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Covariant Wave-Equations, the Galilei Group, and the Magnetic Moment of the Electron

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ABSTRACT. — The construction of covariant unitary irreducible representations of the space-time groups is briefly reviewed, and a slight new degree of flexibility is introduced into the construction. The new flexibility is applied to the Galilei group, for which group it is best exhibited. One of the equations which emerges in a natural way from the formalism is an equation proposed some time ago by Lévy-Leblond in connection with the magnetic moment of the electron. It is argued that in spite of Lévy-Leblond's observation that the value $e/2m$ for the magnetic moment can be derived from this equation, the derivation is more ambiguous than in the original Dirac case.

RÉSUMÉ. — Après avoir été passée en revue, la construction des représentations unitaires irréductibles covariantes des groupes à espace-temps est légèrement modifiée dans le sens d'une flexibilité accrue. Cette dernière se révèle particulièrement avantageuse dans l'application au groupe de Galilée. Une équation proposée il y a quelque temps par Lévy-Leblond, en rapport avec le moment magnétique de l'électron, découle de manière naturelle du formalisme employé. On montre cependant que la dérivation par Lévy-Leblond de la valeur $e/2m$ du moment magnétique à partir de cette équation demeure plus ambiguë que dans le cas original de Dirac.

1. INTRODUCTION

« A free-field equation is nothing but an invariant record of which components are superfluous » (Weinberg [*1*]). This point of view has been taken in ref. [*2*], where it is shown that the continuous unitary irreducible representations (CUIR's) of any space-time group (or indeed of any semi-direct product group T ∧ G where T is abelian and G semi-simple) can be carried by wave-functions $\psi(p)$ which transform covariantly, and carry superfluous components which are eliminated by « free-field » equations. That is to say [*2*], the wave-functions $\psi(p)$ transform covariantly according to

$$(\mathrm{U}(g)\psi)(p) = \mathscr{D}(g)\psi(g^{-1}p), \tag{1.1}$$

where $\mathscr{D}$ is a linear, *p-independent*, representation of G, and satisfy a covariant wave-equation of the form

$$\mathrm{W}(p)\psi(p) = 0, \qquad \mathrm{W}(p) = \mathscr{D}^{-1}(g)\mathrm{W}\mathscr{D}(g), \tag{1.2}$$

where $\mathrm{W} = \mathrm{W}(p)$ is a projection operator for some fixed $\mathring{p}$, and $p = g^{-1}\mathring{p}$. $\mathrm{W}(p)$ depends only on p (not g) and is *covariant* (forminvariant) in the sense that it transforms as

$$\mathscr{D}^{-1}(g)\mathrm{W}(p)\mathscr{D}(g) = \mathrm{W}(g^{-1}p). \tag{1.3}$$

For the space-time groups p is the momentum restricted to G-orbit of $\mathring{p}$, but in general, it is identified with a coset space G/K where K (the little group of $\mathring{p}$) is a closed subgroup of G. The representation $\mathscr{D}$ is assumed to have unitary restriction $\mathscr{D} \downarrow \mathrm{K}$.

The advantage of using covariant wave-functions, apart from the obvious simplicity of the transformation law (1.1), is that the corresponding configuration space functions (Fourier transforms) transform locally. This property is obviously of great importance in local quantum field theory, for example in the implementation of crossing symmetry [*3*]. A disadvantage of the covariant wave-functions is that they introduce an ambiguity [*2*] in the choice of representation $\mathscr{D}$ of G which carries a given CUIR of T ∧ G. A lesser ambiguity is also present in the choice [*2*] of projection W. Although all choices of the pair $\{\mathscr{D}, \mathrm{W}\}$ have the same physical content, the wave-equations obtained with the different choices differ widely in form, in whether they incorporate orbital conditions such as $p^2 = m^2$ or $\mathrm{E} = \boldsymbol{p}^2/2m$, and they lead to physically different results when interactions are introduced.

In a sense the ambiguity present in the choice of $\{\mathscr{D}, \mathrm{W}\}$ may be regarded as an advantage, since it allows a certain degree of flexibility in the choice of wave-functions. Taking this point of view, the first purpose of the present note is to show that the flexibility may actually be increased a little. This is done (section 2) by generalizing the definition of W(p) in (1.2) to

$$W(p) = \Delta^{-1}(g)W\mathscr{D}(g), \tag{1.4}$$

where Δ is any representation of G such that

$$(i) \quad \dim \Delta = \dim \mathscr{D},$$
$$(ii) \quad \Delta(k)W = \mathscr{D}(k)W \qquad (k \in K).$$

The obvious candidates Δ to satisfy (i) are $\mathscr{D}$ itself, $\mathscr{D}^{\dagger -1}$, $\mathscr{D}^*$ and $\mathscr{D}^{*\dagger -1}$, and in many cases of interest at least one of the other three possibilities besides $\mathscr{D}$ satisfies (ii) also. However, as explained in section 2, the choices $\Delta = \mathscr{D}$ and $\Delta = \mathscr{D}^{\dagger -1}$ are particularly favoured.

The second purpose of this note is to apply the above generalization. It will not be applied to the Poincaré group, partly because the wave-functions are already too well-known [*2*], and partly because the pseudo-unitarity of the Lorentz group has the consequence that there is no essential distinction between the two choices of greatest interest, $\mathscr{D}$ and $\mathscr{D}^{\dagger -1}$. Instead, the generalization will be applied to the Galilei group [*4*], for which such a distinction does exist. For simplicity, and because the generalization to other spins is obvious, we consider only the spin $s = 1/2$ in detail. As in well-known [*2*], the 2-component wave function belonging to the non-faithful 2-dimensional representation of the homogeneous Galilei group has no superfluous components, and hence requires no wave-equation other than the orbital condition $E = \boldsymbol{p}^2/2m$, which is just the Schrödinger equation. The interesting case is when $\mathscr{D}$ is required to be *faithful*. Then the lowest dimension possible for $\mathscr{D}$ is 4, and $\psi(p)$ has two superfluous components. To eliminate these components, wave-equations are constructed using the two choices $\Delta = \mathscr{D}$ and $\Delta = \mathscr{D}^{\dagger -1}$. The wave-equations obtained are very different in form. The first one is fairly trivial, but the second one allows the inclusion of the orbital condition and then turns out to be just the wave-equation that was introduced some years ago by Lévy-Leblond [*5*] in connection with the non-relativistic magnetic moment. It is interesting that the Lévy-Leblond equation emerges naturally from the general formalism.

Lévy-Leblond's main motivation for introducing his equation was to argue that the value $e/2m$ for the magnetic moment of the electron could be obtained as readily from a Galilei invariant as from a Lorentz invariant wave-equation. The third purpose of this note is to point out that, while this is true, it is true also of the Schrödinger equation, and that for all Galilei invariant equations, there is an ambiguity which is absent in the Lorentz case. Since the ambiguity can be traced to the structure of the Galilei group, it is concluded that the derivation of $e/2m$ is more compelling in the Lorentz (Dirac) case.

2. GENERALIZATION OF THE WAVE OPERATOR W(p)

The superfluous components of the wave-functions $\psi(p)$ are eliminated [2] by choosing a fixed vector $\mathring{p}$ and projecting them to zero in $\psi(\mathring{p})$ with a suitable projection operator:

$$W\psi(\mathring{p}) = 0. \tag{2.1}$$

The question is only how the equation (2.1) should be made covariant. Usually [2] this is done by noting that (2.1) holds for all $\psi(\mathring{p})$, hence in particular for $(U(g)\psi)(\mathring{p})$ so that also

$$W(U(g)\psi)(\mathring{p}) = W\mathscr{D}(g)\psi(p) = 0, \qquad (p = g^{-1}\mathring{p}), \tag{2.2}$$

and then multiplying the second equation by $\mathscr{D}^{-1}(g)$ to obtain (1.2). The resultant operator $W(p)$ in (1.2) depends only on p because

$$\mathscr{D}^{-1}(kg)W\mathscr{D}(kg) = \mathscr{D}^{-1}(g)\mathscr{D}(k)W\mathscr{D}(k)\mathscr{D}(g) = \mathscr{D}^{-1}(g)W\mathscr{D}(g), \tag{2.3}$$

where k belongs to the little group K of $\mathring{p}$ and therefore commutes with W, and because p is identified with G/K. Furthermore $W(p)$ is covariant (satisfies (1.3)) by definition.

To generalize $W(p)$, we multiply the second equation in (2.2) by $\Delta^{-1}(g)$ instead of $\mathscr{D}^{-1}(g)$, where Δ is any representation of G of the same dimension as $\mathscr{D}$, to obtain

$$\hat{W}(p)\psi(p) = 0, \qquad \hat{W}(p) = \Delta^{-1}(g)W\mathscr{D}(g),$$

and then demand that $\hat{W}(p)$ depend only on p and transform covariantly. In analogy with (2.3), one easily sees that the condition that $\hat{W}(p)$ depend only on p is

$$\Delta^{-1}(k)W\mathscr{D}(k) = W, \qquad (k \in K). \tag{2.4}$$

Since $\mathscr{D}(k)$ and W commute, this is just the condition that $\Delta(k)$ coincide with $\mathscr{D}(k)$ on W, and hence is condition (*ii*) of the introduction. The condition that $\hat{W}(p)$ transform covariantly is

$$\Delta^{-1}(g)\hat{W}(p)\mathscr{D}(g) = \hat{W}(g^{-1}p), \tag{2.5}$$

and is easily seen to be automatically satisfied.

The obvious candidates for representations of G of the same dimension as $\mathscr{D}$ are $\mathscr{D}$ itself, $\mathscr{D}^{\dagger -1}$, $\mathscr{D}^*$ and $\mathscr{D}^{*\dagger -1}$, but, of course, not all of these representations need be inequivalent. There is no *a priori* guarantee that any Δ except $\mathscr{D}$ itself will satisfy the condition (2.4). However, since the restriction $\mathscr{D} \downarrow K$ is assumed to be unitary, the representation $\mathscr{D}^{\dagger -1}$ satisfies (2.4) and can always be added to $\mathscr{D}$ as a possible candidate for Δ.

The candidates $\mathscr{D}$ and $\mathscr{D}^{\dagger -1}$ have other special properties. First, if we note that $W = W(\mathring{p})$ is a projection operator, and therefore satisfies

$$W^2 = W, \qquad W^\dagger = W, \tag{2.6}$$

we see that the choice $\Delta = \mathscr{D}$ preserves the first of these properties,

$$W^2(p) = W(p), \qquad W(p) = \mathscr{D}^{-1}(g)W\mathscr{D}(g), \tag{2.7}$$

while the choice $\Delta = \mathscr{D}^{\dagger -1}$ preserves the second;

$$\hat{W}^\dagger(p) = \hat{W}(p), \qquad \hat{W}(p) = \mathscr{D}^\dagger(g)W\mathscr{D}(g). \tag{2.8}$$

Only if $\mathscr{D}(g)$ is unitary are both properties preserved and is $W(p) = \hat{W}(p)$ a true projection operator. In practice (e. g. in the Lorentz and Galilei cases) $\mathscr{D}$ is not unitary except in the restriction $\mathscr{D} \downarrow K$.

Another property of the choice $\Delta = \mathscr{D}$ is that if we write (2.1) in the more positive form

$$Q\psi(\mathring{p}) = \psi(\mathring{p}), \qquad Q = 1 - W, \tag{2.9}$$

where Q is the projection onto the components of $\psi(\mathring{p})$ which are *not* zero (the notation used in ref. [*2*]), then this form is preserved:

$$Q(p)\psi(p) = \psi(p), \quad Q(p) = \mathscr{D}^{-1}(g)Q\mathscr{D}(g), \qquad p = g^{-1}\mathring{p}. \tag{2.10}$$

On the other hand, a special property of the choice $\Delta = \mathscr{D}^{\dagger -1}$ is that the inner product for the wave-functions, which is [*2*]

$$(\psi_1, \psi_2) = \int d\mu(p)\psi_1^\dagger(p)\mathscr{D}^\dagger(g)\mathscr{D}(g)\psi_2(p), \qquad p = g^{-1}\mathring{p}, \tag{2.11}$$

where $d\mu(p)$ is the invariant measure, can be written in the simpler form

$$(\psi_1, \psi_2) = \int d\mu(p)\psi_1^\dagger(p)\hat{Q}(p)\psi_2(p), \qquad \hat{Q}(p) = \mathscr{D}^\dagger(g)Q\mathscr{D}(g). \tag{2.12}$$

Thus, the choices $\Delta = \mathscr{D}$ and $\Delta = \mathscr{D}^{\dagger -1}$ have special properties which are complementary to each other.

3. APPLICATION TO THE GALILEI GROUP

As in the case of the Poincaré group, the homogeneous and translation parts of the Galilei group $\mathscr{G}$ are parametrized by $(\boldsymbol{v}, R)$ and $(b, \boldsymbol{a})$ respectively, where $\boldsymbol{v}$ are the boosts, R the rotations, and $(b, \boldsymbol{a})$ the time and space translations. Only the group structure is different [*4*]. For the massive orbits, characterized by $E = \boldsymbol{p}^2/2m$, where E is the energy and $\boldsymbol{p}$ the 3-momentum the little group is the rotation group.

To obtain the covariant (projective) CUIR'S of $\mathscr{G}$, one must first choose a representation $\mathscr{D}(\boldsymbol{v}, R)$ of the homogeneous part, and a projection $1 - W$ onto one of the spin representations $D^s(R)$ of the rotation group contained in $\mathscr{D}(0, R)$. Then the covariant transformation law for the wave-functions is

$$(U(b, \boldsymbol{a}, \boldsymbol{v}, R)\psi)(\boldsymbol{p}) = e^{i(Eb - \boldsymbol{p}\cdot\boldsymbol{a})}\mathscr{D}(\boldsymbol{v}, R)\psi(R^{-1}(\boldsymbol{p} - m\boldsymbol{v})), \tag{3.1}$$

the wave operator is

$$W(\boldsymbol{p}) = \mathscr{D}^{-1}(-\boldsymbol{p}/m, 1)W\mathscr{D}(-\boldsymbol{p}/m, 1), \tag{3.2}$$

or

$$\hat{W}(\boldsymbol{p}) = \mathscr{D}^{\dagger}(-\boldsymbol{p}/m, 1)W\mathscr{D}(-\boldsymbol{p}/m, 1), \tag{3.3}$$

according as we choose $\Delta = \mathscr{D}$ or $\mathscr{D}^{\dagger -1}$ respectively, and the inner product is in any case

$$(\psi_1, \psi_2) = \int d^3p\psi_1^{\dagger}(\boldsymbol{p})W(\boldsymbol{p})\psi_2(\boldsymbol{p}). \tag{3.4}$$

Because of the semi-direct product structure of the homogeneous part $\{ \boldsymbol{v}, R \}$ of the Galilei group, the representation D^s of the rotation group is already a representation of $\{ \boldsymbol{v}, R \}$, though, of course, a non-faithful one. Hence, the simplest choice of $\mathscr{D}$ is simply D^s, and this choice requires no wave-equation other than the orbital condition, since then $W(\boldsymbol{p}) = W = 1$. For this reason D^s is the usual choice made in the literature, and for $s = 1/2$, it leads to the 2-component Schrödinger equation as orbital condition.

It is interesting, however, to consider also the case when $\mathscr{D}$ is required to be faithful. Then $\mathscr{D}(0, R)$ is necessarily reducible and a wave-equation becomes necessary. For simplicity, and because the generalization to other spins and other faithful representations will be obvious, we shall consider here only the lowest-dimensional faithful representations for spin 1/2, which are

$$\mathscr{D}(\boldsymbol{v}, R) = \begin{pmatrix} D^{\frac{1}{2}}(R) & 0 \\ \varkappa\boldsymbol{\sigma}\cdot\boldsymbol{v}D^{\frac{1}{2}}(R) & D^{\frac{1}{2}}(R) \end{pmatrix}, \tag{3.5}$$

where $\boldsymbol{\sigma}$ are the Pauli matrices and $\varkappa$ is an arbitrary parameter. The corresponding wave-functions are written as

$$\psi(\boldsymbol{p}) = (\varphi(\boldsymbol{p}), \chi(\boldsymbol{p})), \tag{3.6}$$

where φ and χ are 2-component spinors. One next has to choose a projection onto a single representation $D^{\frac{1}{2}}(R)$ and the obvious choice is

$$W = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}. \tag{3.7}$$

(The other simple choice, $1 - W$, would imply $\chi = 0$ for all $\boldsymbol{p}$ and hence lead back to the non-faithful case.)

Let us now consider what wave-equations we obtain from (3.2) and (3.3). By direct computation, one easily finds that

$$W(\boldsymbol{p})\psi(\boldsymbol{p}) = 0 \Rightarrow \chi(\boldsymbol{p}) = \frac{\varkappa}{m}\boldsymbol{\sigma}\cdot\boldsymbol{p}\varphi(\boldsymbol{p}), \tag{3.8}$$

$$\hat{W}(\boldsymbol{p})\psi(\boldsymbol{p}) = 0 \Rightarrow \begin{pmatrix} -\varkappa p^2/m & \boldsymbol{\sigma}\cdot\boldsymbol{p} \\ \boldsymbol{\sigma}\cdot\boldsymbol{p} & -m/\varkappa \end{pmatrix}\begin{pmatrix} \varphi(\boldsymbol{p}) \\ \chi(\boldsymbol{p}) \end{pmatrix} = 0. \tag{3.9}$$

Note that, since any $\varkappa \neq 0$ can be absorbed by a renormalization of χ, the family of representations characterized by $\varkappa$ all lead to the same two equations. Both equations (3.8) and (3.9) have the same content for the free fields. But they are very different in form, and, in particular, because it contains $\boldsymbol{p}^2$ explicitly, (3.9) allows the inclusions of the orbital condition by changing it to

$$\begin{pmatrix} \mathrm{E} & \boldsymbol{\sigma}\cdot\boldsymbol{p} \\ \boldsymbol{\sigma}\cdot\boldsymbol{p} & 2m \end{pmatrix} \begin{pmatrix} \varphi(\boldsymbol{p}) \\ \chi(\boldsymbol{p}) \end{pmatrix} = 0, \tag{3.10}$$

where we have put $\varkappa = -1/2$.

Equation (3.10) is the equation introduced by Lévy-Leblond [5]. In the sense that it is 4-component and includes the orbital condition, it may be thought of as the Galilean analogue of the Dirac equation. However, it differs from the Dirac equation in one important respect, namely that if the two components χ are eliminated by $\chi = -(1/2m)\boldsymbol{\sigma}\cdot\boldsymbol{p}\varphi$, the remaining two components φ still transform *covariantly:*

$$(\mathrm{U}(\boldsymbol{v}, \mathrm{R})\varphi)(\boldsymbol{p}) = \mathrm{D}^{\frac{1}{2}}(\mathrm{R})\varphi(\mathrm{R}^{-1}(\boldsymbol{p} - m\boldsymbol{v})). \tag{3.11}$$

This follows at once from the triangular nature of the representation (3.5) which in turn follows from the semi-direct product structure of the homogeneous Galilei group. In contrast, if we eliminate two components of the Dirac wave-function (by a Foldy-Wouthuysen transformation [6], for example), then the remaining two components do not transform covariantly, but rather according to

$$(\mathrm{U}(\Lambda)\varphi)(p) = \mathrm{D}^{\frac{1}{2}}(\mathrm{R}(\Lambda, p))\varphi(\Lambda^{-1}p), \tag{3.12}$$

where $\mathrm{R}(\Lambda, p)$ is a Wigner rotation which depends heavily on p. That this difference between (3.10) and the Dirac equation can have a physical significance will be seen in the next section.

Finally, we note that for (3.10) the inner product becomes simply

$$(\psi_1, \psi_2) = \int d^3p\varphi_1^\dagger(\boldsymbol{p})\varphi_2(\boldsymbol{p}), \tag{3.13}$$

and hence does not depend on χ and is a simple local function of the 2-component wave-function φ. In this respect also (3.10) differs from the Dirac equation.

4. THE MAGNETIC MOMENT OF THE ELECTRON

It is well-known that the value $e/2m$ for the magnetic moment of the electron can be obtained by applying the minimal principle

$$(\mathrm{E}, \boldsymbol{p}) \rightarrow (\mathrm{E} - e\Phi, \boldsymbol{p} - e\mathbf{A})$$

to the Dirac equation [7]. Lévy-Leblond has pointed out that the same is true of the Galilei invariant equation (3.10), since, if one applies to it the minimal principle, one obtains

$$\begin{pmatrix} E - e\Phi & \sigma\cdot(p - eA) \\ \sigma\cdot(p - eA) & 2m \end{pmatrix}\begin{pmatrix} \varphi(p) \\ \chi(p) \end{pmatrix} = 0, \tag{4.1}$$

and hence the Pauli equation [8]

$$E\varphi(p) = \left[\frac{1}{2m}(p - eA)^2 - \frac{e}{2m}\sigma\cdot B + e\Phi\right]\varphi(p). \tag{4.2}$$

He concludes that the value $e/2m$ is not, as is often stated, a strictly Lorentzian effect, but is also a Galilean effect. We wish to point out that, while this is true, it is equally true for the Schrödinger equation, and that in both Galilean cases ((3.10) and the Schrödinger equation) there is an ambiguity that is not present in the Dirac case.

The point concerning the Schrödinger equation is rather trivial and consists in observing that the 2-component, spin 1/2, free Schrödinger equation can always be written in the form

$$E\varphi(p) = \frac{1}{2m}(\sigma\cdot p)^2\varphi(p), \tag{4.3}$$

and that if we apply the minimal principle to this equation, we immediately obtain (4.2). This result is hardly surprising since it is easy to see that (4.3) is exactly equivalent to (3.10), but it emphasizes the fact that the real role of the Lévy-Leblond equation (3.10) is to prevent $(\sigma\cdot p)^2$ being set equal to p^2 until *after* the minimal principle has been applied.

Incidentally, the fact that the value $e/2m$ can be obtained from the spin 1/2 Schrödinger equation might suggest that it is even a Euclidean effect. This is not so. The reason is that the vector (Φ, A) of the electromagnetic potential is reducible with respect to the Euclidean group, in contrast to the Lorentz and Galilei group with respect to which it is irreducible and not fully reducible respectively. Hence, the use, in the minimal principle, of the *same* charge e for Φ and A is already demanded by either Lorentz or Galilei invariance but not by Euclidean invariance.

The second point we wish to make, namely, that the Galilei case shows an ambiguity which is not present in the Lorentz case, follows from the fact that the 2-component Schrödinger wave-function φ in (4.1) or (4.3) transforms *covariantly* with respect to the Galilei group and allows the linear implementation of *parity*. That is, φ has the transformation laws

$$\begin{aligned}(U(v, R)\varphi)(p) &= D^{\frac{1}{2}}(R)\varphi(R^{-1}(p - mv))\varphi(p), \\ (\Pi\varphi)(p) &= \varphi(-p),\end{aligned} \tag{4.4}$$

where $D^{\frac{1}{2}}(R)$ is independent of $\boldsymbol{p}$, and Π is the parity operator. Furthermore, both equations in (4.4) are compatible with the replacement $(\boldsymbol{\sigma}\cdot\boldsymbol{p})^2 \to \boldsymbol{p}^2$. But now, according to whether the minimal principle is applied to the 4-component wave-equation (3.10) or to the conventional 2-component Schrödinger equation ((4.3) with $(\boldsymbol{\sigma}\cdot\boldsymbol{p})^2 \to \boldsymbol{p}^2$) the value obtained for the magnetic moment is $e/2m$ or zero, respectively. Thus, in the Galilei case, there are two possible values of the magnetic moment, and each is compatible with covariance and linear parity.

In contrast, in the Lorentz case, if the Dirac wave-function is reduced to two components, then, as discussed in the last section, the resultant 2-component covariant wave-function [9] is used, then it belongs to the D(1/2, 0) or D(0, 1/2) representation of SL(2, C) and does not allow [2] the linear implementation of parity. Thus, the demand that the wave-functions be of minimal spinorial rank compatible with covariant transformation and linear implementation of parity singles out the Dirac equation but, in the Galilei case, does not single out the Lévy-Leblond equation (3.10). Hence, the derivation of the value $e/2m$ for the magnetic moment is more compelling in the Lorentz case than it is in the Galilei case.

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