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**PROBLEMS IN BOOLEAN ALGEBRAS**

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Let  $B = (B, 0, 1, \wedge, \vee, \neg)$  be a Boolean algebra. Let  $L$  be a subset of  $B$ .  $L$  is said to be a discrete set (of  $B$ ) if and only if  $L$  is a set of pairwise incomparable elements of  $B$ , i.e. if  $x$  and  $y$  are distinct members of  $L$  such that  $x \wedge y = y$ , then  $x = y$ . For instance an antichain of  $B$  is a discrete set. There is an uncountable Boolean algebra such that every antichain is countable.

*Theorem 1.* Assume Martin's axiom. There is a Boolean algebra of power  $2^\omega$  such that every discrete set is of power  $< 2^\omega$ .

*Problem 1.* Does there is an uncountable Boolean algebra such that all discrete sets and all chains are countable ?

A subset  $D$  of  $B$  is said to be a *strong generator* of  $B$  iff every element of  $B$  is the infinite supremum of members of  $D$ . A strong generator is a dense subset of  $B$ . There is an uncountable Boolean algebra with a countable strong generator.

*Theorem 2.* Assume C.H. there is an uncountable Boolean algebra with a countable strong generator such that all discrete sets are countable.

*Problem 2.* Is there an uncountable Boolean algebra with a countable strong generator such that all discrete sets and all chains are countable ?