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APPLICATIONS OF DECOMPOSITIONS OF HOLOMORPHIC FUNCTIONS TO
PARTIAL DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

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NOTATIONS.

We consider \mathbb{R}^n endowed with the usual scalar product defined by $\langle y, \xi \rangle = \sum y_j \xi_j$ and the euclidean norm $|y| = \sqrt{\langle y, y \rangle}$ as a closed submanifold of \mathbb{C}^n . We shall denote by S^{n-1} the unit sphere of \mathbb{R}^n and for any cone $\Gamma \subset \mathbb{R}^n$ we define the polar of Γ by $\Gamma^\Delta = \{\xi \in \mathbb{R}^n \setminus \{0\} : \langle y, \xi \rangle \geq 0, \forall y \in \Gamma\}$. By a salient cone, we mean a cone that does not contain any straight line. Given an open subset Ω of \mathbb{R}^n and an open convex cone $\Gamma \subset \mathbb{R}^n$, a subset A of \mathbb{C}^n will be called of profile $\Omega + i\Gamma$ if for every compact sets $K \subset \Omega$ and $\mathcal{K} \subset \Gamma \cap S^{n-1}$, there exists $\rho_0 > 0$ such that the wedge

$$\{x + i\rho y : x \in K, y \in \mathcal{K}, \rho \in]0, \rho_0]\}$$

is contained in A . We are going to represent by \mathcal{D} the ring of linear partial differential operators with constant complex coefficients. It is well known that \mathcal{D} is unitary and noetherian. If $P = \sum_{|\alpha| \leq m} a_\alpha D^\alpha$ belongs to \mathcal{D} , we shall write $\overset{\circ}{P}$ the principal symbol of P and $\text{car}(P)$ the characteristic variety of P , i.e. the set $\{\xi + i\eta \in \mathbb{C}^n : |\xi|^2 + |\eta|^2 = 1, P(\xi + i\eta) = 0\}$. Finally, let us denote by \mathcal{O} the sheaf of holomorphic functions on \mathbb{C}^n and by \mathcal{A} the linear space of \mathbb{C} -valued analytic functions on \mathbb{R}^n .

Let us first recall two decomposition theorems proved in [5] and [6].

Theorem 1: For any $F \in \mathcal{B}$ and any finite family of open convex salient cones Γ_j of \mathbb{R}^n whose polars cover S^{n-1} , there exist domains of holomorphy V_j containing \mathbb{R}^n and an open convex tube $\mathbb{R}^n + i\Omega_j$ of profile $\mathbb{R}^n + i\Gamma_j$ and $F_j \in \mathcal{O}(V_j)$ such that $F = \sum F_j$ holds on a neighborhood of \mathbb{R}^n . Moreover, if the interiors of the polars of the Γ_j 's cover S^{n-1} , given $r \in]0, +\infty[$, one can assume that the V_j 's are open pseudoconvex neighborhoods of the closed tubes

$$\mathbb{R}^n + i\{y \in \bar{\Gamma}_j : |y| \leq r\}.$$

Theorem 2: Let Γ be an open convex cone of \mathbb{R}^n , Ω an open subset of \mathbb{R}^n and V an open subset of \mathbb{C}^n of profile $\Omega + i\Gamma$. For any $r \in]0, +\infty[$, any $F \in \mathcal{O}(V)$ and any open subcone Γ' of Γ whose intersection with S^{n-1} is relatively compact in Γ , there exist an open convex neighborhood Ω' of $\{y \in \bar{\Gamma}' : 0 < |y| \leq r\}$ in \mathbb{R}^n , an open pseudoconvex neighborhood W of Ω contained in $\Omega + i\mathbb{R}^n$, $A \in \mathcal{O}(W)$ and $G \in \mathcal{O}(\mathbb{R}^n + i\Omega')$ such that $W \cap (\mathbb{R}^n + i\Omega') \subset V$ and $F = G + A$ on $W \cap (\mathbb{R}^n + i\Omega')$.

Remark 3: This last statement constitutes in fact a slight improvement of the result obtained in [6]. To establish it, one only needs (besides evident modifications) to remark that lemma 6 of [5] can be precised as follows: if the U_j 's are strictly pseudoconvex tubes with C_2 -boundaries $\partial U_j \supset \mathbb{R}^n$, for any complex neighborhood W of an open set Ω of \mathbb{R}^n , there exists an open pseudoconvex neighborhood V_W of $\Omega \cup (\cap U_j)$ such that $V_W \setminus (\cap U_j) \subset W$.

Lemma 4: Let Γ be an open convex non void cone of \mathbb{R}^n . The polar of $\mathbb{R}^n \times \Gamma$ coincides with $0 \times \Gamma^\perp$ and is a convex salient cone closed in $\mathbb{R}^{n+n'} \setminus \{0\}$. Moreover, for any open cone $\gamma \supset 0 \times \Gamma^\perp$, there exists $\rho > 0$ such that γ^\perp is contained in

$$\tilde{\Gamma}_\rho = \{(x, y) : |x| \leq \frac{|y|}{\rho}, y \neq 0, d(\frac{y}{|y|}, S^{n-1} \setminus \Gamma) \geq \rho\},$$

where d denotes the euclidean distance.

Proof: One obtains immediately the equality $(\mathbb{R}^n \times \Gamma)^\perp = 0 \times \Gamma^\perp$ and as Γ is open and non void, this is a salient cone closed in $\mathbb{R}^{n+n'} \setminus \{0\}$.

For the second part of the statement, let us first choose $\varepsilon > 0$ such that

$$\omega_\varepsilon = \{(\xi, \eta) \in S^{n+n'-1} : |\xi| < \varepsilon, d\left(\frac{\eta}{|\eta|}, \Gamma^\perp \cap S^{n'-1}\right) < \varepsilon\} \subset \gamma,$$

hence such that γ^\perp is contained in

$$\Gamma' = \bigcap_{(\xi, \eta) \in \omega_\varepsilon} \{(x, y) : \langle x, \xi \rangle + \langle y, \eta \rangle \geq 0\}.$$

As it is clear by definition of ω_ε that $\Gamma' \setminus \{0\}$ is disjoint from

$\{(x, 0) : x \in \mathbb{R}^n\}$, we are going to show that the existence of sequences (x_m, y_m) or $(x'_m, y'_m) \in \Gamma' \setminus \{0\}$ such that $|x_m| > m|y_m|$ or such that $|y'_m| = 1$ and

$d(y'_m, S^{n'-1} \setminus \Gamma) < \frac{1}{m}$ leads to a contradiction in each case. As a matter of fact,

for any $\eta_0 \in \Gamma^\perp \cap S^{n'-1}$, the points $(\xi_m, \eta_m) = \frac{1}{\sqrt{1+m^2}} \left(-\frac{x_m}{|x_m|}, m\eta_0\right)$ belong to

ω_ε for m sufficiently large and one obtains

$$0 \leq \langle x_m, \xi_m \rangle + \langle y_m, \eta_m \rangle < -\frac{m|y_m|}{\sqrt{1+m^2}} + \frac{m|y_m|}{\sqrt{1+m^2}} = 0,$$

hence a first contradiction. In the second case, we can find $y''_m \in S^{n'-1} \setminus \bar{\Gamma}$ such

that $|y'_m - y''_m| < \frac{1}{m}$ and by convexity of $\bar{\Gamma}$, there are points $\eta'_m \in \Gamma^\perp \cap S^{n'-1}$

verifying $\langle y''_m, \eta'_m \rangle < 0$.

As the points $(\xi_m, \eta_m) = \frac{1}{\sqrt{1+m^2}} (0, m\eta'_m - y'_m)$ belong to ω_ε for m

sufficiently large, we obtain another contradiction:

$$0 \leq \langle x'_m, \xi_m \rangle + \langle y'_m, \eta_m \rangle \leq \frac{1}{\sqrt{1+m^2}} [m \langle y'_m - y''_m, \eta'_m \rangle + m \langle y''_m, \eta'_m \rangle - 1]$$

$$< \frac{1}{\sqrt{1+m^2}} [m |y'_m - y''_m| - 1] \leq 0.$$

Definition 4: Adapting a definition due to Bony and Schapira (cf. [1]) to the particular situation we have in mind, we shall say that a point z_0 of the boundary of an open subset V of \mathbb{C}^n verifies the condition $c(z_0, \gamma, V)$ with respect to a convex salient cone γ closed in $\mathbb{R}^n \setminus \{0\}$ if for any $\varepsilon > 0$, there exists $\eta > 0$ such that

$$\{z \in V : |z - z_0| < \eta\} - i\{y \in (\gamma_\varepsilon)^\perp : |y| < \eta\}$$

is contained in V , where γ_ε denotes the conic hull of the set of points of S^{n-1} whose distance to $\gamma \cap S^{n-1}$ is less than ε .

Lemma 5: a) If $\Gamma \subset \mathbb{R}^n$ is an open convex non void cone and $V = \mathbb{R}^n + i\Omega$ an open tube of \mathbb{C}^n , then for any $z_0 \in \partial V$, the condition $c(z_0, -\Gamma^\perp, V)$ is equivalent to the cone condition $C(z_0, (0 \times \Gamma^\perp) \cap S^{2n-1})$ stated in 4.1 of [1].

b) Let r belong to $]0, +\infty[$, Γ be an open convex non void cone of \mathbb{R}^n and V denote the open tube $\{z \in \mathbb{R}^n + i\Gamma : |y| < r\}$. Every $x \in \mathbb{R}^n \subset \partial V$ verifies then the condition $c(x, -\Gamma^\perp, V)$. Moreover, if γ is an open conic neighborhood of $(-\Gamma^\perp) \setminus \{0\}$ and if $V' = \mathbb{R}^n + i\Omega$ is an open tube of profile $\mathbb{R}^n + i\Gamma'$, there exists an open convex subcone Γ' of Γ such that $-\Gamma'^\perp \subset \gamma$ and any $x \in \mathbb{R}^n \subset \partial[V' \cap (\mathbb{R}^n + i\Gamma')]$ verifies the condition $c[x, -\Gamma'^\perp, V' \cap (\mathbb{R}^n + i\Gamma')]$

Proof: a) We first show that c implies C . Let I' be an open neighborhood of $(0 \times \Gamma^\perp) \cap S^{2n-1}$ in S^{2n-1} . By Lemma 4, the polar of I' in the sense of Bony and Schapira (which is the opposite of ours) is contained in $-\widetilde{\Gamma}'_\rho$ for some $\rho > 0$. There exists $\varepsilon > 0$ such that $(\Gamma^\perp_\varepsilon)^\perp$ contains $\{y \in S^{n-1} : d(y, S^{n-1} \setminus \Gamma) \geq \rho\}$. Now let η be the number which corresponds to ε by application of $c(z_0, -\Gamma^\perp, V)$. For $V' = \{z : |z - z_0| < \frac{\eta}{2}\}$ and the set A of the points of the polar of I' (in the sense of [1]) whose module is less than $\frac{\eta}{2}$, the inclusion $(V' \cap V) + A \subset V$ is easy to obtain because V is a tube.

The proof of $C \rightarrow c$ is similar and as we shall not use this implication in what follows, we do not give further details.

b) The first assertion follows immediately from the inclusion

$[(\Gamma^{\perp})_{\varepsilon}^{\perp}]^{\perp} \subset \Gamma$, which is easy to obtain. For the second one, let us denote by γ' an open convex cone of \mathbb{R}^n verifying

$$-\Gamma^{\perp} \subset \gamma' \subset \overline{\gamma'} \setminus \{0\} \subset \gamma$$

and set $\Gamma' = -\gamma'^{\perp 0}$. One has evidently $\overline{\Gamma'} = -\gamma'^{\perp} = -\overline{\gamma'}^{\perp} \subset \Gamma^{\perp \perp 0} = \Gamma$ and $-\Gamma'^{\perp} = \overline{\gamma'} \setminus \{0\} \subset \gamma$ and as $\mathbb{R}^n + i\Omega$ is of profile $\mathbb{R}^n + i\Gamma$, there exists $\rho_0 > 0$ such that $\{y \in \Gamma' : |y| < \rho_0\} \subset \Omega$. Hence the conclusion by application of the first part of this result.

Theorem 6: Let $P \in \mathcal{D}$, $r \in]0, +\infty[$ and Γ_j be a finite family of open convex non void cones of \mathbb{R}^n . If $\text{car}(P) \cap [-i \cup \Gamma_j^{\perp}]$ is empty, the equation $Pu = f$ is solvable in the subspace of \mathcal{A} , whose elements can be written $\Sigma F_j|_{\mathbb{R}^n}$ with $F_j \in \mathcal{A} \cap \mathcal{O}[\mathbb{R}^n + i\{y \in \Gamma_j : |y| < r\}]$.

If there moreover exists an open convex conic neighborhood γ of $-U\Gamma_j^{\perp}$ such that $\text{car}(P) \cap i\gamma$ is empty, the same equation is solvable in the subspace of \mathcal{A} , whose elements can be written $\Sigma F_j|_{\mathbb{R}^n}$ with $F_j \in \mathcal{A} \cap \mathcal{O}(V_j)$, where V_j denotes an open convex tube of profile $\mathbb{R}^n + i\Gamma_j$.

In particular, when P is ξ_0 -hyperbolic in the sense that it verifies the following two conditions

- a) $\mathring{P}(\xi_0) \neq 0$
- b) P does not vanish on $\mathbb{R}^n + i\{\lambda\xi_0 : \lambda > c\}$ for some $c \geq 0$,

the second situation occurs if $-U\Gamma_j^{\perp}$ is contained in the open convex cone

$$\gamma_P = \bigcap_{\lambda \geq 0} \{\xi \in \mathbb{R}^n : P(\xi + \lambda\xi_0) \neq 0\}$$

Proof: According to the Malgrange-Ehrenpreis theorem (cf. [4] or [7]), the equations $PU_j = F_j$ are solvable in both cases in a convex tube of profile

$\mathbb{R}^n + i\Gamma_j$. By the precedent lemma, we can apply theorem 4.1 of [1] and therefore suppose that the U_j 's are also holomorphic on a neighborhood of \mathbb{R}^n . Hence the conclusion by linearity of P .

The third assertion is a direct consequence of a slight modification of Gårding's well known result on hyperbolic polynomials (cf. [2]) which asserts under our hypotheses that $\overset{\circ}{P}$ does not vanish on $\mathbb{R}^n + i\gamma_P$.

Remark 7: a) Our definition of ξ_0 -hyperbolicity differs from Gårding's one because we interchange the roles of the real and imaginary parts of the complex directions.

b) When P is ξ_0 -hyperbolic, it is easy to prove by Hurwitz's theorem that γ_P coincides with the connected component of

$$\{\xi \in \mathbb{R}^n : \overset{\circ}{P}(\xi) = 0\}$$

that contains ξ_0 .

c) Combining theorems 1 and 6, one obtains immediately the following well known result:

Corollary 8: If $P \in \mathcal{D}$ is elliptic, one has $P(D)\mathcal{A} = \mathcal{A}$.

Proof: One only needs to point out that $\text{car}(P)$ does not meet $i\mathbb{R}^n$.

Proposition 9: Let $\Gamma \subset \mathbb{R}^n$ be a closed convex cone with non void interior and

\mathcal{M} be a finitely generated \mathcal{D} -module. For any $r \in]0, +\infty[$, one has

$\text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \mathcal{O}_{(F_{\Gamma,r})}] = 0$ for all $j \geq 2$, where $F_{\Gamma,r}$ denotes

$$\{x + iy \in \mathbb{R}^n + i\Gamma : |y| \leq r\}.$$

Proof: The $K_m = \{x + iy \in \mathbb{R}^n + i\Gamma : |x| \leq m, |y| \leq r\}$ ($m \in \mathbb{N}$) form a sequence of compact convex sets which increases towards $F_{\Gamma,r}$ in such a way that one has

$$\mathcal{O}_{(F_{\Gamma,r})} = \varprojlim_m \mathcal{O}_{(K_m)} \text{ and consequently } \varprojlim_m \text{Hom}_{\mathcal{D}}[\mathcal{D}^N, \mathcal{O}_{(K_m)}] =$$

$= \text{Hom}_{\mathcal{D}}[\mathcal{D}^N, \mathcal{O}_{(F_{\Gamma,r})}]$ for any $N \in \mathbb{N}$. Let

$$0 \leftarrow \mathcal{M} \leftarrow \mathcal{D}^{r_0} \xleftarrow{t_{\psi_0}} \mathcal{D}^{r_1} \xleftarrow{t_{\psi_1}} \dots \xleftarrow{t_{\psi_{q-1}}} \mathcal{D}^{r_q} \leftarrow 0 \quad (q \leq n)$$

be a free projective resolution of \mathcal{M} and consider the commutative diagram

$$\begin{array}{ccccccc} 0 \rightarrow \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_0}, \mathcal{O}_{(\Gamma, r)}] & \rightarrow & \dots & \rightarrow & \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_q}, \mathcal{O}_{(\Gamma, r)}] & \rightarrow & 0 \\ \downarrow \vdots & & & & \downarrow \vdots & & \\ 0 \rightarrow \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_0}, \mathcal{O}_{(K_{m+1})}] & \rightarrow & \dots & \rightarrow & \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_q}, \mathcal{O}_{(K_{m+1})}] & \rightarrow & 0 \\ \downarrow & & & & \downarrow & & \\ 0 \rightarrow \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_0}, \mathcal{O}_{(K_m)}] & \rightarrow & \dots & \rightarrow & \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_q}, \mathcal{O}_{(K_m)}] & \rightarrow & 0 \\ \downarrow \vdots & & & & \downarrow \vdots & & \end{array}$$

A well known result (cf. [3], p. 410, for example) asserts that the canonical maps

$$\phi^j : \text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \mathcal{O}_{(\Gamma, r)}] \rightarrow \varprojlim_m \text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \mathcal{O}_{(K_m)}] , \quad \forall j \geq 2$$

are isomorphisms because Mittag-Leffler's condition is satisfied since we have for every $j \geq 1$ and $m \in \mathbb{N}$

$$H^j(0 \rightarrow \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_0}, \mathcal{O}_{(K_m)}] \rightarrow \dots \rightarrow \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_q}, \mathcal{O}_{(K_m)}] \rightarrow 0) = \text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \mathcal{O}_{(K_m)}] = 0$$

by virtue of the Malgrange-Ehrenpreis theorem. Hence the conclusion.

Notation 10: If Γ is an open convex non void cone of \mathbb{R}^n , we set

$$\tilde{\mathcal{A}}_{\Gamma} = \varinjlim_{V} \mathcal{O}_{(V)} ,$$

where V runs over the open subsets of \mathbb{C}^n of profile $\mathbb{R}^n + i\Gamma$. We shall also denote by \mathcal{F}_{Γ} the family of open convex non void subcones Γ' of Γ such that $\Gamma' \cap S^{n-1}$ is relatively compact in Γ .

Theorem 11: For any open convex non void cone Γ of \mathbb{R}^n and any finitely generated \mathcal{D} -module \mathcal{M} , one has

$$\varprojlim_{\Gamma' \in \mathcal{F}_\Gamma} \text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \tilde{\mathcal{A}}_{\Gamma', \mathcal{A}}] = 0, \quad \forall j \geq 1.$$

Proof: Given any $\Gamma' \in \mathcal{F}_\Gamma$, we are going to establish that for every $j \geq 1$ and every $\Gamma'' \in \mathcal{F}_\Gamma$ such that $\Gamma' \in \mathcal{F}_{\Gamma''}$, the canonical operator

$$\text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \tilde{\mathcal{A}}_{\Gamma'', \mathcal{A}}] \rightarrow \text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \tilde{\mathcal{A}}_{\Gamma', \mathcal{A}}]$$

vanishes. Using the same notations as in the precedent proof to denote the free projective resolution of \mathcal{M} , we are lead to prove that the image of

$$(\tilde{\mathcal{A}}_{\Gamma'', \mathcal{A}})^{r_{j-1}} \xrightarrow{\psi_{j-1}} (\tilde{\mathcal{A}}_{\Gamma'', \mathcal{A}})^{r_j} \xrightarrow{\psi_j} (\tilde{\mathcal{A}}_{\Gamma'', \mathcal{A}})^{r_{j+1}}$$

in the complex

$$(\tilde{\mathcal{A}}_{\Gamma', \mathcal{A}})^{r_{j-1}} \xrightarrow{\psi_{j-1}} (\tilde{\mathcal{A}}_{\Gamma', \mathcal{A}})^{r_j} \xrightarrow{\psi_j} (\tilde{\mathcal{A}}_{\Gamma', \mathcal{A}})^{r_{j+1}}$$

is exact. In other words, we have to prove that given any $F \in \tilde{\mathcal{A}}_{\Gamma''}^{r_j}$ verifying

$$\psi_j F = 0 \text{ mod } \mathcal{A}, \quad (1)$$

there exists $U \in \tilde{\mathcal{A}}_{\Gamma'}^{r_{j-1}}$ such that

$$\psi_{j-1} U = F \text{ mod } \mathcal{A}. \quad (2)$$

By theorem 2, we can decompose F in $G + A$ with

$G \in \mathcal{O}[\mathbb{R}^n + i\{y \in \bar{\Gamma}' : 0 < |y| \leq r\}]$ and $A \in \mathcal{A}$. Therefore (1) and (2)

become respectively

$$\psi_j G \in \mathcal{O}_{(\bar{\Gamma}', r)}^{r_{j+1}} \quad (3)$$

$$\psi_{j-1} U = G \text{ mod } \mathcal{A}. \quad (4)$$

As we have trivially $\psi_{j+1} \psi_j G = 0$ the precedent lemma assures the existence of $H \in \mathcal{O}_{(F_{\Gamma', r})}^{r^j}$ such that $\psi_j H = \psi_j G$ and we can replace (3) and (4) respectively by

$$\begin{aligned} \psi_j(G - H) &= 0 \text{ on } F_{\Gamma', r} \\ \psi_{j-1}U &= G - H \text{ mod } \mathcal{A}. \end{aligned}$$

Since $\mathbb{R}^n + i\{y \in \Gamma' : |y| < r\}$ is an open convex subset of $F_{\Gamma', r}$, another application of the Malgrange-Ehrenpreis theorem allows to conclude.

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R E F E R E N C E S

- [1] J.M. BONY et P. SCHAPIRA, Existence et prolongement des solutions des équations aux dérivées partielles, *Inventiones Math.* 17 (1972), pp. 95-105.
- [2] L. GÄRDING, Linear hyperbolic partial differential equations with constant coefficients, *Acta Math.* 85 (1951), pp. 1-62.
- [3] A. GROTHENDIECK, *Eléments de Géométrie Algébrique III*, *Publ. Math. I.H.E.S.*, 11, (1961).
- [4] H. KOMATSU, Relative cohomology of sheaves of solutions of differential equations, *Lecture Notes in Math.* 287, Springer Verlag, Berlin (1973), pp. 192-259.
- [5] J.L. LIEUTENANT, Decomposition of analytic functions on \mathbb{R}^n in sum of holomorphic functions in conic tubes, *Bull. Soc. Roy. Sc. Liège*, 49 (1980), pp. 347-357.
- [6] J.L. LIEUTENANT, Decomposition of functions holomorphic in tuboids, *Bull. Soc. Roy. Sc. Liège*, 49 (1980).

- [7] B. MALGRANGE, Systèmes différentiels à coefficients constants, Séminaire Bourbaki, 246, Paris, (1962).

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