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The canonical structure of generalized non-linear sigma models in constrained Hamiltonian formalism

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ABSTRACT. — The canonical structures of a generalized non-linear σ model and a generalized $\mathbb{C}P^{n-1}$ model are investigated in the constraint Hamiltonian formalism due to Dirac. The real and complex scalar fields are defined over the Grassmann manifolds $O(N)/O(p) \otimes O(N-p)$ and $U(N)/U(p) \otimes U(N-p)$, respectively. The Dirac brackets between canonical variables and the total Hamiltonian density are computed in the axial gauge as well as in the unitary gauge. The canonical and path integral quantization prescriptions are discussed.

RÉSUMÉ. — On étudie les structures canoniques d'un modèle σ non linéaire généralisé et d'un modèle $\mathbb{C}P^{n-1}$ généralisé, dans le formalisme Hamiltonien avec contraintes dû à Dirac. On définit les champs scalaires réel et complexe sur les variétés Grassmanniennes $O(N)/O(p) \otimes O(N-p)$ et $U(N)/U(p) \otimes U(n-p)$ respectivement. On calcule les crochets de Dirac entre les variables canoniques et la densité de Hamiltonien totale en jauge axiale ainsi qu'en jauge unitaire. On discute les prescriptions de quantification canonique et par intégrale de chemins.

1. INTRODUCTION

The chiral models [1] in two space-time dimensions possess several interesting characteristics similar to those of Yang-Mills field theories,

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e. g. asymptotic freedom, confinement, topological charge, and non-perturbative particle spectrum. Furthermore, the chiral models admit an infinite sequence of conservation laws [*2*] and consequently they are known to be completely integrable systems at the classical level. The loop-space formulation of non-Abelian gauge theory [*3*] and the self-dual Yang-Mills theory [*4*] have been investigated in detail and they possess an infinite set of conserved currents. These developments bring the two theories even closer. Recently, it has been claimed [*5*] that the hidden symmetries responsible for the existence of the set of non-local charges have been identified.

The chiral models are constraint systems and they are described by singular Lagrangians. Therefore, the conventional method of quantization is not applicable. However, the constraint Hamiltonian formalism due to Dirac [*6*] is a very elegant technique to investigate systems with constraints in general and gauge theories [*7*] (such as Yang-Mills fields, strings, gravitation, etc.) in particular. The first step is to determine all constraints of the given system and then choose an appropriate quantization procedure — either canonical quantization or path integral quantization.

The purpose of this article is to investigate the constraints and canonical structure of the generalized non-linear σ model in one-space and one-time dimensions. The Lagrangian densities describing the models (see below) not only possess global symmetries but also are invariant under local non-Abelian gauge transformations [*8*]. Furthermore, the gauge fields are introduced as the composite (auxiliary) fields and they acquire kinetic energy dynamically owing to quantum fluctuations [*9*]. First we determine all constraints; primary and secondary. Then it is found that the generators of the non-Abelian gauge transformations form a set of first-class constraints and the rest are all second-class ones.

Our main purpose is to study the canonical structure [*10*] of the models in various gauges. Therefore, we introduce the primary Dirac brackets to eliminate the second-class constraints. Now the primary constraints together with the gauge conditions form a set of second-class constraints. We investigate the canonical structure of the models in the axial gauge as well as in the unitary gauge and obtain the total Hamiltonian explicitly in both the gauges. We find that the Hamiltonian density is quadratic in the momenta in the two gauges mentioned above. Therefore, in the path integral quantization the momenta can be integrated.

The rest of the paper is arranged as follows: we introduce the model in Section 2. The Dirac constraint formalism is applied to the models in Section 3 and in Section 4. The quantization schemes are discussed in Section 5. Finally, Section 6 contains discussions and the summary of our results. There is an appendix which lists some of the useful formulae.

2. THE MODELS

The chiral model Lagrangian density is

$$\mathscr{L} = \frac{1}{2} \operatorname{Tr} \left[\partial_\mu g(x) \partial_\mu g^{-1}(x) \right] \tag{1}$$

where $g(x)$ belongs to some compact simple Lie groupe G in a matrix representation. When $g(x)$ varies over the whole group G the action is invariant under the global transformations of $G \otimes G$: $g(x) \rightarrow h_1 g(x) h_2$.

There is another class of models defined as follows: let the matrices vary over a subset F of the group G such that a given element $g_0 \in G$ satisfies the condition

$$g_0 g_0^* = \lambda \mathbb{1} \tag{2}$$

where λ is either real or a complex number. The star operation is an involutive automorphism of the group such that $(g_1 g_2)^* = g_1^* g_2^*$ and $(g^*)^* = g$. The subset F is defined in the following way: If $g(x) \in F$ and $U(x)$ is an arbitrary element of G then:

$$g(x) = U^{-1*}(x) g_0 U(x) . \tag{3}$$

It follows that $g(x)g^*(x) = \lambda H$. Let H be a subgroup of G, $h(x) \in G$, such that $h(x)$ leaves g_0 invariant, i. e. $g_0 = h^{-1*} g_0 h \Leftrightarrow h \in H$. The elements $g(x)$ are invariant under left multiplication of $U(x)$ by $h(x) \in H$ and they belong to the coset space G/H. H is also a subgroup of G under involutive automorphism $U \rightarrow \tilde{U} = g_0^{-1} U^* g_0$. The Lagrangian density on G/H is

$$\mathscr{L} = \frac{1}{2} \operatorname{Tr} \left[\partial_\mu g(x) \partial_\mu g^{-1}(x) \right] \tag{4}$$

The two models under consideration are defined as follows:

i) $G = O(N)$, $\lambda = 1$; choose g_0 to be diagonal such that it has p eigenvalues $+1$ and $N - p$ eigenvalues equal to -1. The subgroup that leaves g_0 invariant is $O(p) \otimes O(N - p)$. Notice that the case $p = 1$ or $p = N - 1$ corresponds to the well-known non-linear σ model. It is useful to introduce a set of projectors $P(x)$ such that $g(x) = 2P(x) - 1$. We can write $P(x)$ as

$$P^{\alpha\beta}(x) = \phi_i^\alpha(x) \phi_i^\beta(x) , \qquad \alpha, \beta = 1, 2, \ldots N \quad i = 1, 2, \ldots p \tag{5}$$

where $\phi_i^\alpha(x)$ are a set of real orthonormal basis vectors with the property

$$\phi_i^\alpha(x) \phi_j^\alpha(x) = \delta_{ij} \tag{6}$$

We adopt the convention of summation over repeated indices throughout this paper. Notice that the right-hand side of Eq. (6) carries dimension since the fields $\phi_i^\alpha(x)$ do. Conventionally a coupling constant g/p is introduced, where g carries dimension, besides the factor δ_{ij} on the right-hand side of Eq. (6) in usual 1/N ($1/p$ in our case) expansions. However, we do not introduce a coupling constant explicitly for the sake of simplicity.

The Lagrangian density is

$$\mathscr{L} = \frac{1}{2}\left[\partial_\mu\phi_i^\alpha(x)\partial_\mu\phi_i^\alpha(x) - \partial_\mu\phi_i^\alpha(x)\partial_\mu\phi_i^\beta(x)\phi_j^\alpha(x)\phi_j^\beta(x)\right] \tag{7}$$

The fields are subject to the constraint (6).

ii) The second model corresponds to the choice G = U(N), and the subgroup that leaves g_0 invariant is $U(p) \otimes U(N-p)$ (again we choose g_0 to be diagonal with $|\lambda| = 1$).

$$P^{\alpha\beta}(x) = Z_i^{\alpha*}(x)Z_i^\beta(x), \qquad \alpha, \beta = 1, 2, \ldots N \quad i = 1, 2, \ldots p \tag{8}$$

Here $Z_i^\alpha(x)$ are complex scalar fields subject to the constraints

$$Z_i^{\alpha*}(x)Z_j^\alpha(x) = \delta_{ij} \tag{9}$$

and the Lagrangian density is

$$\mathscr{L} = \partial_\mu Z_i^{\alpha*}(x)\partial_\mu Z_i^\alpha(x) - Z_i^\beta(x)\partial_\mu Z_j^{\beta*}(x)Z_i^{\alpha*}(x)\partial_\mu Z_j^\alpha(x) \tag{10}$$

The Lagrangian density (7) is invariant under global O(N) transformations and local $O(p)$ gauge transformations, where as the Lagrangian of Eq. (10) possesses global U(N) and local $U(p)$ invariance.

The constraints (6) and (9) can be incorporated in the Lagrangians (7) and (10) by introducing Lagrangian multiplier fields and treating the field variables $\phi_i^\alpha(x)$, $Z_i^\alpha(x)$, and $Z_i^{\alpha*}(x)$, as unconstrained variables. Thus we rewrite the Lagrangian densities as

$$\mathscr{L} = \frac{1}{2}\left[\partial_\mu\phi_i^\alpha(x)\partial_\mu\phi_i^\alpha(x) - \partial_\mu\phi_i^\alpha(x)\partial_\mu\phi_i^\beta(x)\phi_j^\alpha(x)\phi_j^\beta(x)\right] - \lambda_{ij}(x)\left[\phi_i^\alpha(x)\phi_j^\alpha(x) - \delta_{ij}\right] \tag{11}$$

and

$$\mathscr{L} = \partial_\mu Z_i^{\alpha*}(x)\partial_\mu Z_i^\alpha(x) - Z_i^\beta(x)\partial_\mu Z_j^{\beta*}(x)Z_i^{\alpha*}(x)\partial_\mu Z_j^\alpha(x) - \chi_{ij}(x)\left[Z_i^{\alpha*}(x)Z_j^\alpha(x) - \delta_{ij}\right] \tag{12}$$

where $\lambda_{ij}(x)$ and $\chi_{ij}(x)$ are the Lagrangian multiplier fields.

3. CONSTRAINT HAMILTONIAN FORMALISM

We follow the general prescriptions of constraint Hamiltonian formalism due to Dirac [6] in order to investigate the properties of the two models described by the Lagrangians of the last section. Let us first examine the case of real scalar fields defined over the Grassmann manifold $O(N)/O(p) \otimes O(N-p)$. Parallel developments will follow for the complex scalar fields defined over the Grassmann manifold $U(N)/U(p) \otimes U(N-p)$.

3.1 Constraints and the canonical structure.

First we determine all the canonical momenta from the given Lagrangian density

$$\pi_i^\gamma(x) \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}_i^\gamma(x)} = M_{\alpha\beta}\dot{\phi}_i^\beta(x) \tag{13}$$

$\dot{\phi}_i^\beta(x)$ is the time derivative of the field and

$$M_{\alpha\beta} = \delta_{\alpha\beta} - \phi_j^\alpha(x)\phi_j^\beta(x) \tag{14}$$

Since the time derivative of the Lagrange multiplier field does not appear in the Lagrangian density (11), the corresponding canonical momenta vanish:

$$\pi_\lambda^{ij}(x) \equiv \frac{\partial \mathcal{L}}{\partial \dot{\lambda}_{ij}} = 0 \tag{15}$$

The canonical Hamiltonian density is

$$\begin{aligned}\mathcal{H}_c &= \pi_i^\gamma(x)\dot{\phi}_i^\gamma(x) - \mathcal{L}\\ &= \frac{1}{2}\left[\pi_i^\gamma(x)\pi_i^\gamma(x) + \phi_i'^\alpha(x)\phi_i'^\alpha(x) - \phi_i'^\alpha(x)\phi_i'^\beta(x)\phi_j^\alpha(x)\phi_j^\beta(x)\right]\\ &+ \lambda_{ij}(x)\left[\phi_i^\alpha(x)\phi_j^\alpha(x) - \delta_{ij}\right]\end{aligned} \tag{16}$$

where $\phi_i'^\alpha(x)$ is the space derivative of $\phi_i^\alpha(x)$. It follows from expressions (13), (14), and (6) that $\pi_k^\gamma(x)\phi_l^\gamma(x) = 0$ for all k and l.

The primary constraints are

$$\Omega_1 \equiv \pi_\lambda^{ij}(x) \approx 0 \tag{17}$$

$$\Omega_2 \equiv \pi_k^\gamma(x)\phi_l^\gamma(x) - \pi_l^\gamma(x)\phi_k^\gamma(x) \approx 0 \tag{18}$$

$$\Omega_3 \equiv \pi_k^\gamma(x)\phi_l^\gamma(x) + \pi_l^\gamma(x)\phi_k^\gamma(x) \approx 0 \tag{19}$$

We have suppressed the indices of Ω_1, Ω_2, and Ω_3 (for notational convenience), which are $p \times p$ matrices, and $\approx$ denotes weak equality.

The total Hamiltonian is defined as

$$H_T \approx \int \mathcal{H}_c dx + \sum_{a=1}^{3}\int v_i(x)\Omega_i(x)dx \tag{20}$$

where $v_i(x)$ are arbitrary functions of space-time.

Now we demand that all primary constraints (17)-(19) should hold good for all time, i. e. the Poisson brachets (PB) of all the constraints with H_T vanish. As a consequence we generate new constraints: the secondary constraints. Then we further require that the PB of the secondary constraints with H_T vanishes so that the secondary constraints hold good for all times. We continue this process until no new constraints are generated. The secondary constraints are

$$\Omega_4 \equiv \phi_i^\alpha(x)\phi_j^\alpha(x) - \delta_{ij} \approx 0 \tag{21}$$

and

$$\Omega_5 = \pi_k^\rho(x)\pi_l^\rho(x) - \phi_k'^\alpha \phi_l'^\alpha + 2\phi_i'^\alpha \phi_i'^\beta \phi_k^\alpha \phi_l^\beta - 2\lambda_{kl} = 0 \tag{22}$$

It is easy to check that Ω_2 are the set of generators of the local O(p) gauge transformations. Furthermore,

$$[\Omega_2, H_T]_{PB} \approx 0 \tag{23}$$

which is a consequence of the fact that H_T is gauge invariant. Our next task is to determine all the first-class and second-class constraints. It is found that only Ω_2 form a set of first-class constraints and the rest are all second class. We denote the second-class constraints as $\{\psi_i\}$, $i = 1 \ldots 4$, with $\psi_1 \equiv \Omega_3$, $\psi_2 \equiv \Omega_5$, $\psi_3 \equiv \Omega_1$, and $\psi_4 \equiv \Omega_4$.

3.2 The primary Dirac brackets.

We introduce the primary Dirac brackets in the presence of the secondary constraints $\{\psi_i\}$ as follows:

$$[f(x), g(y)]' \approx [f(x), g(x)]_{PB} - \sum_{a,b=1}^{4} \int du dv \, [f(x), \psi_a(u)]_{PB} \mathbb{D}_{ab}^{-1}(u, v) [\psi_b(v), g(y)]_{PB} \tag{24}$$

where

$$\int \mathbb{D}_{ab}(u, v)\mathbb{D}_{bd}^{-1}(v, w) dv \approx \delta_{ab}\delta(u - w) \tag{25}$$

and the non-singular antisymmetric matrix $\mathbb{D}(u, v)$ is defined as

$$\mathbb{D}_{ab}(u, v) \equiv [\psi_a(u), \psi_b(v)]_{PB} \tag{26}$$

$\mathbb{D}$ and $\mathbb{D}^{-1}$ have the following explicit form for the model under consideration.

$$\mathbb{D}(u, v) = \begin{bmatrix} 0 & D_{12}(u, v) & 0 & -2(\delta_{lj}\delta_{ki} + \delta_{li}\delta_{kj}) \times \delta(u-v) \\ -D_{12}(u, v) & 0 & -\delta_{pi}\delta_{qj}\delta(u-v) & 0 \\ 0 & \delta_{pi}\delta_{qj}\delta(u-v) & 0 & 0 \\ 2(\delta_{lj}\delta_{ki} + \delta_{li}\delta_{kj}) \times \delta(u-v) & 0 & 0 & 0 \end{bmatrix} \tag{27}$$

and

$$\mathbb{D}^{-1}(u, v)$$

$$= \begin{bmatrix} 0 & 0 & 0 & (\delta_{lj}\delta_{ki} + \delta_{li}\delta_{kj}) \times \dfrac{\delta(u-v)}{2(p+1)^2} \\ 0 & 0 & \delta_{pi}\delta_{qj}\dfrac{\delta(u-v)}{p^2} & 0 \\ 0 & -\delta_{pi}\delta_{qj}\dfrac{\delta(u-v)}{p^2} & 0 & -\dfrac{1}{2p(p+1)} D_{12}(u, v) \\ -(\delta_{lj}\delta_{ki} + \delta_{li}\delta_{kj}) \times \dfrac{\delta(u-v)}{2(p+1)^2} & 0 & \dfrac{D_{12}(u, v)}{2p(p+1)} & 0 \end{bmatrix} \quad (28)$$

Notice that the elements of $D_{12}(u, v)$ are actually objects with four indices, i. e. they are PB of ψ_1 and ψ_2; but we suppress the indices for notational convenience.

The following remark is appropriate here. The set of constraints $\{ \psi_i \}$, Ω_2 and the gauge conditions will constitute a set of second-class constraints, and one can compute the relevant Dirac brackets directly. However, we are interested in investigating the canonical structure of the model for different choices of gauge conditions. Therefore, we have adopted this two-step process of first introducing the primary Dirac brackets to eliminate the second-class constraints $\{ \psi_i \}$ and then introducing the gauge conditions and computing the Dirac brackets.

The relevant primary Dirac brackets are

$$[\phi_i^\alpha(x), \phi_j^\beta(y)]' = 0 \quad (29)$$

$$[\phi_i^\alpha(x), \pi_j^\beta(y)]' = \delta_{\alpha\beta}\delta_{ij}\delta(x-y) - \frac{2\delta(x-y)}{(p+l)^2}\left[\phi_j^\alpha(x)\phi_i^\beta(x) + \phi_k^\alpha(x)\phi_k^\beta(x)\delta_{ij}\right]. \quad (30)$$

$$[\pi_i^\alpha(x), \pi_j^\beta(y)]'$$
$$= \frac{2\delta(x - y)}{(p + 1)^2}\left[\phi_j^\alpha(x)\pi_i^\beta(x) + \phi_k^\alpha(x)\pi_k^\beta(x)\delta_{ij} - \pi_j^\alpha(x)\phi_i^\beta(x) - \pi_k^\alpha(x)\phi_k^\beta(x)\delta_{ij}\right]. \quad (31)$$

Notice the appearance of extra terms in Eqs. (30) and (31) in contrast to the standard PB relations; they are a consequence of the fact that we are dealing with a constraint system.

3.3 The gauge condition.

The model is invariant under local non-Abelian gauge transformations as stated earlier. The Lagrangian density (11) can be rewritten as:

$$\mathscr{L} = \frac{1}{2}\left[\partial_\mu \phi_i^\alpha(x)\partial_\mu \phi_i^\alpha(x) + 2A_\mu^{ij}(x)\phi_i^\alpha(x)\partial_\mu\phi_j^\alpha(x) + A_\mu^{ij}(x)A_\mu^{ij}(x)\right] - \lambda_{ij}(x)\left[\phi_i^\alpha(x)\phi_j^\alpha(x) - \delta_{ij}\right] \quad (32)$$

The kinetic energy terms corresponding to the gauge fields, A_μ^{ij}, are not introduced explicitly in the Lagrangian density ; but they are generated dynamically (owing to the effects of quantum fluctuations) in this model. Thus at the semi-classical level (tree level) the Euler-Lagrange equations for the gauge fields are merely constraint equations

$$A_\mu^{ij}(x) = \phi_j^\alpha(x)\partial_\mu\phi_i^\alpha(x). \quad (33)$$

Notice that Eq. (33) can be re-expressed as

$$A_\mu^{ij}(x) = \frac{1}{2}\left[\phi_j^\alpha(x)\partial_\mu\phi_i^\alpha(x) - \phi_i^\alpha(x)\partial_\mu\phi_j^\alpha(x)\right]. \quad (34)$$

The gauge conditions together with the set of first-class constraints Ω_2 form a set of second-class constraints and the Dirac brackets (DB) are to be computed in the presence of these constraints. In what follows, we present the DB structure of the model in various gauges.

3.3.1. *The axial gauge* $A_1^{ij} = 0$.

The choice of the axial-gauge condition corresponds to the constraints

$$\chi_1 \equiv \phi_i^\alpha(x)\partial_{|x}\phi_j^\alpha(x) \approx 0 \quad (35)$$

(again we have suppressed the indices of χ_1), and $\chi_2 \equiv \Omega_2$.

The Dirac bracket by definition is

$$\{ F(x), G(y)_{DB} \approx [F(x), G(y)]' - \sum_{a,b=1}^{2} \int du dv\, [F(x), \chi_a(u)]' \times \mathbb{B}_{ab}(u, v)[\chi_b(v), G(y)]', \quad (36)$$

where $\mathbb{B}_{ab}(u, v)$ is the inverse of the antisymmetric non-singular 2×2 matrix $\mathbb{B}(u, v)$ defined as

$$\mathbb{B}_{ab}(u, v) = [\chi_a(u), \chi_b(v)]'. \quad (37)$$

It follow from the properties of the matrix $\mathbb{B}$ that $\mathbb{B}_{11} = \mathbb{B}_{22} = 0$ and

$\mathbb{B}_{12} = - \mathbb{B}_{21}$. Thus it suffices to compute only one non-trivial matrix element

$$\mathbb{B}_{12}^{ijkl}(u, v) = [\phi_i^\rho(u)\phi_l^\rho(v)\delta_{kj} - \phi_i^\rho(u)\phi_k^\rho(v)\delta_{jl} - \phi_j^\rho(u)\phi_l^\rho(v)\delta_{ki} + \phi_k^\rho(v)\phi_j^\rho(u)\delta_{li}]\partial_{|u}\delta(u - v). \quad (38)$$

The relevant canonical Dirac brackets are computed in a straightforward manner, using Eq. (36).

$$\{ \phi_i^\alpha(x), \phi_j^\beta(y) \}_{\mathrm{DB}} = 0. \quad (39)$$

$$\{ \phi_i^\alpha(x), \pi_j^\beta(y) \}_{\mathrm{DB}} = \left[\delta_{\alpha\beta}\delta_{ij} - \frac{2}{(p+1)}(\phi_j^\alpha(x)\phi_i^\beta(x) + \phi_k^\alpha(x)\phi_k^\beta(x)\delta_{ij}) \right]\delta(x-y) - \frac{2\varepsilon(x-y)}{p(p-1)} \times [\phi_j^\alpha(x)\partial_{|y}\phi_i^\beta(y) - \phi_k^\alpha(x)\partial_{|y}\phi_k^\beta(y)\delta_{ij}]. \quad (40)$$

$$\begin{aligned}\{ \pi_i^\alpha(x), \pi_j^\beta(y) \}_{\mathrm{DB}} &= \frac{2\delta(x-y)}{(p+1)^2}[\pi_j^\alpha(x)\phi_i^\beta(x) + \pi_k^\alpha(x)\phi_k^\beta(x) \times \delta_{ij} - \phi_j^\alpha(x)\pi_i^\beta(x) - \phi_k^\alpha(x)\pi_k^\beta(x)\delta_{ij}] \\ &- \frac{2\varepsilon(x-y)}{p(p-1)}(\phi_j^\alpha(x)\partial_{|y}\phi_i^\beta(y) - \phi_k^\alpha(x)\partial_{|y}\phi_k^\beta(y)\delta_{ij} \\ &- \partial_{|x}\phi_j^\alpha(x)\phi_i^\beta(y) + \partial_{|x}\phi_k^\alpha(x)\phi_k^\beta(y)\delta_{ij}). \quad (41)\end{aligned}$$

where

$$\begin{aligned}\varepsilon(x - y) &= x - y \quad \text{for} \quad x > y \\ &= y - x \quad \text{for} \quad y > x. \end{aligned} \quad (42)$$

Now we are in a position to determine the total Hamiltonian since we can set all the constraints equal to zero strongly:

$$\mathrm{H_T} = \frac{1}{2}\int [\pi_i^\alpha(x)\pi_i^\alpha(x) + \phi_i'^\alpha(x)\phi_i'^\alpha(x)]dx. \quad (43)$$

3.3.2. *The unitary gauge* [9].

In order to implement the U-gauge condition, we decompose the fields $\phi_i^\alpha(x)$ into two sets

$$\phi_i^\alpha(x) = \begin{pmatrix} \phi_i^j(x) \\ \hline v_i^\alpha(x) \end{pmatrix}, \qquad \begin{aligned} i, j &= 1, 2, \dots p \\ \alpha &= p + 1, \dots \mathrm{N}. \end{aligned} \quad (44)$$

Let us define the matrix $\mathrm{U}_{ij} = \phi_i^j(x)$. The choice of unitary gauge corresponds to the constraints

$$\chi_1 \equiv \phi_i^j(x) + \phi_j^i \approx 0 \quad (45)$$

The orthonormality condition (6) is expressed as

$$\sum_{k=1}^{p} \phi_i^k(x)\phi_j^k(x) = \delta_{ij} - \sum_{\alpha=p+1}^{N} v_i^\alpha(x)v_j^\alpha(x) . \tag{46}$$

The form of the matrix element $\mathbb{B}_{12}$ in the U gauge is determined with $\chi_2 \equiv \Omega_2$

$$\mathbb{B}_{12}^{ijkl}(u, v) = \left(\phi_l^j(x)\delta_{ki} - \phi_k^j(x)\delta_{li} + \phi_l^i\delta_{ki} - \phi_k^i\delta_{lj}\right) \times \delta(u - v) . \tag{47}$$

The Dirac brackets are

$$\{ \phi_i^\alpha(x), \phi_j^\beta(y) \}_{\rm DB} = 0 . \tag{48}$$

$$\begin{aligned}\{ \phi_i^\alpha(x), \pi_j^\beta(y) \}_{\rm DB} \\ = \Bigg[\delta_{\alpha\beta}\delta_{ij} - \frac{2}{(p+1)^2}\left(\phi_j^\alpha(x)\phi_i^\beta(x) + \phi_k^\alpha(x)\phi_k^\beta(x)\delta_{ij}\right) \\ - \frac{4}{\det \mathbb{B}}\{ \phi_k^\alpha(x)\phi_k^\beta(x) \times \delta_{ij} + \phi_k^\alpha(x)\phi_k^j(x)\delta_{i\beta} - \phi_j^\alpha(x)\phi_i^\beta(x) \\ - \phi_k^\alpha(x)\phi_i^j(x)\delta_{\rm PB} - (\phi_l^\alpha(x)\phi_l^k(x)\phi_j^k(x)\phi_i^\beta(x) + \phi_l^\alpha(x)\phi_l^k(x)\phi_k^\beta(x)\phi_j^i(x) \\ + \phi_l^\alpha(x)\phi_l^k(x)\phi_m^k(x)\phi_m^\beta(x)\delta_{ij} + \phi_l^\alpha(x)\phi_l^j(x)\phi_m^i(x)\phi_m^\beta(x) \\ - \{ \alpha \leftrightarrow \beta, i \leftrightarrow j \})/(p+1)^2 \} \Bigg]\delta(x - y) .\end{aligned} \tag{49}$$

The Dirac brackets between $\pi_i^\alpha(x)$ and $\pi_j^\beta(y)$ can be computed in a straightforward manner.

In the unitary gauge it is possible to write the Lagrangian density in terms of the fields $v_i^\alpha(x)$ only ; however, the Lagrangian density is an infinite power series in these fields. We write the total Hamiltonian density, $\mathscr{H}_{\rm T}$, in the following form:

$$\begin{aligned}\mathscr{H}_{\rm T} = \frac{1}{2}\, [\pi_i^\alpha(x)\pi_i^\alpha(x) + v_i'^\alpha(x)v_i'^\alpha(x) + v_i'^\alpha(x)v_j^\alpha(x) \times v_j'^\beta(x)v_i^\beta(x) \\ + v_i^\alpha(x)v_j'^\alpha(x)(\phi_i'^l(x)\phi_j^l(x) - \phi_j'^l(x)\phi_i^l(x)) - \phi_j'^k(x)\phi_j'^k(x)v_l^\alpha(x)v_l^\alpha(x)]\end{aligned} \tag{50}$$

Notice that $\mathscr{H}_{\rm T}$ is quadratic in the canonical momenta in axial gauge as well as in the U gauge.

4. COMPLEX SCALAR FIELDS

The model is described by the Lagrangian density (12), where we have already introduced the Lagrangian multiplier fields $\chi_{ij}(x)$ and $Z_i^\alpha(x)$ are treated as unconstrained variables. Since the techniques involved are similar to those of the last section we shall list the corresponding results only.

i) Canonical momenta and canonical Hamiltonian density:

$$\pi_j^\alpha(x) \equiv \frac{\partial \mathscr{L}}{\partial \dot{Z}_j^\gamma(x)} = (\delta_{\beta\gamma} - Z_i^{\gamma *}(x) Z_i^\beta(x))\dot{Z}_j^{\beta *}(x) . \tag{51}$$

$$\pi_j^{\gamma *}(x) \equiv \frac{\partial \mathscr{L}}{\partial \dot{Z}_j^{\gamma *}(x)} = (\delta_{\gamma\alpha} - Z_i^\gamma(x) Z_i^{\alpha *}(x))\dot{Z}_j^\alpha(x) . \tag{52}$$

$$\pi_\chi^{ij}(x) \equiv \frac{\partial \mathscr{L}}{\partial \dot{\chi}_{ij}} = 0 \tag{53}$$

$$\mathscr{H}_c = \pi_i^{\alpha *}(x)\pi_i^\alpha(x) + Z_i'^{\alpha *}(x) Z_i'^\alpha(x) - Z_i^\beta(x) Z_j'^{\beta *}(x) Z_i^{\alpha *}(x) Z_i'^\alpha(x) + \chi_{ij} x)(Z_i^{\alpha *}(x) Z_j^\alpha(x) - \delta_{ij}) . \tag{54}$$

ii) Primary constraints and the total Hamiltonian:

$$\Omega_1 \equiv \pi_\chi^{ij}(x) \approx 0 \tag{55}$$

$$\Omega_2 \equiv \pi_k^\gamma(x) Z_l^\gamma(x) - \pi_l^{\gamma *}(x) Z_k^{\gamma *}(x) \approx 0 \tag{56}$$

$$\Omega_3 \equiv \pi_k^\gamma(x) Z_l^\gamma(x) + \pi_l^{\gamma *}(x) Z_k^{\gamma *}(x) \approx 0 \tag{57}$$

$$\mathscr{H}_T = \int \mathscr{H}_c dx + \sum_{i=1}^{3} \int w_i(x)\Omega_i(x) dx , \tag{58}$$

where $w_i(x)$ are arbitrary functions of space-time.

iii) Secondary constraints:

$$\Omega_4 \equiv Z_i^{\alpha *}(x) Z_j^\alpha(x) - \delta_{ij} \approx 0 \tag{59}$$

$$\Omega_5 \equiv \pi_k^\rho(x)\pi_l^{\rho *}(x) + Z_j'^{\rho *}(x) Z_l'^\alpha(x) Z_l^\rho(x) Z_k^{\alpha *}(x) - \chi_{kl}(x) - Z_k'^\alpha(x) Z_l^{x'\alpha}(x) \approx 0, \tag{60}$$

and

$$[\Omega_2, H_T]_{PB} \approx 0 . \tag{61}$$

Equation (61) is a mere reflection of the fact that H_T is gauge invariant. The Ω_2 form a set of first-class constraints, whereas $\{ \psi_i \}$, $i = 1 \dots 4$ with $\psi_1 = \Omega_3$, $\psi_2 \equiv \Omega_5$, $\psi_3 \equiv \Omega_1$, and $\psi_4 \equiv \Omega_4$ form a set of second-class constraints.

The primary Dirac brackets can be computed once the secondary constraints have been identified as in the case of Section 3. We have listed all the primary Dirac brackets in the Appendix and now proceed to compute the Dirac brackets in the two gauges as was done in the previous section.

4.1 Gauge conditions.

There are no kinetic energy terms for the gauge fields in this case. The equations of motion for the gauge fields A_μ^{ij} are

$$A_\mu^{ij}(x) = \frac{i}{2}\left[Z_j^{\alpha *}(x)\partial_\mu Z_i^\alpha(x) - Z_i^\alpha(x)\partial_\mu Z_j^{\alpha *}(x) \right] . \tag{62}$$

i) *The axial gauge* $A_1^{ij} = 0$.

The corresponding constraints are

$$\chi_1 \equiv Z_j^{\alpha *}(x)\partial_{|x} Z_i^{\alpha}(x) \approx 0 . \tag{63a}$$

$$\chi_2 \equiv \Omega_2 = \pi_k^{\gamma}(x) Z_l^{\gamma}(x) - \pi_l^{\gamma *}(x) Z_k^{\gamma *}(x) \approx 0 . \tag{63b}$$

The antisymmetric non-singular 2 × 2 matrix $\mathbb{B}$ is completely determined once we specify one non-trivial matrix element (since $\mathbb{B}_{11} = \mathbb{B}_{22} = 0$).

$$\mathbb{B}_{12}^{ijkl}(u, v) = \left(Z_j^{\rho *}(u) Z_l^{\rho}(v)\delta_{ik} + Z_i^{\rho}(u) Z_k^{\rho *}(v)\delta_{lj}\right)\partial_{|u}\delta(u - v) . \tag{64}$$

The Dirac brackets are as follows:

$$\{ Z_r^{\alpha}(x), Z_s^{\beta}(y) \}_{DB} = 0 \tag{65}$$

$$\{ Z_r^{\alpha}(x), Z_s^{\beta *}(y) \}_{DB} = 0 \tag{66}$$

$$\{ Z_r^{\alpha}(x), \pi_s^{\beta}(y) \}_{DB} = \left(\delta_{\alpha\beta}\delta_{rs} - \frac{1}{p^2} Z_n^{\alpha}(x) Z_n^{\beta *}(s)\delta_{rs} \right) \times \delta(x-y) + \frac{\varepsilon(x-y)}{p^2} Z_l^{\alpha}(x)\partial_{|y} Z_l^{\beta *}(y)\delta_{rs} . \tag{67}$$

$$\{ Z_r^{\alpha}(x), \pi_s^{\beta *}(y) \}_{DB} = -\frac{1}{2p^2} Z_s^{\alpha}(x) Z_r^{\beta}(x)\delta(x-y) - \frac{\varepsilon(x-y)}{p^2} Z_s^{\alpha}(x)\partial_{|y} Z_r^{\beta}(y) . \tag{68}$$

$$\{ \pi_r^{\alpha}(x), \pi_s^{\beta}(y) \}_{DB} = \frac{\delta(x - y)}{2p^2}\left(\pi_s^{\alpha}(x) Z_r^{\beta *}(x) - Z_s^{\alpha *}(x)\pi_r^{\beta}(x)\right) + \frac{\varepsilon(x - y)}{p^2}\left(\partial_{|x} Z_s^{\alpha *}(x)\pi_r^{\beta}(y) - \pi_s^{\alpha}(x)\partial_{|y} Z_r^{\beta *}(y)\right) \tag{69}$$

$$\{ \pi_r^{\alpha}(x), \pi_s^{\beta}(y) \}_{DB} = \frac{\delta(x - y)}{2p^2}\left(\pi_m^{\alpha}(x) Z_m^{\beta}(x)\delta_{rs} - Z_s^{\alpha *}(x)\pi_r^{\beta}(x)\right) - \frac{\varepsilon(x - y)}{p^2}\left(\partial_{|x} Z_l^{\alpha *}(x)\pi_l^{\beta *}(y)\delta_{rs} - \pi_i^{\alpha}(x)\partial_{|y} Z_i^{\beta}(y)\delta_{rs}\right) . \tag{70}$$

The total Hamiltonian in this gauge has the following simple form:

$$H_T = \int \left[\pi_i^{\alpha *}(x)\pi_i^{\alpha}(x) + Z_i'^{\alpha}(x) Z_i'^{\alpha}(x)\right] dx . \tag{71}$$

ii) *The unitary gauge* [9]

We decompose the fields into two sets as we did in the case of real scalar fields.

$$Z_i^{\alpha}(x) = \begin{pmatrix} S_i^j(x) \\ \hline v_i^{\alpha}(x) \end{pmatrix}, \qquad \begin{matrix} i, j = 1, 2, \dots p \\ \alpha = p + 1, \dots N \end{matrix} \tag{72}$$

Define the matrix $U_{ij}(x) = S_i^j(x)$. The choice of the U gauge corresponds to

$$U = U^{\dagger} \tag{73}$$

The non-trivial matrix element of $\mathbb{B}$ is

$$\mathbb{B}_{12}^{ijkl}(u, v) = \left(\delta_{ik} S_l^j(x) + S_k^{i *}(x)\delta_{jl}\right)\delta(x - y) . \tag{74}$$

The Dirac brackets are

$$\{ Z_r^\alpha(x), Z_s^\beta(y) \}_{DB} = 0 \tag{75}$$

$$\{ Z_r^\alpha(x), Z_s^{\beta*}(y) \}_{DB} = 0 \tag{76}$$

$$\begin{aligned} \{ Z_r^\alpha(x), \pi_s^\beta(y) \}_{DB} = \Big\{ & \delta_{\alpha\beta}\delta_{rs} - \frac{1}{2p^2} Z_n^\alpha(x) Z_n^{\beta*}(x) \delta_{rs} \\ & - \frac{1}{\det \mathbb{B}} \Big[Z_l^\alpha(x) Z_l^\beta(x) \delta_{rs} + Z_s^\alpha(x) Z_r^{\beta*}(x) - \frac{1}{2p^2} (Z_l^\alpha(x) S_l^j(x) S_n^j(x) Z_n^{\beta*}(x) \delta_{rs}) \\ & + Z_l^\alpha(x) S_l^j(x) S_s^{r*}(x) Z_j^{\beta*}(x) + Z_s^\alpha(x) S_r^{j*}(x) S_n^j(x) Z_n^{\beta*}(x) \\ & + Z_i^\alpha(x) S_s^{j*}(x) S_n^j(x) Z_j^{\beta*}(x) \Big] \Big\} \, \delta(x-y). \end{aligned} \tag{77}$$

$$\begin{aligned} \{ \pi_r^\alpha(x), \pi_s^\beta(y) \}_{DB} = & \frac{1}{2p^2} [\pi_s^\alpha(x) Z_r^{\beta*}(x) - Z_s^{\alpha*}(x) \pi_r^\beta(y)] \delta(x-y) \\ & + \frac{1}{\det \mathbb{B}} \Big[Z_s^\alpha(x) \pi_r^\beta(x) + S_k^r(x) \pi_k^\beta(x) \delta_{\alpha s} - \frac{1}{2p^2} (S_n^j(x) Z_n^{\alpha*}(x) S_s^j(x) \pi_r^\beta(x)) \\ & + S_n^s(x) Z_n^{\alpha*}(x) S_k^{r*}(x) \pi_k^\beta(x) + S_r^i(x) Z_j^{\alpha*}(x) S_s^j(x) \pi_i^\beta(x) \\ & - \{ (r \leftrightarrow s), (\alpha \leftrightarrow \beta) \} \Big] \delta(x-y). \end{aligned} \tag{78}$$

All other brackets can be computed in a straight forward manner. The Lagrangian density can be written in terms of the fields $v_i^\alpha(x)$ alone; but it is an infinite power seri s in these fields as has been noted by Brézin *et al.* [9]. We write

$$\mathscr{H}_T = \pi_i^{\alpha*}(x)\pi_i^\alpha(x) + P(v_i^\alpha(x), v_i^{\alpha*}(x)), \tag{79}$$

where

$$\begin{aligned} P(v_i^\alpha, v_i^\alpha) = v_i'^{\alpha*}(x) v_i'^\alpha(x) - \frac{1}{2} (v_i'^{\alpha*}(x) v_j^\alpha(x) v_j'^{\beta*}(x) v_i^\beta(x)) \\ + v_i^{\alpha*}(x) v_j'^\alpha(x) v_j^{\beta*}(x) v_i'^\beta(x) + \text{higher powers of } v. \end{aligned} \tag{80}$$

We are in a position now to discuss quantization of the models.

5. QUANTIZATION

The theory can be quantized in two different ways. If we follow the prescription of canonical quantization, then the Dirac brackets fix all the relevant canonical commutation relations. In this procedure one confronts the usual problems of operator ordering while going from Dirac brackets to commutation relations [11]. However, this problem can be resolved by demanding that the canonical momenta $\pi_i^\alpha(x)$ are now Hermitian operators $\hat{\pi}_i^\alpha(x)$ and the commutator $[\hat{\pi}_i^\alpha(x), \hat{\pi}_j^\beta(y)]$ be consistent with the

Hermiticity of $\hat{\pi}_i^\alpha(x)$. Notice that the operator ordering ambiguity, i. e. the product of ϕ_i^α and π_j^β, arises only in the abnormal commutation relations [see Eqs. (41), (69), (70), and (78)] such as the commutator between two momenta and never in a commutation relation between a field operator and its canonically conjugate momenta. Then the total Hamiltonian is expressed in terms of the operators and the equations of motion are obtained through appropriate Hamilton's equations of motion in the operator form (commutation relations instead of Dirac brackets).

The other approach is to adopt the path-integral quantization [*12*]. The standard method can be applied to obtain the evolution operator. The Hamiltonian is quadratic in momenta in the gauges we have considered; therefore the functional integrals over π's can be carried out. Therefore, we obtain the standard form. For the real scalar fields

$$Z = \int \pi\mathscr{D}[\phi_i^\alpha(x)]\pi\mathscr{D}[\lambda_{ij}(x)]\pi\mathscr{D}[A_\mu^{ij}(x)] \times \exp\left(iS[\phi_i^\alpha, \lambda_{ij}, A_\mu^{ij}]\right), \quad (81)$$

where

$$S = \int (\mathscr{L} + \mathscr{L}_{GF} + \mathscr{L}_{ghost})d^2x \quad (82)$$

$\mathscr{L}_{GF}$ is the gauge-fixing term and $\mathscr{L}$ is given by Eq. (11). If we work in the U gauge [*9*] or in any other ghost-free gauge then $\mathscr{L}_{ghost}$ is absent in Eq. (82). We may remark that starting from Z we can calculate the effective action as has been done in Ref. 9 and/or define a generating functional for the Green's functions by introducing c-number source terms for each field.

6. CONCLUSIONS

We have studied the canonical structures of two models, i. e. real scalar fields and complex scalar fields defined over the Grassmann manifolds $O(N)/O(p) \otimes O(N-p)$ and $U(N)/U(p) \otimes U(N-p)$, respectively. These models were investigated in the axial gauge as well as in the unitary gauge in the framework of constraint Hamiltonian formalism due to Dirac. We computed the Dirac brackets among various canonical variables and obtained the total Hamiltonian for the two models in both the gauges. The Hamiltonian density is found to be quadratic in canonical momenta in the two gauges.

Next we considered the quantization of the models in the canonical formalism. The problem of operator ordering was discussed. It should be noted that as $p \to \infty$, keeping N/p constant, all the abnormal Dirac brackets between canonical variables vanish as $1/p^2$ [refer to Eqs. (41), (68), (69), (70), and (78)] and the Dirac brackets between fields and their canonically conjugate momenta have the form $\delta_{\alpha\beta}\delta_{ij}\delta(x-y)$ [additional terms are $O(1/p^2)$]; see Eqs. (40), (49), (67), and (77). Then in the limit $p \to \infty$, we recover free-field canonical quantization when Dirac brackets are replaced by quantum commutator brackets.

The evolution operator is obtained in a straightforward manner. The generating functional for the Green's functions can be defined using standard techniques.

We conclude this article with the following remarks.

i) The chiral models admit infinitely many conserved currents. In the case of the non-linear σ model the conservation laws survive quantization [*13*], whereas in the case of the $\mathbb{C}P^{n-1}$ model the conservation of the non-local charges is destroyed owing to the presence of anomalies [*14*]. It is not known whether the currents of the generalized non-linear σ model are conserved in the quantized theory. This problem is under investigation using the techniques due to Slavnov [*15*].

ii) A super-symmetric extension of the generalized CP^{n-1} model has been considered [*16*]. It will be interesting to investigate the model in the constraint Hamiltonian formalism [*17*].

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APPENDIX

Here we list all relevant formulae for the complex scalar fields and calculate the primary Dirac brackets. We need the inverse of the non-singular antisymmetric matrix $\mathbb{D}$ defined as

$$\mathbb{D}_{mn}(u, v) = [\psi_m(u), \psi_n(v)]_{PB}, \qquad m, n = 1, \dots 4. \tag{A.1}$$

$$\mathbb{D}^{-1}(u, v) = \begin{bmatrix} 0 & 0 & 0 & \dfrac{\delta_{mj}\delta_{ni}\delta(u-v)}{2p^2} \\ 0 & 0 & \dfrac{\delta_{kp}\delta_{lq}\delta(u-v)}{2p^2} & 0 \\ 0 & -\dfrac{\delta_{kp}\delta_{lq}\delta(u-v)}{2p^2} & 0 & -\dfrac{D_{12}\delta_{mj}\delta_{ni}\delta_{kp}\delta_{lq}}{4p^4} \\ -\dfrac{\delta_{mj}\delta_{ni}\delta(u-v)}{2p^2} & 0 & \dfrac{D_{12}\delta_{mj}\delta_{ni}\delta_{kp}\delta_{lq}}{4p^4} & 0 \end{bmatrix} \tag{A.2}$$

(Indices of D_{12} are suppressed).

Primary Dirac brackets

$$[Z_r^\alpha(x), Z_s^\beta(y)]' = [Z_r^\alpha(x), Z_s^\beta(y)]_{PB} - \sum_{a,b=1}^{4} \int dudv [Z_r^\alpha(x), \psi_a(u)] \times \mathbb{D}_{ab}^{-1}(u, v)[\psi_b(v), Z_s^\beta(y)] = 0. \tag{A.3}$$

$$[Z_r^\alpha(x), Z_s^{\beta*}(y)]' = 0 \tag{A.4}$$

$$[Z_r^\alpha(x), \pi_s^\beta(y)]' = \left(\delta_{\alpha\beta}\delta_{rs} - \frac{1}{2p^2} Z_n^\alpha(x) Z_n^{\beta*}(x)\delta_{rs}\right)\delta(x-y) \tag{A.5}$$

$$[Z_r^\alpha(x), \pi_s^{\beta*}(y)]' = \frac{-1}{2p^2} Z_s^\alpha(x) Z_r^\beta(x)\delta(x-y) \tag{A.6}$$

$$[\pi_r^\alpha(x), \pi_s^\beta(y)]' = \frac{1}{2p^2}(\pi_s^\alpha(x) Z_r^{\beta*}(x) - Z_s^{\alpha*}(x)\pi_r^\beta(x))\delta(x-y) \tag{A.7}$$

$$[\pi_r^\alpha(x), \pi_s^{\beta*}(y)]' = \frac{1}{2p^2}(\pi_m^\alpha(x) Z_m^\beta(x)\delta_{rs} - Z_s^{\alpha*}(x)\pi_r^{\beta*}(x))\delta(x-y). \tag{A.8}$$

All other primary Dirac brackets can be obtained from the above brackets.

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