Astérisque

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Introduction to stochastic field theory (abstract)

INTRODUCTION TO STOCHASTIC FIELD THEORY [*]

(Abstract)

Francesco Guerra

We consider classical dynamical systems with a finite or infinite
number of degrees of freedom and with Lagrangian of canonical type
which arise in many areas of physics. Typical examples are the har-
monic and anharmonic oscillator, the free Klein-Gordon scalar field,
the vector meson Proca field, the electromagnetic Maxwell field, the
Klein-Gordon field with polynomial self-interaction, etc. To each of
these classical dynamical systems we associate some stochastic pro-
cesses following the prescriptions of Nelson stochastic mechanics
[9,10] extended to general Lagrangian systems [7,4]. In particular
we focus our attention on the ground state stochastic process of
lowest energy and study its properties.

As a rather unexpected and surprising result [7] we find that
the ground state stochastic process so constructed coincides with the
stochastic process introduced in the so-called Euclidean formulation
of quantum field theory. We recall here that the Euclidean theory,
initiated by Schwinger [14] and Nakano [8],after the work of Symanzik
[17] and Nelson [11,12] has become an essential tool [18,15] for the
constructive quantum field theory program of Glimm and Jaffe and

their followers [3, 18]. In the usual interpretation the Euclidean
theory is introduced starting from quantum field theory in the
Wightman formulation[16] and then performing an analytic continuation
in the time variables from real Minkowski time to the imaginary
"Euclidean" time [14, 17]. The inverse analytic continuation is neces-
sary in order to recover Wightman field theory from the Euclidean
theory [11,13,15,1]. According to the proposed new interpretation [7]
we can introduce "Euclidean"-Markov fields directly inside the frame-
work of physical space-time and no analytic continuation is necessary
both for the mathematical construction and the physical interpreta-
tion.

An attractive feature of our way of introducing Euclidean fields
via stochastic mechanics is a simple derivation [7] of the Feynman-
Kac-Nelson formula (see [6]) from the theory of stochastic differen-
tial equations [2].

The role of Euclidean field theory and stochastic mechanics as
possible tools for the quantization of general relativity is stressed
in [5].

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