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The Moment-Condition for the Free Boundary Problem for CR Functions

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1. - Statements

We are interested in studying a *free boundary problem* for holomorphic functions: a continuous function f is given on an open piece Γ of the boundary of an open set in $\mathbb{C}^n$ and we want to find a holomorphic function in the open set which is continuous up to Γ and takes the value f there. We shall say then that f extends holomorphically to the original open set.

If $n \geq 2$ then f must satisfy a well known necessary condition, i.e., f must be a CR function on Γ.

This problem has been studied in [1], [7], [10], [12], [14], [15], [16], [17] (for a survey see [2] Section 27). In those papers rather complicated conditions are given with exception of [7]. Recently, using a simple theorem in one variable (Theorem A below), in the spirit of [7], simpler results have been obtained.

Later, very general results appeared ([16], [9]) but they are hard to use. A generalization of the one-variable-theorem A is in [5]. We shall consider it later in Theorem B.

We are indebted to Lee Stout who pointed out an error in a former version of Theorem 2. We also thank Peter Plug for useful conversations.

Let Ω be an open set in $\mathbb{C}_z \times \mathbb{C}^n_w$, $\Gamma \subset \partial\Omega$ a hypersurface of class C^1, transversal to the z-direction. Write $L_w \equiv \mathbb{C}_z \times \{w\}$.

Assume that, for all $w \in \mathbb{C}^n$ that

$$(1) \qquad L_w \cap \Omega \text{ is connected and } L_w \cap \Omega \neq \emptyset \Longrightarrow L_w \cap \Gamma \neq \emptyset.$$

Next fix, for any w with $L_w \cap \Omega \neq \emptyset$, a conformal mapping $\varphi_w : D \to L_w$,

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injective, with non-vanishing derivative, with these properties:

(2) $$\varphi_w(0) \notin \overline{\Omega}, \quad (\Omega \cap \Gamma) \cap L_w \text{ is a closed subset of } \varphi_W(D).$$

The inverse of φ_w defines a function $\psi : \Omega \cup \Gamma \to D$. Let now f be a continuous, CR function on Γ such that, for any w, $f \circ \varphi_w$ is a L^1 function on the curve $\psi(L_w \cap \Gamma) \subset D$. Set

(3) $$a_k(w) = \int_{L_w \cap \Gamma} \frac{f}{\psi^{k+1}} \, d\psi.$$

THEOREM 1. *f extends holomorphically to Ω, continuous up to Γ if and only if*

(4) $$\overline{\lim_{k \to \infty}} |a_k(w)|^{1/k} \leq 1, \qquad \forall w \in \mathbb{C}^n.$$

This will be obtained as a consequence of two other results: Theorem A and Theorem 2 below.

THEOREM A. (Aĭzenberg [3]). *Let γ be a C^1, simple curve in the unit disc D, closed as a subset of D. Let D^+, D^- be the components of $D \backslash \gamma$, assume $0 \in D^-$. Let f be a continuous function on γ, $f \in L^1(\gamma)$. Set*

(5) $$a_k = \int_\gamma \frac{f(\varsigma)}{\varsigma^{k+1}} \, d\varsigma.$$

Then f extends as a holomorphic function on D^+, continuous up to γ if and only if

(6) $$\overline{\lim_{k \to \infty}} |a_k|^{1/k} \leq 1.$$

A simple proof of this Theorem, renewed for the necessity side, can be found in Section 3.

THEOREM 2. *Let $\Omega \subset \mathbb{C}_z \times \mathbb{C}^n_w$ and $\Gamma \subset \partial\Omega$ satisty* (1) *and let $f : \Omega \cup \Gamma \to \mathbb{C}$ be a function which is holomorphic with respect to the z-variable and, when restricted to $(\Omega \cup \Gamma) \cap L_w$, is continuous up to $\Gamma \cap L_w$. Assume that $f|_\Gamma$ is* CR *and continuous. Then f is holomorphic in Ω with respect to all variables and continuous on $\Omega \cup \Gamma$.*

Notice that the CR-hypothesis is essential in the last theorem: if this hypothesis fails, then f might be even non-locally bounded as we show in the next

EXAMPLE. $\Omega \equiv \{(z,w) | \, |z| < 1/2, |w| < 1\}$, $\Gamma \equiv \{(z,w) \in \partial\Omega | \, |z-1| < 2/3\}$, $f(z,w) = w(z-1)^{1/|w|}$ for $w \neq 0$, $f(z,0) = 0$. There is $f \in C^0(\Gamma)$ because there $|z-1| < 2/3 < 1$, but near $\Omega \cap \{|z-1| > 1\} \cap \{w=0\}$ f is unbounded (local boundedness of f would at least ensure its continuity by a well known Lindelöf procedure).

PROOF OF THEOREM 1. Set $\gamma_m = \psi(L_w \cap \Gamma) \subset D$ so that, for $a_k(w)$ defined in (3), we have

$$a_k(w) = \int_{\gamma_w} \frac{f \circ \varphi_w}{\varsigma^{k+1}} \, d\varsigma.$$

Thus we can apply Theorem A to $f \circ \varphi_w$ on γ_w and this gives that (4) is equivalent to the fact that $f \circ \varphi_w$ extends holomorphically to $\psi(\Omega \cap L_w)$, continuous up to γ_w, for any w.

If we go back to $\Omega \cap L_w$ with φ_w, we obtain that (4) is equivalent to the fact that f is holomorphic with respect to the z-variable and, when restricted to $(\Omega \cup \Gamma) \cap L_w$, it is continuous up to $\Gamma \cap L_w$. Thus the necessity of condition (4) follows immediately. For the sufficiency we have only to apply Theorem 2. □

REMARK 1. No assumption is made on the dependence of the φ_w's on w and their choice is suggested by the various situations where the theorem applies. See "applications" below.

REMARK 2. The L^1 assumption in Theorem 1 and Theorem A can be forgotten when they are used as extendability theorems: one shrinks $\Omega \cup \Gamma$ a little bit...

REMARK 3. No assumption is necessary on the boundary behaviour of γ in Theorem A and, correspondingly, on $\psi(L_w \cap \Gamma) \subset D$ in Theorem 1. For different values of w this last can have very different shapes; an arc joining two points, a closed curve, a curve which is ergodic to the boundary $\partial\Delta$....

For instance if γ of Theorem A (or $\psi(L_w \cap \Gamma)$ in Theorem 1, for all w) is a closed curve, then those theorems yield the solvability of an *exterior problem.*

REMARK 4. Instead of taking $\Gamma \subset \partial\Omega$, one can take $\Gamma \subset \Omega$, disconnecting Ω in two parts Ω^+, Ω^- and Theorem 1 might be applicable twice: to $\Omega^+ \cup \Gamma$ and to $\Omega^- \cup \Gamma$ so that it yields a necessary and sufficient condition for the solution of an *interior problem.* We shall see such a situation in the applications below. See Remark 6 in Section 2.

REMARK 5. The proof of Theorem 2 can become very easy if stronger hypothesis are imposed. For instance if $f \in C^2(\Omega) \cap C^1(\Omega \cup \Gamma)$ then the proof goes as follows: for any $j = 1, \dots, n$, $f_{\overline{w_j}}$ is z-holomorphic and vanishes on Γ and hence everywhere in Ω.

Also if Γ contains no complex hypersurface, then the application of Trepreau's well known theorem yields quickly to the statement. Nevertheless, if really *nothing* is assumed on the behaviour of f out of Γ (such as local boundedness or L^1_{loc}) and on the complex structure of Γ, then Theorem 2 becomes a rather delicate matter.

Further possible Developments. We obtained Theorem 1 applying Theorem 2 to Theorem A. One could start with the n- dimensional analogue of Theorem A which we state now.

THEOREM B. (Aĭzenberg, Kytmanov [5]). *Let Ω be a Reinhardt domain in $\mathbb{C}^n$, $|\partial\Omega|$ the image of $\partial\Omega$ by $(z_1,\dots,z_n) \mapsto (|z_1|,\dots,|z_n|)$, b a $C^2(n-2)$-form in the space of the moduli, with support on $|\partial\Omega|$, $\psi(\varsigma) = \psi(|\varsigma_1|,\dots,|\varsigma_n|)$ the Minkowski functional of Ω. Let Γ be a smooth, relatively closed hypersurface in Ω which divides Ω in two domains Ω^-, Ω^+ and let 0 be in Ω^-. Then for the holomorphic extension of a* CR *function $f \in C^0(\Gamma) \cap L^1(\Gamma)$ it is necessary and sufficient that*

$$\overline{\lim_{|\alpha|\to\infty}} (|c_\alpha|/d_\alpha(\Omega))^{1/|\alpha|} \leq 1$$

where

$$c_\alpha = \int\limits_\Gamma f(\varsigma) \left(\frac{\bar{\varsigma}}{\psi^2([\varsigma])}\right)^\alpha b\left(\frac{[\varsigma]}{\psi([\varsigma])}\right) \wedge \frac{d\varsigma}{\varsigma}$$

and $[\varsigma]$ stands for $(|\varsigma_1|,\dots,|\varsigma_n|)$, usual notation for the multi-index is employed and $d_\alpha(\Omega) = \max_{\overline{\Omega}} |z^\alpha|$.

Other developments of Theorem A are in [4].

2. - Applications of Theorem 1

The very general set up of Theorem 1 has been conceived in view of its simple formulation for some important classes of domains or manifolds.

We list some of them.

(A) Ω *is contained in a complete 1-circular domain* Δ.

$\Delta \subset \mathbb{C}_z \times \mathbb{C}^n_w$ is such that $\lambda \in \mathbb{C}$, $|\lambda| < 1$ and $(z,w) \in \Delta$ imply $(\lambda z, w) \in \Delta$. Those domains can be given the form

$$|z| < \rho(w), \qquad w \in B. \tag{7}$$

$B \subset \mathbb{C}^n$ is an open subset and ρ is a positive, lower semi-continuous function on B. The map φ_w of Theorem 1 is defined by $\varsigma \mapsto \varsigma\rho(w)$, hence $\psi(z,w) = z\rho(w)^{-1}$. Let Ω be defined by $\phi > 0$, where $\phi \in C^1(\Delta)$ is a real function, Γ by the equation $\phi = 0$. Our requirements (1) and (2) on the position of Γ with respect to Ω and to the maps φ_w become $\phi(0,w) < 0$, $\phi_z \neq 0$ on Γ and for no $w \in B$, $\phi(z,w)$ is positive for all $z \in \mathbb{C}$, $|z| < \rho(w)$.

For any $f \in C^0(\Gamma) \cap \mathrm{CR}(\Gamma)$, set

$$\alpha_k(w) = \int\limits_{(\varsigma,w)\in\Gamma} \frac{f(\varsigma,w)}{\varsigma^{k+1}} d\varsigma \tag{8}$$

and

(9) $$r_f(w) = \left(\overline{\lim_{k\to\infty}}|\alpha_k(w)|^{1/k}\right)^{-1}$$

$\tilde{\Delta} \equiv \{|z| \leq r_f(w)\}$ is the maximal circular set such that f extends to $\tilde{\Delta} \cap \Omega$. Hence the necessary and sufficient condition (4) for f to be the trace of a function holomorphic in Ω, continuous up to Γ, becomes

(10) $$r_f \geq \rho \qquad \text{on } B,$$

and this is very easy to handle in concrete cases.

(B) Ω *is contained in a complete circular domain* (Cartan domain) Δ.

Here Δ is a domain in $\mathbb{C}^n_z$ with the property that $\lambda \in \mathbb{C}$, $|\lambda| < 1$, $z \in \Delta$ imply $\lambda z \in \Delta$.

If $B \subset \mathbb{P}_{n-1}(\mathbb{C})$ is the open set of those complex lines which have non-empty intersection with $\Delta\backslash\{0\}$ and $\pi : \mathbb{C}^n\backslash\{0\} \to P_{n-1}(\mathbb{C})$ is the usual projection, then $\Delta\backslash\{0\}$ can be given the equation

$$0 < |z| < \rho(\pi z), \qquad \pi z \in B,$$

where ρ is again any lower semicontinuous, positive function on B. The rest is very similar to the case (A): let Ω be $\phi > 0$, $0 \notin \Omega$, and $\Gamma \equiv \{\phi = 0\}$ with $\sum_{j=1}^{n} z_j \phi_{z_j}(z) \neq 0$ on Γ. For any $w \in B$ let L_w be the complex line through the origin corresponding to w ($L_w = \pi^{-1}w \cup \{0\}$) and $a(w) \in L_w$, with $|a(w)| = 1$, be arbitrarily chosen. (If $0 \in \Delta$, we have $B = \mathbb{P}_{n-1}(\mathbb{C})$ and a continuous $a(w)$ does not exist, but this displays no role).

The function

(11) $$\alpha_k(w) = \int\limits_{\varsigma a(w) \in \Gamma} \frac{f(\varsigma a(w))}{\varsigma^{k+1}}\, \mathrm{d}\varsigma, \qquad w \in B,$$

may be non-continuous. Nevertheless, if we set again

(12) $$r_f(w) = \left(\overline{\lim_{k\to\infty}}|\alpha_k(w)|^{1/k}\right)^{-1},$$

the condition

(13) $$r_f \geq \rho \qquad \text{on } B$$

turns out again to be necessary and sufficient for f to be extendable to Ω as we want.

(C) Ω *is a part of a disc bundle* Δ.

Although Theorem 1 is stated for open sets, its method works for manifolds as well. Let be $\Delta \equiv \{z \in L, |z| < 1\}$, where L is a holomorphic line bundle over a complex manifold B and $|\cdot|$ a lower semicontinuous hermitian metric on the fibers L_w of L, $(w \in B)$. Now $\psi : \Delta \to D$ is the complex gauge function of Δ. Further details are similar to the preceding cases.

(D) Ω *is contained in a domain* Δ *with simply connected sections.*

In this case $\Omega \subset \Delta \subset \mathbb{C}_z \times \mathbb{C}^n_w$ and for each w, $\Delta_w \equiv \Delta \cap \{\mathbb{C}_z \times \{w\}\}$, there is a Riemann map $\Delta_w \to D$ which defines a function $\psi : \Delta \to D$. If Γ is unbounded then the L^1 assumption on f becomes an asymptotic condition. Note that if Ω is pseudoconvex then the simply-connectedness hypothesis is useless.

REMARK 6. (*Inner problems*). This is a continuation of Remark 4 in Section 1. Let $\Delta \equiv \{|z| < \rho(w)\}$ be as in (A) or (B) of Section 1, $\Gamma \equiv \{\phi = 0\}$ with $\phi_z \neq 0$ on Γ. For each $w \in B$, pick arbitrary numbers $a_1(w)$, $a_2(w)$, with $|a_j(w)| < \rho(w)$, $(j = 1,2)$, and $\phi[a_1(w), w] < 0$, $\phi[a_2(w), w] > 0$. Set $\varphi_j(z,w) = \dfrac{z - a_j(w)}{1 - \overline{a}_j(w)\rho^2(w)\overline{z}}$, $j = 1,2$, and define, as in (A), for $j = 1,2$,

$$r_j(w) = \left\{ \overline{\lim_{k\to\infty}} \left| \int\limits_{\Phi(\varsigma,w)=0} \frac{f(\varsigma)\, \mathrm{d}_\varsigma \varphi_j(\varsigma, w)}{[\varphi_j(\varsigma, w)]^{k+1}} \right|^{1/k} \right\}^{1}.$$

Then the following condition, similar to (10)

$$\rho \leq r_1, \qquad \rho \leq r_2 \qquad \text{on } B$$

is necessary and sufficient for f to be extended holomorphically *to the whole set* Δ.

3. - Complexified double layer potential or Plemelj integral and proof of Theorem A

Let γ be a C^1 arc of curve in the complex plane $\mathbb{C}$ such that γ has two endpoints and let f be a complex function on γ. For $z \in \mathbb{C}\setminus\gamma$, we consider the integral

$$F(z) = \frac{1}{2\pi i} \int\limits_\gamma \frac{f(\varsigma)}{\varsigma - z} \mathrm{d}\varsigma. \tag{14}$$

This is a holomorphic function on $\mathbb{C}\backslash\overline{\gamma}$. The usual context for studying F is when f is Hölder continuous in γ, continuous on $\overline{\gamma}$.

In this case, F is continuous up to γ and the jump of F across γ is exactly equal to f.

If f is not Hölder but only continuous on $\overline{\gamma}$ (or continuous on γ, L^1 on $\overline{\gamma}$) then the continuity of F up to γ no longer holds and the relation

$$F^+ - F^- = f \qquad \text{on } \gamma$$

must be taken in weaker sense, i.e., on dipoles. If z^+, z^- are two points on the normal to γ at some point $z \in \gamma$, then we have

$$\lim_{z^\pm \to z} [F(z^+) - F(z^-)] = f(z)$$

(z^+ and z^- are taken at the left and right side respectively of the point moving positively on γ).

In particular if F is continuous up to γ from one side, then it is continuous from the other side too. For all that we refer to [11] or to the original proof of Plemelj [13]. More about this kind of integrals can be found in [11], see also [13].

We are now in a position for proving Theorem A.

PROOF OF THEOREM A. Let $F^\pm$ be defined by the integral (13) according to $z \in D^+$ or $z \in D^-$. Since $O \in D^-$ we can write in the integral (13), $(\varsigma - z)^{-1} = \varsigma^{-1} \sum_0^\infty (z/\varsigma)^k$ and obtain that the left hand side of (6) is the inverse of the radius of convergence of F^- at the origin. Thus condition (6) is equivalent to saying that F^- extends holomorphically to the whole disc and in particular is continuous on γ. Hence, if (6) holds, then F^+ is continuous up to γ, and $F^+ - F^-$ is holomorphic in D^+, continuous up to γ and equal to f there.

Conversely, if f extends in this way, then the path of integration in the integral defining F^- can be deformed only in a compact subset of D, going inside in D^+ so that the new set D^- includes any given point of D and so F^- is holomorphic in D and (6) holds. □

4. - Proof of Theorem 2

By Hartogs' separate holomorphicity theorem there is no restriction if we assume to be in $\mathbb{C}^2 = \mathbb{C}_z \times \mathbb{C}_w$. Also we can use a classical theorem of Hartogs ([10], Lemma 2.2.11) which says that if $\emptyset \neq E \subset A \subset \mathbb{C}_z$ and $B \subset \mathbb{C}_w$ are open sets and A is connected, then any z-holomorphic function in $A \times B$, which is w-holomorphic in $E \times B$, is also (z, w)-holomorphic in $A \times B$.

Therefore, using the hypothesis (1), we only need to prove the statement near a point $p \in \Gamma$ and choose two small discs $D \subset \mathbb{C}_z$, $\Delta \subset \mathbb{C}_w$ so that $p \in D \times \Delta$ and Γ cuts $D \times \Delta$ in two open components $(D \times \Delta)^+ = \Omega \cap (D \times \Delta)$ and $(D \times \Delta)^-$ so that, for each $w \in \Delta$, the disc $D_w \equiv D \times \{w\}$ is divided in two parts $D_w^\pm$ by a C^1 curve $\gamma_w = \Gamma \cap D_w$.

For any fixed $z^\pm \in (D \times \Delta)^\pm$, we consider the Plemelj integral

$$F^\pm(z^\pm, w) = \frac{1}{2\pi i} \int\limits_{\gamma_w} \frac{f(\varsigma, w)}{\varsigma - z^\pm} \, \mathrm{d}\varsigma.$$

By dominated convergence the functions $F^\pm$ are continuous and holomorphic with respect to $z^\pm$ in $(D \times \Delta)^\pm$ respectively. We shall now prove that they are also holomorphic with respect to w in $(D \times \Delta)^\pm$ respectively.

For we fix $(z^\pm, w^\pm) \in (D \times \Delta)^\pm$ and shall prove the w-holomorphicity near this point. Take a smooth, compactly supported function $\psi(w)$ whose support is so small that $\{z^\pm\} \times \operatorname{supp} \psi \subset (D \times \Delta)^\pm$. If we consider ψ as a function in $D \times \Delta$, constant with respect to z, then $\psi|_\Gamma$ is compactly supported.

There is

$$\int F^\pm(z^\pm, w) \psi_{\overline{w}} \, \mathrm{d}w \wedge \mathrm{d}\overline{w} = \frac{1}{2\pi i} \int\limits_\Gamma \frac{f(\varsigma, w)}{\varsigma - z^\pm} \overline{\partial}(\psi \, \mathrm{d}\varsigma \wedge \mathrm{d}w) = 0$$

because $f(\varsigma, w)(\varsigma - z^\pm)^{-1}$ is a CR function on Γ and the holomorphicity of $F^\pm$ follows.

Observe incidentally that, till now, we only dealt with the restriction of f to Γ, thus we proved the well known Andreotti-Hill theorem [6] about the representation of CR functions as a difference of two holomorphic functions. The z-holomorphic function f will be considered now.

There is $\overline{(D \times \Delta)}^+ \setminus \Gamma \subset \Omega$, thus, by moving the integration defining F^-, we have

$$F^-(z, w) = \frac{1}{2\pi i} \int\limits_{\partial^+ D_w} \frac{f(\varsigma, w)}{\varsigma - z} \, \mathrm{d}\varsigma$$

where $\partial^+ D_w = \partial D_w \cap \partial D_w^+$.

F^- is now z-holomorphic in $D \times \Delta$, thus, again by the classical theorem of Hartogs, we conclude that F^- extends to a function in $D \times \Delta$ which is holomorphic with respect to both variables and in particular is continuous on Γ, hence F^+ is also continuous up to Γ as well as $f = F^+ - F^-$. □

REFERENCES

[1] L.A. AĬZENBERG, *On the possibility of holomorphic continuation from a part of the boundary of a domain to the whole domain. A generalization of the theorem of Fock-Kuni*, Complex analysis and mathematical physic, Krasnoyarsk, 5-11 (1980) (Russian).

[2] L.A. AĬZENBERG, *Carleman formulas in complex analysis. First application "Nauka"*, Novosibirsk 1990, (Russian).

[3] L.A. AĬZENBERG - A.M. KYTMANOV, *On the holomorphic extendability of functions given on a connected part of a boundary*, Mat. Sb., **4**, 490-507 (1991) (Russian).

[4] L.A. AĬZENBERG, *Simple conditions for holomorphic continuation from a part of the boundary of a convex domain to the whole domain*, preprint, Dept. of Math., Royal Institute of technology, 5-100 44 Stockholm, Sweden, 7 (1991).

[5] L.A. AĬZENBERG - A.M. KYTMANOV, *On the holomorphic extendability of functions given on a connected part of a boundary*, II, preprint, Roma Tor Vergata, 14 (1991).

[6] A. ANDREOTTI - D. HILL, *F.E. Levi convexity and the Hans Levy problem*, Part I and II, Ann. Scuola Norm. Sup. Pisa, Cl. Sci., **26**, 325-363 and 747-806 (1972).

[7] V.A. FOCK - F.M. KUNI, *On the introduction of a "annihilating" function in the dispersion equations of gases*, Dokl. Akad. Nauk SSSR, **127**:6, 1195-1198 (1959) (Russian).

[8] L. HÖRMANDER, *An Introduction to complex analysis in several variables*, Van Nostrand 1966.

[9] O.V. KAREPOV - N.N. TARKHANOV, *The Cauchy problem for holomorphic functions of Hardy class H^2*, preprint 53M, Institut Fiziki 50 AN SSSR, Krasnoyarsk, 21 (1990) (Russian).

[10] M.G. KREĬN - P. YA NUDELMAN, *On some new problems for Hardy class functions and continuous families of functions with double orthogonality*, Soviet Math. Dokl. **14**:2 (1973).

[11] N.I. MUSKHELISHVILI, *Singular integral equations*, P. Noordhoff N.V. - Groningen - Holland, 435, (1953).

[12] D.I. PATIL, *Representation of H^p functions*, Bull. Amer. Math. Soc., **78**:4, 617-620 (1972).

[13] J. PLEMELJ, *Ein Ergänzungssatz für Cauchyschen Integraldarstellung analytischer Funktionen, Randwerte betreffend*, Monatshefte für Math. u. Phys. XIX, 211-245 (1908).

[14] B.A. SHAIMKULOV, *Conditions for the solution of the Cauchy problem for holomorphic functions*, Proc. Conf. of geometric theory of functions, Novosibirsk, 1988.

[15] A. STEINER, *Abschnitte von Randfunktionen beschränkter Analytischer Funktionen*, Lecture Notes in Math. **419**, 342-351, (1974).

[16] L.N. ZNAMENSKAYA, *Conditions for the holomorphic extension of functions of class L^2 given on a part of the Shilov boundary of circular, starshaped domains*, Sibirsk. Mat. Zh., **31**:3 (1990).

[17] G. ZIN, *Esistenza e rappresentazione di funzioni analitiche, le quali, su una curva di Jordan, si riducono ad una funzione assegnata*, Ann. Mat. Pura Appl. **34**, 365-405 (1953).

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