ANNALES SCIENTIFIQUES
DE L'UNIVERSITÉ DE CLERMONT-FERRAND 2
Série Mathématiques

R. BONNET

Problems in Boolean algebras

Annales scientifiques de l'Université de Clermont-Ferrand 2, tome 60, série *Mathématiques*, nº 13 (1976), p. 75

<http://www.numdam.org/item?id=ASCFM_1976__60_13_75_0>

Ann. Sci. Univ. Clermont, Sér. Math., fasc. 13, 1976, pp. 75

PROBLEMS IN BOOLEAN ALGEBRAS

R. BONNET

Université Claude Bernard, LYON, France

Let $B = (B, 0, 1, \wedge, \vee, \neg)$ be a Boolean algebra. Let L be a subset of B. L is said to be a discrete set (of B) if and only if L is a set of pairwise incomparable elements of B, i.e. if x and y are distinct members of L such that $x \wedge y = y$, then $x = y$. For instance an antichain of B is a discrete set. There is an uncountable Boolean algebra such that every antichain is countable.

Theorem 1. Assume Martin's axiom. There is a Boolean algebra of power 2^{ω} such that every discrete set is of power $< 2^{\omega}$.

Problem 1. Does there is an uncountable Boolean algebra such that all discrete sets and all chains are countable ?

A subset D of B is said to be a *strong generator* of B iff every element of B is the infinite supremum of members of D. A strong generator is a dense subset of B. There is an uncountable Boolean algebra with a countable strong generator.

Theorem 2. Assume C.H. there is an uncountable Boolean algebra with a countable strong generator such that all discrete sets are countable.

Problem 2. Is there an uncountable Boolean algebra with a countable strong generator such that all discrete sets and all chains are countable ?