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Non-Locality in the Stochastic Interpretation of the Quantum Theory

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ABSTRACT. — This article reviews the stochastic interpretation of the quantum theory and shows in detail how it implies non-locality.

RÉSUMÉ. — Cet article présente une revue de l'interprétation stochastique de la théorie quantique et montre en détail en quoi elle implique une non-localité.

1. INTRODUCTION

It gives me great pleasure on the occasion of Jean-Pierre Vigier's retirement to recall our long period of fruitful association and to say something about the stochastic interpretation of the quantum theory, in which we worked together in earlier days. Because of requirements of space, however, this article will be a condensation of my talk (*) which was in fact based on a much more extended article by Basil Hiley and me [*1*].

It is well known that all the commonly accepted interpretations of the quantum theory, including the causal (or pilot wave) interpretation imply non-locality [*2-8*]. However, because of its very name, the stochastic interpretation seems to imply a theory in which quantum mechanics could be explained in terms of particles undergoing independent random processes, so that at least in this interpretation, one might hope that there

(*) Delivered on February 19th 1988 at the « Journée Séminaire » in honour of Jean-Pierre Vigier.

would be no need for non-locality. The main purpose of this paper is, on the basis of a systematic and coherent account of the stochastic interpretation, to make it clear that the latter also necessarily involves non-locality.

2. THE STOCHASTIC INTERPRETATION: THE SINGLE PARTICLE

The stochastic interpretation of the quantum theory was first introduced by Bohm and Vigier [*9*]. Later Nelson and others [*10-13*] developed a somewhat similar model which was, however, different in certain significant ways. (This model was also discussed in some detail in the talk, and in the paper [*11*].

The basic ideas were first applied to a single electron. In particular Bohm and Vigier assumed that the electron is a particle suspended in a Madelung fluid whose general motion is determined by the Schrödinger equation with the density $\rho(x) = |\psi(x)|^2$ and the local velocity $\underline{v} = \frac{\nabla S}{m}$. The particle suspended in this fluid would be carried along with the local velocity. It was then assumed that the fluid has a further random component to its local velocity which could arise from a level below that of the quantum mechanics. This random motion will also be communicated to the particle so that it will undergo a stochastic process with a trajectory having the average local velocity and a random component. No detailed assumptions were made about this random component but it was shown that under certain fairly general conditions an arbitrary probability distribution, $P(\underline{x}, t)$, would approach the quantum mechanical distribution $P = |\psi(\underline{x})|^2$ as a limit.

As pointed out by Bohm [*14*], this general result does not require the assumption of a Madelung fluid. All that is needed is to suppose that there is a mean velocity given by $\underline{v} = \nabla S/m$, together with an additional stochastic contribution. For example, this latter may come, as proposed by Nelson [*11*], from a randomly fluctuating background field. Or else it may be regarded as an implication of some kind of « vacuum fluctuations » similar in some ways, to the effects of a space-filling medium or « ether » which is undergoing internal random motions. This notion has been emphasised by De Witt [*15*] and has also been used by Vigier [*16*] in the context of the stochastic interpretation.

The original model of Bohm and Vigier [*9*] was not developed in further detail as this did not seem to be called for at that time. However, it now seems appropriate to examine this again. To this end we assume that whatever the origin of the random motion may be, it can be represented by a simple diffusion process.

To illustrate this, let us consider the Brownian motion of particles in a gravitational field using the simple theory of Einstein. If P is the probability density of particles, then there is a diffusion current,

$$\underline{J}^{(d)} = - D\underline{\nabla}P \tag{1}$$

where D is the diffusion coefficient. If this were all there was, the conservation equation would be

$$\frac{\partial P}{\partial t} = - D\nabla^2 P \tag{2}$$

and clearly this would lead to a uniform equilibrium distribution. However, as Einstein showed, in a gravitational field there is what he called an osmotic velocity,

$$u_0 = D\frac{mg}{kT}\underline{z}. \tag{3}$$

The conservation equation then becomes

$$\frac{\partial P}{\partial t} = - D\nabla\left[\frac{mg}{kT}zP + \nabla P\right]. \tag{4}$$

For the equilibrium distribution, $\frac{\partial P}{\partial t} = 0$, and

$$\frac{\underline{\nabla}P}{P} = -\frac{mg}{kT}\underline{z} + \text{const.} \tag{5}$$

or

$$P = Ae^{-\frac{mgz}{kT}}$$

which is the well-known Boltzmann factor.

The picture implied by the Einstein model of diffusion is that the particle is drifting downward in the gravitational field and that the net upward diffusive movement balances this to produce equilibrium.

It is worth while to provide a still more detailed picture of this process. To do this let us consider a simple one dimensional model in which there is a unique free path λ and a unique speed $\overline{v}$. We assume, further that after collisions the velocities are randomly distributed in positive and negative directions while the speed is still $\overline{v}$.

Let us now consider two layers which are separated by λ. The net diffusion current between these layers will be

$$J^{(d)} = \frac{\overline{v}}{2}[P_z - P_{z-\lambda}]$$

$$\cong -\frac{\overline{v}}{2}\frac{\partial P}{\partial z}\cdot\lambda. \tag{6}$$

Between collisions the average velocity gained from the gravitational field is

$$u_0 = -\frac{g}{2}\frac{\lambda}{\bar{v}}. \tag{7}$$

The above is evidently the osmotic velocity. The net current is

$$\begin{aligned} J &= -\frac{g}{2}\frac{\lambda}{\bar{v}}P - \frac{\bar{v}}{2}\frac{\partial P}{\partial z}\lambda \\ &= -\frac{\lambda\bar{v}}{2}\left[\frac{g}{\bar{v}^2}P + \frac{\partial P}{\partial z}\right]. \end{aligned} \tag{8}$$

Writing $\bar{v}^2 = \frac{kT}{m}$ and $D = \frac{\bar{v}\lambda}{2}$ we have

$$J = -D\left[\frac{mg}{kT}\cdot P + \frac{\partial P}{\partial z}\right] \tag{9}$$

and this is just the equation that Einstein assumed.

It is important to emphasise that at least in this case the osmotic velocity is produced by a field of force. Without such a field there would be no reason for an osmotic velocity.

For the stochastic interpretation of the quantum theory we would like to have a random diffusion process whose equilibrium state corresponds to a probability density $P = |\psi|^2 = \rho$ and to a mean current $j = \rho\bar{v} = \rho\frac{\nabla S}{m}$. Such a state is a consistent possibility if $\psi = \sqrt{\rho}e^{iS/t}$ satisfies Schrödinger's equation because this implies the conservation equation

$$\frac{\partial p}{\partial t} + \underline{\nabla}\cdot\underline{j} = 0.$$

In order to have $\rho = |\psi|^2$ as an equilibrium density under such a random process, we will have to assume a suitable osmotic velocity. We do not have to suppose, however, that this osmotic velocity is necessarily produced by a force field, similar to that of the gravitational example, but rather it may have quite different causes. (Thus as suggested by Nelson [*16*], some kind of background field that would produce a systematic drift as well as a random component of the motion.) At this stage it will be sufficient simply to postulate a field of osmotic velocities, $\underline{u}_0(\underline{x}, t)$, without committing ourselves as to what is its origin. We therefore assume an osmotic velocity

$$\underline{u}_0 = D\frac{\underline{\nabla}\rho}{\rho} \tag{10}$$

and a diffusion current $\underline{j}^{(d)} = -D\underline{\nabla}P$. (11)

The total current will be

$$\underline{j} = \frac{P}{m}\underline{\nabla}S + DP\frac{\underline{\nabla}\rho}{\rho} - D\underline{\nabla}P. \tag{12}$$

The conservation equation is then

$$\frac{\partial P}{\partial t} + \underline{\nabla}\cdot\left(P\frac{\underline{\nabla}S}{m} + DP\frac{\underline{\nabla}\rho}{\rho} - D\underline{\nabla}P\right) = 0. \tag{13}$$

In the above equations there is a systematic velocity $\underline{v}s = \frac{\underline{\nabla}S}{m} + D\frac{\underline{\nabla}\rho}{\rho}$. This is made up of two parts, the mean velocity $\underline{\bar{v}} = \frac{\underline{\nabla}S}{\rho}$ and the osmotic velocity $\underline{u}_0 = D\frac{\underline{\nabla}\rho}{\rho}$. The mean velocity $\underline{\bar{v}}$ may be thought to arise from de Broglie's guidance condition. As explained earlier the osmotic velocity will arise from some other source, but the main point is that it is derivable from a potential $D\ln\rho$ where ρ is a solution of the conservation eqn. $\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\frac{\nabla S}{m}\right) = 0$.

It follows from (13) that there is an equilibrium state in which the osmotic velocity is balanced by the diffusion current so that the mean velocity is $\underline{\bar{v}} = \frac{\underline{\nabla}S}{m}$.

But now we must raise the crucial question as to whether this equilibrium is stable. In other words will an arbitrary distribution P always approach ρ? To simplify this discussion let us first consider the case of a stationary state in which $\underline{\nabla}S = 0$. Writing $P = F\rho$ we obtain the following equation

$$\rho\frac{\partial F}{\partial t} = D\,\mathrm{div}\,(\rho\nabla F). \tag{14}$$

Let us now multiply the equation by F and integrate over all space

$$\frac{1}{2}\int\rho\frac{\partial F^2}{\partial t}d\tau = D\int F\,\mathrm{div}\,(\rho\nabla F)d\tau = -D\int\rho(\nabla F)^2 d\tau. \tag{15}$$

The right hand side is clearly negative unless $F = 1$ everywhere. On the left hand side we have an average of $\frac{\partial F^2}{\partial t}$ weighted with the probability density ρ. Clearly unless the right hand side is zero F must be decreasing somewhere. This decrease will only cease when $F = 1$ everywhere.

This proof can be extended to the case where $\underline{\nabla}S \neq 0$. To do this, let us first note that, as has been shown earlier, $P = \rho$ is still an equilibrium distribution. It has been shown by Bohm and Vigier [9] however that if $P = \rho$ is such an equilibrium distribution and if ρ satisfies a conservation equation, then for the general case, P approaches ρ.

For times much longer than the relaxation time, τ, of this process we will therefore quite generally encounter the usual quantum mechanical probability distribution. It is implied, of course, that at shorter times this need not be so and therefore the possibility of a test to distinguish this theory from the quantum theory is in principle opened up. (As to the conditions under which such a test may be possible is discussed in the article [1].)

For the equilibrium case the mean velocity is, as has been demonstrated, $\bar{v} = \frac{\nabla S}{m}$. Clearly this relation by itself does not determine the acceleration. To find this we need a differential equation for S. Thus, if $\psi = Re^{iS/\hbar}$ satisfies Schrödinger's equation then, as pointed out, for example in Bohm and Hiley [18], S. will satisfy an extended Hamilton-Jacobi equation and from this it follows that the mean acceleration will be

$$m \frac{d\bar{v}}{dt} = - \nabla(V + Q). \tag{16}$$

where Q is the quantum potential, which is

$$Q = - \frac{\hbar^2}{2m} \frac{\nabla^2 | \psi |}{| \psi |}. \tag{17}$$

It is clear that in the present approach the quantum potential is actually playing a secondary role. The fundamental dynamics is determined by the guidance condition and the osmotic velocity along with the effects of the random diffusion. All these work together to keep a particle in a region in which $\rho = | \psi |^2$ is large and where its average velocity fluctuates around $\frac{\nabla S}{m}$. The quantum potential merely represents the mean acceleration of the particle implied by the de Broglie guidance conditions, and this will be valid only if ψ satisfies Schrödinger's equation. If ψ had satisfied another wave equation, the mean acceleration would have been different. In fact in the article [1] this theory is extended so that the wave function satisfies the Dirac equation, for which a very different mean acceleration is implied and in which the quantum potential does not apply.

To illustrate the stochastic model, let us consider the two slit interference experiment. A particle undergoing random motion will go through one slit or the other, but it is affected by the Schrödinger field coming from both slits. In the causal interpretation this effect was expressed primarily through the quantum potential. In the stochastic interpretation it is primarily expressed through the osmotic velocity which reflects the contributions to the wave function ψ coming from both slits. Near the zeros of the wave function the osmotic velocity approaches infinity and is directed away from the zeros. Thus a particle diffusing randomly and approaching a zero is certain to be turned around before it can reach this zero. This explains why no particle ever reaches the points where the wave function is zero. And, as we have indeed already pointed out, the osmotic velocity is constantly pushing the particle to the regions of highest $|\psi|^2$ and this explains why most particles are found near the maxima of the wave function.

Without assuming an osmotic velocity field of this kind, there would be no way of explaining such phenomena. As a result of random motions for example a particle just undergoing a random process on its own would have no way of « knowing » that it should avoid the zeros of the wave function.

To obtain a consistent picture we must consider the random background field. This is assumed, as we have already pointed out, to be the source of the random motions of the particle, but in addition it must determine a condition in space which gives rise to the osmotic velocity. Indeed, a single particle in random motion cannot contain any information capable of determining, for example, that every time it approaches the zero of a wave function it must turn around. This information could be contained only in the background field itself. Therefore, as has already been pointed out in section 1, the stochastic model does not fulfil the expectations that would at first signt be raised by its name, that is to provide an explanation solely in terms of the random movements of a particle without reference to a quantum mechanical field (which may be taken as ρ and S or as $\psi = \rho^{1/2}e^{iS/\hbar}$).

In the causal interpretation this field has the property that its effect does not depend on its amplitude. As has been suggested elsewhere [*18*], this behaviour can be understood in terms of the concept of active information. i. e. that the movement comes from the particle itself, which is however « informed » or « guided » by the field. In the stochastic interpretation there is a further effect of the quantum field through the osmotic velocity, which is also independent of its amplitude. We can therefore say that along with the mean velocity field, $\frac{\nabla S}{m}$, the osmotic velocity field constitutes active information which determines the average movement of the particle. This latter is however modified by a completely random component due to the fluctuations of the background field.

Clearly then, there are basic similarities between the causal interpretation and the stochastic interpretation (some of which will however be discussed only later). Nevertheless, there are also evidently important differences. One of the key differences can be seen by considering a stationary state with S = const. e. g. an *s*-state. In the stochastic model the particle is executing a random motion which would bring about diffusion into space, but the osmotic velocity is constantly drawing it back so that we obtain the usual spherical distribution as an average. But now the basic process is one of dynamic equilibrium the average velocity, which is zero, is the same as the actual velocity in the causal interpretation. Such a view of the s-state as one of dynamic equilibrium seems to fit in with our physical intuition better than one in which the particle is at rest.

3. THE MANY PARTICLE SYSTEM

The extension of this model to the many particle system is straightforward. The wave function, $\psi(x_{in})$, which is defined in a 3N-dimensional configuration space, satisfies the many-body Schrödinger equation. We assume that the mean velocity of nth particle is

$$v_{in} = \frac{1}{m} \frac{\partial}{\partial x_{in}} \mathrm{S}(x_{in}) \,. \tag{18}$$

In addition we assume an arbitrary probability density $\mathrm{P}(x_{in})$ and a random diffusion current of the nth particle

$$j_{in}^{(d)} = - \mathrm{D} \frac{\partial \mathrm{P}}{\partial x_{in}} \,. \tag{19}$$

We then make the further key assumption that the osmotic velocity components of the nth particle is

$$u_{in}^{(0)} = \frac{\mathrm{D}}{\rho} \frac{\partial \rho}{\partial x_{in}} \tag{20}$$

where $\rho = |\,\psi\,|^2$.

From here on the theory will go through as in the one-particle case and it will follow that the limiting distribution will be $\mathrm{P} = |\,\psi\,|^2$.

Equation (19) describes a stochastic process in which the different particles undergo statistically independent random fluctuations. However, in equation (20) we have introduced an important connection between the osmotic velocities of different particles. For the general wave function that does not split into independent factors, the osmotic velocities of different particles will be related and this relationship may be quite strong even though the particles are distant from each other. This means that we cannot eliminate quantum non-locality by going to the stochastic model. For in this model there is tacitly assumed an effectively instantaneous non-local connection which brings about the related osmotic velocities of distant particles (and in addition, of course, the mean acceleration equation (which will be given by an extension of eqn. (17)) will still be determined by a non-local quantum potential).

To bring out the full meaning of this non-locality it is useful to discuss the Einstein, Podolski and Rosen experiment in terms of the stochastic interpretation. This requires, however, that we extend the theory of the measurement process which was developed in connection with the causal interpretation to the stochastic model. We therefore first give a brief resume of how the causal interpretation treats the measurement process [18].

The point that is essential here is that in a measurement process the « particles » constituting the measurement apparatus can be shown to enter a definite « channel » corresponding to the actual result of the measurement. After this they cannot leave the channel in question because the wave function is zero between the channels. From this point on the wave function of the whole system effectively reduces to a product of the wave function of the apparatus and of the observed system. It was demonstrated that thereafter, the remaining « empty » wave packets will not be effective. It follows that the net result is the same as if there had been a collapse of the wave function to a state corresponding to the result of the actual measurement.

It is clear that a similar result will follow for the stochastic interpretation because here too there is no probability that a particle can enter the region between the « channels » in which the wave function is zero.

Let us now consider the EPR experiment in the stochastic interpretation. We recall that in this experiment we have to deal with an initial state of a pair of particles in which the wave function is not factorizable. A measurement is then made determining the state of one of these particles, and it is inferred from the quantum mechanics that the other will go into a corresponding state even though the Hamiltonian contains no interaction terms that could account for this. In the causal interpretation the behaviour of the second particle was explained by the non-local features of the quantum potential which could provide for a direct interaction between the two different particles that does not necessarily fall off with their separation and that can be effective even when there are no interaction terms in the Hamiltonian.

In the stochastic interpretation all the above non-local effects of the quantum potential are still implied, but in addition there is a further non-local connection through the osmotic velocities. And when the properties of the first particle are measured the osmotic velocity will be instantaneously affected in such a way as to help bring about the appropriate correlations of the results.

It is clear that the stochastic interpretation and the causal interpretation treat the non-local EPR correlations in a basically similar way. The essential point is that in an independent disturbance of one of the particles, the fields acting on the other particle (osmotic velocities and quantum potential) respond instantaneously even when the particles are far apart. It is as if the two particles were in instantaneous two-way communication exchanging active information that enables each particle to « know » what has happened to the other and to respond accordingly.

Of course in a non-relativistic theory it is consistent to assume such instantaneous connections. In the talk, as well as in the article [*1*], it was shown that this approach can be extended to the Dirac equation, thus permitting the development of a consistent relativistic stochastic interpretation of the quantum theory in a basically similar way.

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