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CORRECTIONS TO MY PAPER « A STURM-LIOUVILLE THEOREM FOR NONLINEAR ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS »

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By MELVYN S. BERGER

The author is grateful to Professor Colin Clark for pointing out a number of omissions and incorrect statements in the above paper. In particular in order to obtain certain uniform estimates needed in the proofs of Lemmas 1.3.2 and IV. 3.4 it should be *explicitly* assumed that all variational operators A of the various classes considered satisfy a uniform Lipschitz condition on bounded subsets of X, i. e.

$$\| Ax - Ay \| \leq c(K) \| x - y \| \text{ whenever } \| x \|, \| y \| \leq K$$

where $c(K)$ is a constant independent of x and y. Thus for the partial differential operators considered we must assume throughout that $A_\alpha(x, z)$ and $B_\alpha(x, z)$ satisfy a local Lipschitz condition in the z variables for $x \in G$.

Furthermore the following changes are also necessary :

Page 549 (Last line) The extra coerciveness assumption should be

$$(u, Au) \longrightarrow \infty \text{ as either } \| u \| \longrightarrow \infty \text{ or } \int_0^1 (u, A(su))\, ds \longrightarrow \infty$$

Page 552 (Last line) add after on « compact subsets of »

Page 554 (Last line in Lemma 1.3.1) add « and R sufficiently large ». In the subsequent proof read $-a'(t)$ in place of $a'(t)$.

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Page 555 In the statement of I. 3.2 replace θ by $-\theta$ and K_ε by $K\varepsilon$. The proof of the lemma is incorrect and should be altered as follows:

By virtue of Lemma 1.3.1

$$\frac{dv}{dt} = Cu + a'(t)\,u = Cu - \frac{(Cu, Av)}{(u, Av)}\,u \tag{1}$$
$$v(0) = u$$

defines an intial value problem for an ordinary differential equation in X such that for $u \in \partial A_R$, $v(t)$ lies on ∂A_R for $|t|$ sufficiently small. As A is locally Lipschitz continuous, $a'(t)$ is Lipschitz continuous in v for $\|v-u\| \leq \varepsilon$ where ε is a small constant independent of $u \in \partial A_R$. Hence by standard results for such equations (1) has a solution $v(t) = f(u, t)$, which is uniformly continuous with respect to $v(0) = u$, belonging to the compact set a, and with common domain of existence $[-t_\varepsilon, t_\varepsilon]$ independent of $u \in a$. Then set $\theta(t, u) = a(t)\,t^{-1} \cdot \theta(t, u)$ is a continuous function of t and u; for at $t = 0$,

$$\lim_{t \to 0} \theta(t, u) = \lim_{t \to 0} t^{-1} a(t) = a'(0) = -\frac{(Cu, Au)}{(u, Au)}.$$

Furthermore, by the Mean Value Theorem and Lipschitz continuity of $a'(t)$:

$$\left| \theta(t,u) + \frac{(Cu, Au)}{(u, Au)} \right| = |a'(\theta) - a'(0)| \text{ where } 0 < \theta < 1$$
$$\leq K \| v(\theta) - v(0) \| \leq K\varepsilon$$

Page 557 in formulae (1) and (2) replace D^α by $(-1)^{|\alpha|} D^\alpha$.

Page 558 in the formulae for A_{p_α} and B_{p_α} omit the factor $(-1)^{|\alpha|}$.

Page 562 in Lemma II. 3.1 the ellipticity hypothesis (i) should be replaced by its algebraic analogue, namely

$$\text{(i}'\text{)} \qquad \sum_{|\alpha| \leq m} \{A_\alpha(x, z') - A_\alpha(x, z)\}\{z'_\alpha - z_\alpha\} > 0$$
$$\text{for all } z, z' \text{ with } z' \neq z.$$

Page 563 in Lemma II. 3.2, the ellipticity hypothesis (iv) should be replaced by its algebraic analogue

$$\text{(iv}'\text{)} \qquad \sum_{|\alpha| = m} \{A_\alpha(x, y, z') - A_\alpha(x, y, z)\}\{z'_\alpha - z_\alpha\} > 0$$

for $z' \neq z$ where $z = (z_{\alpha_1}, \ldots, z_{\alpha_m})$ and

$|\alpha_i| = m$ and furthermore for each fixed y, z

and any $s \in [0, 1], f(1) \geq f(s)$ where

$$f(s) = \sum_{|\alpha| = m} \{A_\alpha(x, sy, sz) - A_\alpha(x, sy, 0)\} z_\alpha$$

Page 564 (Last equation) We add the details to prove

$$(*) \qquad \lim_{n \to \infty} \int_G \sum_{|\alpha| \leq m} A_\alpha(x, su_n, \ldots, D^m su_n) D^\alpha u_n = \int_G \sum_{|\alpha| \leq m} A_\alpha(x, su, \ldots, D^m su) D^\alpha u.$$

First, from the abstract viewpoint, note that $u_n \to u$ weakly in X implies

$$(R(su_n), u_n) \to (R(su), u) \text{ and } (P(su_n, 0), u_n) \to (P(su, 0), u).$$

Thus $(*)$ will hold if we show

$$(**) \qquad \lim_{n \to \infty} (P(su_n, su_n) - P(su_n, 0), u_n) = (P(su, su) - P(su, 0), u).$$

To demonstrate $(**)$ we remark that if $Au_n \to Au$ strongly $(**)$ holds for $s = 1$, i. e. for the elliptic operators under consideration

$$\lim_{n \to \infty} \int_G \sum_{|\alpha| = m} \{A_\alpha(x, u_n, \ldots, D^m u_n) - A_\alpha(x, u_n, \ldots, 0)\} D^\alpha u_n$$
$$= \int_G \sum_{|\alpha| = m} \{A_\alpha(x, u, \ldots, D^m u) - A_\alpha(x, u, \ldots, 0)\} D^\alpha u.$$

By the ellipticity hypothesis (iv′), the integrands in the above integrals are positive and thus the associated integrals are uniformly absolutely continuous. Furthermore by (iv′) for $0 \leq s \leq 1$

$$0 \leq \sum_{|\alpha| = m} \{A_\alpha(x, su_n, \ldots, D^m(su_n)) - A_\alpha(x, su_n, \ldots, 0)\} D^\alpha u_n$$
$$\leq \sum_{|\alpha| = m} \{A_\alpha(x, u_n, \ldots, D^m u_n) - A_\alpha(x, u_n, \ldots, 0)\} D^\alpha u_n.$$

Hence the integrals associated with the expression $(P(su_n, su_n) - P(su_n, 0), u_n)$ are uniformly absolutely continuous for $0 \leq s \leq 1$, and so $(**)$ holds as required.

Page 574 Equation (ii) add $\int_0^1$ after $\lim_{n \to \infty}$. In the proof note that δ can be chosen independent of $v \in [v]_N$.

Page 576 (Last line) should be

$$\| (1 - t) u + tP^{(n)} u \|^2 = \| P^{(n)} u \|^2 + (1 - t)^2 \| P_n u \|^2$$

$$\geq \| P^{(n)} u \|^2 \geq \delta^2$$

Page 579 Equation (i), in place of $\frac{N + 2m}{N - 2m}$ substitute $\frac{N + 2m}{N - 2m} - \varepsilon$ for any small $\varepsilon > 0$.

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