} b_N(\mathbf{q}, t) \, a(\mathbf{k}, t).$$

Now, following [4], let us divide equation (38) by $\delta\nu_V$. Then, taking the limit $\delta\nu_V \to \infty$ we see that

$$(39) \quad b_V(\mathbf{p}, t) = \hat{b}_V(\mathbf{p}, t) = \frac{g}{4 \pi^{3/2} \delta\nu_V} \int d^3 q \, d^3 k \, \delta(\mathbf{p} - \mathbf{q} - \mathbf{k}) \frac{f(\omega_{\mathbf{k}})}{\sqrt{\omega_{\mathbf{k}}}} b_N(\mathbf{q}, t) \, a(\mathbf{k}, t)$$

if $(\delta\nu_V)^{-1} F(t) \to 0$, where F (t) is the left-hand side of the equation of motion (38). One should notice that this derivation must be understood in the weak sense that (39) is valid for matrix elements between vectors $|\,\alpha\rangle$ and $|\,\beta\rangle$ such that $(\delta\nu_V)^{-1} \langle \alpha \,|\, F(t) \,|\, \beta \rangle \to 0$. If one wants to use (39) to replace b_V by $\hat{b}_V$ in a product, for instance in the computation of $\langle 0 \,|\, \{ b_V(\vec{p}, t), b_V^+(\vec{p}, t') \} \,|\, 0 \rangle$, our last criterion must be modified accordingly. We have studied in detail this last problem [9] because it is essential to the understanding of some discussions of the $Z_V = 0$ condition as related to the $\delta\nu_V = \infty$ condition. The same argument we have just made can also be used to derive from (38) the equality $b_V = \hat{b}_V$ when $Z_V = 0$, and of course the same restrictions are valid.

Let us make some remarks now concerning polynomial relations between fields. We shall still work in the Lee model to fix ideas although the results also apply to our model of section 1. From our results it follows that both independent conditions $\delta\nu_V = \infty$ and $Z_V = 0$ imply the polynomial relation (39) between the three fields in the Yukawa Lee model (it is a polynomial relation in a local theory such as that of section 1). Because of this some attempts have been made to recover the condition $Z_V = 0$ from (39) ([10], [11], [12]), of course this is not possible since (39) may correspond either to $Z_V = 0$ or to $\delta\nu_V = \infty$. The paradoxical result arising in these proofs was pointed out by Brandt et al. [7] and we have shown in [9] that it can be understood precisely by taking into account our remark after formula (39), i. e. that (39) holds only in a weak sense.

We consider now the interpretation of the polynomial relation (39). As it has been remarked in [4] in the case $\delta\nu_V = \infty$ relation (39) holds simultaneously with the field equation (38) for $b_V(p, t)$. Indeed we have seen that it is not possible in this case to eliminate b_V and construct an equivalent Fermi theory. On the contrary in the $Z_V = 0$ case one can interpret (39) as an equation replacing the equation of motion for the field b_V in the precise sense that it is possible here to construct an equivalent Fermi theory as we have explained in I. In this sense we can say that the $Z_V = 0$ condition is related to compositeness (the field for the V-particle is constructed in terms of the N and θ fields in the Fermi model and has no independent equation of motion) while the $\delta\nu_V = \infty$ condition is not. We also remark that if it is true that both the $\delta\nu_V = \infty$ and the $Z_V = 0$ condition allow the replacement in the τ-functions (with the exception of the V-propagator in the second case) of b_V by $\hat{b}_V$, the values of these τ-functions are different in both cases since we have shown that when $Z_V = 0$ they take the values of the τ-functions of the Fermi theory while this is not the case when $\delta\nu_V = \infty$.

We have shown then that in the Yukawa Lee model (*mutatis mutandis* in the model of section 1) the field b_V can be replaced by $\hat{b}_V$ in the τ-functions (as it follows from the results in section 1), and in matrix elements satisfying certain conditions (as it follows from the derivation from the equation of motion). Let us finally remark that if both conditions $Z_V = 0$ and $\delta\nu_V = \infty$ are satisfied simultaneously, then all the τ-functions of the Yukawa model become equal to the corresponding τ-functions of the Fermi model since the term $(\delta\nu_V)^{-1}$ in the propagator vanishes.

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