M. D. Missarov

Random fields on the adele ring and Wilson's renormalization group

Annales de l'I. H. P., section A, tome 50, nº 3 (1989), p. 357-367

<http://www.numdam.org/item?id=AIHPA_1989__50_3_357_0>

Ann. Inst. Henri Poincaré,

Vol. 49, n° 3, 1989, p. 357-367. *Physique théorique*

Random fields on the adele ring and Wilson's renormalization group

by

M. D. MISSAROV (1) (2)

Centre de Physique Théorique, (3)
C.N.R.S., Luminy, Case 907,
13288 Marseille Cedex 09

ABSTRACT. – We give the definitions of generalized random field on the adele ring, corresponding renormalization group and discuss the discretization procedure. The gaussian translation and scaling invariant random fields are described.

RÉSUMÉ. – Nous définissons les champs aléatoires sur le corps des adèles, le groupe de renormalisation correspondant et nous discutons la procédure de discrétisation. Nous décrivons les champs gaussiens invariants par translation et par dilatation.

(1) This work was supported in part by the French Government.

(2) Permanent address: Department of Applied Mathematics, Kazan State University, Kazan, U.S.S.R.

(3) Laboratoire Propre LP. 7061, Centre National de la Recherche Scientifique.

1. INTRODUCTION

Recently there a number of papers which discuss a possible use of p-adic numbers and adeles in string theory, statistical physics, quantum mechanics (*see* [1]-[13]) have been published.

We have shown previously (*see* [1], [2]) that the discretization of some scalar p-adic field models are a models of Dyson's type, and we have also described non-gaussian branch of fixed points of p-adic renormalization group. As in the real case (*see* [14]), the hamiltonian of non-gaussian fixed point in p-adic case is an analytic renormalization of projection of φ^4-hamiltonian on the unit ball. Therefore it is natural to generalize this theory on the adelic case. In the paragraph 2 we give the definition of Wilson's renormalization group for generalized random fields on the adele group and describe the gaussian branch of translation and scaling (self-similar) invariant random fields. In the paragraph 3 the discretization procedure is discussed. As a result we have a random fields on the set of rational numbers and definition of renormalization group in this case. This set is "non-distributive ring" with unusual addition and multiplication.

2. GENERALIZED RANDOM FIELDS ON THE ADELE GROUP AND WILSON'S RENORMALIZATION GROUP

Let Q be the field of rational numbers, R a real completion of Q, Q_p a p-adic completion of Q, where p is a prime number, Z_p a ball of p-adic innteegers

$$Z_p = \{ a \in Q_p : \| a \|_p \leqq 1 \}, \tag{2.1}$$

and U_p a multiplicative group of units

$$U_p = \{ a \in Q_p : \| a \|_p = 1 \}, \tag{2.2}$$

where $\| \ \|_p$ is a p-adic norm. The restricted direct product of additive groups R^+, Q_p^+, $p=2, 3, 5, \ldots$ with respect to Z_p^+, $p=2, 3, 5, \ldots$ is called the adele group of Q and is denoted by A. The restricted direct product of multiplicative groups R, Q_p, $p=2, 3, 5, \ldots$ with respect to the units U_p, $p=2, 3, 5, \ldots$ is called the idele group of Q and is denoted by A^*. In other words, A is a set of all sequences of the form

$$a = (a_\infty, a_2, \ldots, a_p, \ldots),$$

where $a_\infty \in R$, $a_p \in Q_p$, $p=2, 3, \ldots$ and $a_p \in Z_p$ for almost all p. The set of all such sequences forms a ring under componentwise addition and multiplication. The additive group of this group is called the group of adeles. The elements of the group of ideles are sequences

$$\lambda=(\lambda_\infty, \lambda_2, \ldots, \lambda_p, \ldots),$$

where $\lambda_\infty \in \mathrm{R}$, $\lambda_\infty \neq 0$, $\lambda_p \in \mathrm{Q}_p$, $\lambda_p \neq 0$, $p=2, 3, \ldots$ and $\|\lambda_p\|_p=1$ for almost all p. A* is a set of adeles that have an inverse.

A sequence of adeles $a^{(n)}=(a_\infty^{(n)}, \ldots, a_p^{(n)}, \ldots)$ converges to the adele $a=(a_\infty, \ldots, a_p, \ldots)$ if it converges to a componentwise and if there is an N such that for $n \geqq \mathrm{N}$ $a_p-a_p^{(n)}$ are p-adic integers.

The group of adeles A, being locally compact, has an invariant measure which we denote by da. This measure will be normalized by the following condition

$$\int_\Omega da=1, \tag{2.3}$$

where the integral is taken over the compact set Ω of adeles $a=(a_\infty, \ldots, a_p, \ldots)$ for which

$$0 \leqq a_\infty < 1, \quad \|a_p\|_p \leqq 1, \qquad p=2, 3, \ldots$$

There is a well known Schwartz space of functions on the field of real numbers.

There is a space of test functions $f(a_p)$ on Q_p satisfying the following two requirements:

1. The function $f(a_p)$ is finite, *i. e.* supp f is a compact set.
2. There exists n (depending on f) such that $f(a_p)=f(b_p)$ if $\|a_p-b_p\|_p \leqq p^n$.

We say that a sequence of functions $f_i(a_p)$ tends to zero if:

1. The functions $f_i(a_p)$ are zero outside some fixed compact set (independent on i).
2. There exists a positive integer N such that all the functions $f_i(a_p)$ satisfy the condition

$$f_i(a_p)=f_i(b_p) \qquad \text{if} \quad \|a_p-b_p\|_p \leqq p^n.$$

3. The sequence $f_i(a_p)$ tends to zero uniformly in a_p, as i tends to infinity.

We denote this spase of functions as $\mathrm{S}(\mathrm{Q}_p)$. Now we consider functions $f(a)$ on the adele group that are representable in the form of an infinite product

$$f(a)=f_\infty(a_\infty) f_2(a_2) \ldots f_p(a_p) \ldots, \tag{2.4}$$

where $f_\infty \in \mathrm{S}(\mathrm{R})$, $f_p(a_p) \in \mathrm{S}(\mathrm{Q}_p)$, $p=2, 3, \ldots$, and for all p, except a finite number, $f_p(a_p)=1$, when a_p is a p-adic integer, and $f_p(a_p)=0$ otherwise. Thus, the product (2.4) converges, and it is easy to see that $f(a)$ is a continuous function on A. The functions of the form (2.4) are called elementary functions on A. The space S(A) of all functions $f(a)$ on A that are representable as finite linear combinations of elementary functions is called Schwartz-Bruhat space (*see* [15]). Finally, let some topology on S(A) be given. For example, $f_n(a) \to 0$, $n \to \infty$, if there exists such representations

$$f_n(a) = \sum_{k=1}^{m} f_{k,\infty}^n f_{k,2}^n \dots f_{k,p}^n \dots,$$

m does not depend on n, and such N that $f_{k,p}^n$ are a characteristic function of the ball Z_p for all $p > N$, all n and

$$f_{k,\infty}^n \to 0 \quad \text{in the topology } S(R),$$
$$f_{k,p}^n \to 0 \quad \text{in the topology } S(Q_p).$$

Let a generalized random field $P(\varphi)$ in the A be given, *i. e.* a system of probability distributions $P\{(\varphi, f_1), \dots, (\varphi, f_m)\}$ with the usual conditions of accordance (*see* [16]). Here $f_1 = f_1(a), \dots, f_m = f_m(a)$ are arbitrary test functions in Schwartz-Bruhat space.

We introduce two groups of continuous transformations on random fields. The group of shift transformations $T - \{t_b, b \in A\}$ consists of

$$t_b P(\varphi) = P(t_b \varphi), \tag{2.5}$$

where $(t_b \varphi(a), f(a)) = (\varphi(a), f(a+b))$ and

$$P\{(t_b \varphi, f_1), \dots, (t_b \varphi, f_m)\}$$
$$= P\{(\varphi(a), f_1(a+b)), \dots, (\varphi(a), f_m(a+b))\}. \tag{2.6}$$

Fixing the real number α, $1 < \alpha < 2$, we define the scaling operator

$$R_\lambda^\alpha P(\varphi) = P(|\lambda|^{1-(\alpha/2)} \varphi_\lambda), \tag{2.7}$$

where $\lambda \in A^*$, $\lambda = (\lambda_\infty, \lambda_2, \dots, \lambda_p, \dots)$,

$$|\lambda| = \|\lambda_\infty\|_\infty \|\lambda_2\|_2 \dots \|\lambda_p\|_p \dots, \tag{2.8}$$

$\| \;\|_\infty$ is a usual absolut value on R,

$$(\varphi_\lambda, f) = (\varphi(\lambda a), f(a)) = \|\lambda\|^{-1} (\varphi(a), f(a\lambda^{-1})), \tag{2.9}$$

and $P(|\lambda|^{1-(\alpha/2)} \varphi_\lambda)$ is a generalized random field with probability distributions

$$P\{(|\lambda|^{1-(\alpha/2)} \varphi_\lambda, f_1), \dots, (|\lambda|^{1-(\alpha/2)} \varphi_\lambda, f_m)\}.$$

A generalized random field is translation and scaling invariant if

$$t_b P(\varphi) = P(\varphi), \tag{2.10}$$
$$R_\lambda^\alpha P(\varphi) = P(\varphi) \tag{2.11}$$

for all $b \in A$, $\lambda \in A^*$.

If φ is a translation and scaling invariant random field on A, then its binary correlation function must satisfy the following condition

$$d(a-b) = \langle \varphi(a)\varphi(b) \rangle$$
$$= \langle R_\lambda^\alpha \varphi(a), R_\lambda^\alpha \varphi(b) \rangle = |\lambda|^{2-\alpha} d(\lambda(a-b)) \quad (2.12)$$

for all a, $b \in A$, $\lambda \in A^*$. It is natural to propose the following possible solution of (2.12):

$$d(a-b) = \text{Const.}\ |a-b|^{\alpha-2}. \quad (2.13)$$

But unfortunately the function $|a|$ is well defined only on the idele group. Therefore we introduce the following family of distributions on S(A):

$$g_\gamma(a) = c(\gamma)|a|^\gamma, \quad (2.14)$$

where γ is a real number, $\gamma \neq -1$, and

$$c(\gamma) = \prod_p c_p(\gamma), \quad (2.15)$$

$$(c_p(\gamma))^{-1} = \int_{\|a_p\|_p \leq 1} da_p = \frac{p-1}{p} \frac{1}{1-p^{-\alpha-1}}, \quad (2.16)$$

the product is taken over all prime p. Note, that the product diverges:

$$c(\gamma) = \frac{\zeta(\gamma+1)}{\zeta(1)}, \quad (2.17)$$

where $\zeta(s)$ is the Riemann Zeta-function. Moreover, $|a| = \|a_\infty\|_\infty \|a_2\|_2 \dots \|a_p\|_p \dots$ is equal to zero for every $a \in A \setminus A^*$. Newertheless, g_γ is well defined as a distribution on S(A). If $f(a) = f_\infty(a_\infty) f_2(a_2) \dots f_p(a_p) \dots$ is an elementary function, then

$$(g_\gamma, f) = \int f(a_\infty)\|a_\infty\|_\infty^\gamma da_\infty \prod_p (c_p(\gamma)\int f_p(a_p)\|a_p\|_p^\gamma da_p), \quad (2.18)$$

where da_∞ is the the Lebesgue measure on R. For all p, except a finite number

$$c_p(\gamma)\int f_p(a_p)\|a_p\|_p^\gamma da_p = c_p(\gamma)\int_{\|a_p\|_p<1} \|a_p\|_p^\gamma da_p = 1 \quad (2.19)$$

and then all the factors in the product (2.18) from a certain p onward are equal to 1. To the whole space S(A) the integral (2.18) is extended by linearity.

Now we can define the gaussian random field with binary correlation function

$$\langle (\varphi, f_1)(\varphi, f_2) \rangle = \int_A \int_A f_1(a) f_2(b) g_{\alpha-2}(a-b)\, da\, db. \quad (2.20)$$

For elementary functions

$$f_1(a)=f_{1,\infty}(a_\infty)f_{1,2}(a_2)\dots f_{1,p}(a_p)\dots,$$
$$f_2(a)=f_{2,\infty}(a_\infty)f_{2,2}(a_2)\dots f_{2,p}(a_p)\dots$$

the integral (2.20) must be understood in the following sense:

$$\langle(\varphi, f_1)(\varphi, f_2)\rangle = \int f(a_\infty)f(b_\infty)\|a_\infty-b_\infty\|_\infty^{\alpha-2}\,da_\infty\,db_\infty \times \prod_p \Big(c_p(\alpha-2)\int f_p(a_p)f_p(b_p)\|a_p-b_p\|_p^{\alpha-2}\,da_p\,db_p\Big). \quad (2.21)$$

For all p with a finite exception

$$c_p(\alpha-2)\int f_p(a_p)f_p(b_p)\|a_p-b_p\|_p^{\alpha-2}\,da_p\,db_p = c_p(\alpha-2)\int_{\|b_p\|_p\leq 1}\int_{\|a_p\|_p\leq 1}\|a_p-b_p\|_p^{\alpha-2}\,da_p\,db_p=1. \quad (2.22)$$

Thus, the product (2.21) converges.

It is easy to verify that this correlation function really determines the gaussian translation and scaling invariant random field (note, that the set of elementary functions is invariant under the shift and scaling transformations).

Now we define the Wilson's renormalization group transformation for the random fields in momentum space representation. Note, that the Schwartz-Bruhat space is invariant under Fourier transformation: if $f(a)\in S(A)$, then $(Ff)(k)\in S(A)$,

$$(Ff)(k)=\int_A \exp\{2\pi i(-k_\infty a_\infty+k_2a_2+\dots+k_pa_p+\dots)\}f(a)\,da. \quad (2.23)$$

Let a generalized random field $P(\sigma)$ in the domain

$$\Omega=\{k\in A: 0\leq k_\infty<1,\ \|k\|_p\leq 1,\ p=2, 3, 5, \dots\} \quad (2.24)$$

be given, *i. e.* a system of probability distributions $P\{(\sigma, f_1), \dots, (\sigma, f_m)\}$ with the usual conditions of accordance. Here $f_i(k)\in S(A)$, $\operatorname{supp} f_i\subset\Omega$, $i=1, 2, \dots, m$.

If $P(\sigma)$ is a generalized random field in the domain $\lambda\Omega$,

$$\lambda\in\Lambda=\{\lambda\in A^*: \lambda_\infty>1,\ \|\lambda\|_p\geq 1,\ p=2, 3, 5, \dots\}, \quad (2.25)$$

we denote by $S_{\Omega,\lambda}$ the operator of restriction on the domain Ω,

$$S_{\Omega,\lambda}P=P|_\Omega. \quad (2.26)$$

The scaling operator in momentum space representation is defined as

$$R_\lambda^\alpha P(\sigma)=P(|\lambda|^{1-(\alpha/2)}\sigma_{\lambda^{-1}}), \quad (2.27)$$

where $(\sigma_{\lambda^{-1}}, f)=(\sigma(\lambda^{-1}k), f(k))=|\lambda|(\sigma(k), f(\lambda k))$.

The Wilson's renormalization transformation $R^{\alpha}_{\Omega,\lambda}$, $\lambda\in\Lambda$, is a composition of the transformations R^{α}_{λ} and $S_{\Omega,\lambda}$:

$$R^{\alpha}_{\Omega,\lambda}=S_{\Omega,\lambda}R^{\alpha}_{\lambda}. \tag{2.28}$$

It is easy to see, that

$$R^{\alpha}_{\Omega,\lambda}R^{\alpha}_{\Omega,\mu}=R^{\alpha}_{\Omega,\lambda\mu}, \qquad \lambda, \mu\in\Lambda. \tag{2.29}$$

A generalized random field is scaling invariant in Ω, if

$$R^{\alpha}_{\Omega,\lambda}P(\sigma)=P(\sigma) \tag{2.30}$$

for all $\lambda\in\Lambda$.

We introduce the Fourier transform of the translation operator as

$$t_a:\quad \sigma(k)\to\exp\{2\pi i(-k_\infty a_\infty+k_2a_2 +\ldots+k_pa_p+\ldots)\}\sigma(k), \qquad a\in A. \tag{2.31}$$

A random field is called translation invariant if $P(\sigma)=P(t_a\sigma)$ for any $a\in A$.

A generalized random field with zero mean and binary correlation function

$$\langle\sigma(k)\sigma(k')\rangle=\delta(k+k')g_{1-\alpha}(k)\chi_\Omega(k) \tag{2.31}$$

is translation and scaling invariant in Ω random field. Here

$$\delta(k+k')=\delta(k_\infty+k'_\infty)\delta(k_2+k'_2)\ldots\delta(k_p+k'_p)\ldots, \tag{2.32}$$

$\chi_\Omega(k)$ is a characteristic function of the Ω, and for elementary functions

$$f_1(k)=f_{1,\infty}(k_\infty)f_{1,2}(k_2)\ldots f_{1,p}(k_p)\ldots,$$
$$f_2(k)=f_{2,\infty}(k_\infty)f_{2,2}(k_2)\ldots f_{2,p}(k_p)\ldots$$

$$\langle\sigma(f_1)\overline{\sigma(f_2)}\rangle=\int f_{1,\infty}(k_\infty)\overline{f_{2,\infty}(k_\infty)}\,\|k_\infty\|_\infty^{1-\alpha}\chi_\infty(k_\infty)\,dk_\infty \times\prod_p(c_p(\alpha-2)\int f_{1,p}(k_p)\overline{f_{2,p}(k_p)}\,\|k_p\|_p^{1-\alpha}\chi_p(k_p)\,dk_p), \tag{2.33}$$

$\chi_\infty(k_\infty)$ is a characteristic function of the $\Omega_\infty=\{k_\infty\in R: 0\leqq k_\infty<1\}$, $\chi_p(k_p)$ is a characteristic function of the

$$\Omega_p=\{k_p\in Q_p: \|k_p\|\leqq 1\}, \qquad p=2, 3, \ldots$$

The family of binary correlation functions, which is given by (2.31), describes the gaussian brunch of fixed points of the Wilson's renormalization group. The very interesting, but quite more complicated case of adelic nongaussian branch which bifurcates from the gaussian one is under present investigation.

3. DISCRETIZATION OF THE RANDOM FIELDS ON THE ADELE GROUP

As is known (*see* [14]), the field of rational numbers Q can be isomorphically embedded in the ring of adeles A. With every rational number r is associated the sequence

$$(r, r, \ldots, r, \ldots) \in \mathrm{A}. \qquad (3.1)$$

This sequences are adeles, which are called principal adeles. Moreover, the ring Q of principal adeles is discrete in A. The set Ω of adeles $a=(a_\infty, \ldots, a_p, \ldots)$ for which $0 \leqq a_\infty < 1$, $\|a_p\|_p \leqq 1$, $p=2, 3, \ldots$ is a fundamental domain of the additive group of A relative to the subgroup of principal adeles Q.

But this natural lattice in A is not adequate to the discretization procedure because Q is not invariant under the multiplication by the idele $\lambda \in \Lambda$ [with only trivial exception $\lambda=(1, 1, \ldots, 1, \ldots)$]. Therefore we introduce other lattice as follows. Let us consider the set of adeles

$$\mathrm{D}=\{a=(a_\infty, \ldots, a_p, \ldots) \in \mathrm{A}: a_\infty \in \mathrm{Z},\ a_p=\{a_p\}_p,\ p=2, 3, \ldots\}. \qquad (3.2)$$

Here Z is a ring of rational integer numbers and the function $\{a_p\}_p$ is defined as follows: if

$$a_p=\sum_{i=n}^{\infty} c_i p^i, \qquad 0 \leqq c_i < p-1, \quad c_n \neq 0, \qquad (3.3)$$

then

$$\{a_p\}_p=\sum_{i=n}^{-1} c_i p^i, \quad \text{if } n<0 \qquad \text{and} \qquad \{a_p\}_p=1, \quad \text{if } n \geqq 0. \qquad (3.4)$$

Note, that if $a \in \mathrm{D}$ and $a_\infty \neq 0$, then $a \in \mathrm{A}^*$. Every adele $a \in \mathrm{D}$ has a form

$$a=\left(n, \frac{n_2}{2^{m_2}}, \ldots, \frac{n_p}{p^{m_p}}, \ldots, 1, 1, \ldots\right), \qquad (3.5)$$

where $n, n_p, m_p \in \mathrm{Z}$, $0 \leqq m_p$, $1 \leqq n_p \leqq p^{m_p}$, $p=2, 3, \ldots$ The set of all such adeles forms a "non-distributive ring" under the addition

$$a \oplus b=(a_\infty+b_\infty, \{a_2+b_2\}_2, \ldots, \{a_p+b_p\}_p, \ldots) \qquad (3.6)$$

and multiplication

$$ab=(a_\infty b_\infty, a_2 b_2, \ldots, a_p b_p, \ldots). \qquad (3.7)$$

Note, that the addition $\oplus$ does not coinside with the usual addition on A. There is one-to-one correspondence between the set D and the set of rational numbers Q. To every $r \in \mathrm{Q}$ corresponds the sequence

$$d(r)=([r]_\infty, \{r\}_2, \ldots, \{r\}_p, \ldots) \in \mathrm{D}, \qquad (3.8)$$

where [ ] is a usual integer part in Q. On the other hand, to every adele $a \in D$ corresponds the rational number

$$d^{-1}(a) = a_\infty + \{a_2 + \dots a_p + \dots\}, \tag{3.9}$$

where

$$\{a_2 + \dots a_p + \dots\} = (a_2 + \dots a_p + \dots) \bmod 1. \tag{3.10}$$

Since all the numbers a_p beginning with sufficiently large p are equal to 1, (3.9) is well defined. For $a \in D$ $a_q \in Z_p$, if $q \neq p$, $p = 2, 3, \dots$, $q = \infty, 2, 3, \dots$ and therefore $\{d^{-1}(a)\}_p = a_p$. So, $d(d^{-1}(a)) = a$ and d is an isomorphism between the "ring" D and the "ring" Q of rational numbers with non-usual addition and multiplication

$$r \oplus s = [r] + [s] + \{r + s\}, \tag{3.11}$$

$$r \otimes s = [r][s] + \{r_2 s_2 + \dots + r_p s_p + \dots\}, \tag{3.12}$$

where $r_p = \{r\}_p$, $p = 2, 3, \dots$

Let $a = (a_\infty, a_2, \dots, a_p, \dots\}$ be an arbitrary adèle. Then a can be represented in the form

$$a = r(a) + b, \qquad r(a) \in D, \quad b \in \Omega, \tag{3.13}$$

where $r(a) = ([a_\infty], \{a_2\}_2, \dots, \{a_p\}_p, \dots)$, $b = a - r(a)$. We see that A is a union of pairwise disjoint sets $r + \Omega$, where r ranges over the D.

Let φ be a generalized random field on A (in coordinate space representation). The discretization of φ is defined as a random field ξ on D such that

$$\xi = \{\xi(r) = (\varphi, \chi_r), r \in D\}, \tag{3.14}$$

where $\chi_r(a)$ is a characteristic function of the set $r + \Omega$.

The action of shift operation t_s on ξ is defined as

$$(t_s \xi)(r) = \xi(r \oplus s), \qquad s \in D. \tag{3.15}$$

The renormalization group transformation is defined as

$$(r_\lambda^\alpha \xi)(r) = \int_{r+\Omega} |\lambda|^{1-(\alpha/2)} \varphi(\lambda a)\, da = |\lambda|^{-(\alpha/2)} \int_{\lambda(r+\Omega)} \varphi(a)\, da = |\lambda|^{-(\alpha/2)} \sum_{s \in D_\lambda} \xi(r\lambda \oplus s), \tag{3.16}$$

where $\lambda \in D$, $\lambda_\infty \neq 0$, $|\lambda| = \|\lambda_\infty\|_\infty \|\lambda_2\|_2 \dots \|\lambda_p\|_p \dots$,

$$D_\lambda = \{s \in D : 0 \leqq s_\infty < \|\lambda_\infty\|_\infty, 1 \leqq \|s_p\|_p \leqq \|\lambda_p\|_p\}. \tag{3.17}$$

Here we use that $\lambda\Omega = \bigcup (s + \Omega)$, s goes over D_λ. Note also that if $\lambda \in D$, $\lambda_\infty \neq 0$, then $\lambda \in \Lambda$. The definition (3.17) is a generalization of a block-spin transformation on the lattice of integer numbers in real case or on the hierarchical lattice in p-adic case.

If a generalized random field φ is a translation and scaling invariant, then its discretization is invariant relative to actions t_s, $s \in \mathrm{D}$ and r_λ^α, $\lambda \in \mathrm{D}$, $|\lambda| \geqq 1$.

Let us consider the gaussian translation and scaling invariant random field φ, which is given by the correlation function $g_{\alpha-2}(a-b)$. The binary correlation function of its discretization ξ is equal to

$$\langle \xi(r) \xi(s) \rangle = \int_{r+\Omega} \int_{s+\Omega} g_{\alpha-2}(a-b)\, da\, db$$

$$= \int_{r_\infty+\Omega_\infty} \int_{s_\infty+\Omega_\infty} \| a_\infty - b_\infty \|_\infty^{\alpha-2} da_\infty\, db_\infty$$

$$\times \prod_p (c_p(\alpha-2) \int_{r_p+\Omega_p} \int_{s_p+\Omega_p} \| a_p - b_p \|_p^{\alpha-2} da_p\, db_p), \quad (3.18)$$

where $\Omega_\infty = \{ a_\infty \in \mathrm{R} : 0 \leqq a_\infty < 1 \}$, $\Omega_p = \mathrm{Z}_p$, $r = (r_\infty, r_2, \ldots, r_p, \ldots)$, $s = (s_\infty, s_2, \ldots, s_p, \ldots)$. It is easy to see that the function

$$\Psi_\infty(r_\infty - s_\infty) = \int_{r_\infty+\Omega_\infty} \int_{s_\infty+\Omega_\infty} \| a_\infty - b_\infty \|_\infty^{\alpha-2} da_\infty\, db_\infty$$

$$= \int_{\Omega_\infty} \int_{\Omega_\infty} \| r_\infty - s_\infty + a_\infty - b_\infty \|_\infty^{\alpha-2} da_\infty\, db_\infty \quad (3.19)$$

coinsides with the binary correlation function of the gaussian translation and scaling invariant (self-similar) random process on the lattice of rational integer numbers (*see* [16]), and the function

$$\Psi_p(r_p - s_p) = \int_{r_p+\Omega_p} \int_{s_p+\Omega_p} \| a_p - b_p \|_p^{\alpha-2} da_p\, db_p$$

$$= \int_{\Omega_p} \int_{\Omega_p} \| r_p - s_p + a_p - b_p \|_p^{\alpha-2} da_p\, db_p = \| \{ r_p - s_p \}_p \|_p^{\alpha-2} \quad (3.20)$$

coinsides with the binary correlation function of the gaussian translation and scaling invariant random field on the hierarchical lattice of p-adic purely fraction numbers (see [1], [2], [18]-[21]). As $\{ r_p - s_p \}_p = 1$ for sufficiently large p, the product (3.18) converges. So,

$$\langle \xi(r) \xi(s) \rangle = \Psi(r-s) = \Psi_\infty(r_\infty - s_\infty) \Psi_2(r_2 - s_2) \ldots \Psi_p(r_p - s_p) \ldots \quad (3.21)$$

exists as a usual function. Note, that $r \in \mathrm{D}$, $s \in \mathrm{D}$ and $r-s$ must be understood in the sense of the "non-distributive" ring D.

ACKNOWLEDGEMENTS

This work was done at the Centre de Physique Théorique, C.N.R.S.-Luminy, Marseille. The author would like to thank Prof. J. Bellisard for useful discussions.

REFERENCES

[1] E. Y. LERNER and M. D. MISSAROV, The Scalar Models of p-Adic Quantum Field Theory and Hierarchical Model of Dyson, submitted to *Teor. Mat. Fiz.*
[2] E. Y. LERNER and M. D. MISSAROV, P-*Adic Feynman and String Amplitudes*, preprint CPT-88/P 2126, C.N.R.S.-Luminy.
[3] I. V. VOLOVICH, *Number Theory as the Ultimate Physical Theory*, preprint C.E.R.N.-TH. 4781/87.
[4] I. V. VOLOVICH, *Teor. Mat. Fiz.*, Vol. **71**, 1987, p. 337; *Classical and Quantum Gravity*, Vol. **4**, 1987, p. L83.
[5] B. GROSSMAN, *Phys. Lett. B*, Vol. **197**, 1987, p. 101.
[6] P. G. O. FREUND and E. WITTEN, *Phys. Lett. B*, Vol. **199**, 1987, p. 191.
[7] L. BREKKE, P. G. O. FREUND, M. OLSON and E. WITTEN, *Non-Archimedian String Dynamics*, University of Chicago, Preprint EFI-87-101.
[8] I. YA. AREF'EVA, B. G. DRAGOVIC and I. V. VOLOVICH, *Phys. Lett. B*, Vol. **200**, 1988, p. 512.
[9] J. L. GERVAIS, *Phys. Lett. B*, Vol. **201**, 1988, p. 306.
[10] E. MARINARI and G. PARISI, *Phys. Lett. B*, Vol. **203**, 1988, p. 52.
[11] I. YA. AREF'EVA, B. G. DRAGOVIC and I. V. VOLOVICH, *Phys. Lett. B*, Vol. **209**, 1988, p. 445.
[12] B. L. SPOKOINY, *Phys. Lett. B*, Vol. **209**, 1988, p. 401.
[13] C. ALACOQUE, P. RUELLE, E. THIRAN, D. VERSTEGEN and J. WEYERS, *Phys. Lett. B*, Vol. **211**, 1988, p. 59.
[14] P. M. BLEHER and M. D. MISSAROV, *Commun. Math. Phys.*, Vol. **74**, 1980, p. 255.
[15] I. M. GEL'FAND, M. I. GRAEV and I. I. PYATETSKII-SHAPIRO, *Representation Theory and Automorphic Functions*, Saunders, London, 1966.
[16] I. M. GEL'FAND and N. Y. VILENKIN, *Applications of Harmonic Analysis*, Academic Press, 1964.
[17] Ya. G. SINAI, *Theory of prob. and its appl.*, Vol. **21**, 1976, p. 63.
[18] P. M. BLEHER and Y. G. SINAI, *Commun. Math. Phys.*, Vol. **45**, 1975, p. 247.
[19] P. COLLET and J. P. ECKMANN, A Renormalization Group Analysis of the Hierarchical Model in Statistical Mechanics, *Lect. Notes Phys.*, Vol. **74**, 1978, pp. 1-199.
[20] P. M. BLEHER, *Commun. Math. Phys.*, Vol. **84**, 1982, p. 557.
[21] K. GAWEDZKI and A. KUPIAINEN, *Journal of Stat. Physics*, Vol. **29**, 1982, p. 683.

(*Manuscrit reçu le* 27 *septembre* 1988.)