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ASTÉRIQUE

1992

**AN EXTENSION OF A THEOREM
BY CHEEGER AND MÜLLER**

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(with and appendix by François LAUDENBACH)**

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This volume is dedicated to Jeff Cheeger and Werner Müller

Contents

Introduction	7
I - Reidemeister metrics and Milnor metrics	21
a) A metric on the determinant of the cohomology of a finite dimensional chain complex	21
b) The Reidemeister metric on the determinant of the cohomology of a simplicial complex	23
c) The Thom-Smale complex of the gradient field of a Morse function	26
d) Milnor metrics on the determinant of the cohomology of a flat vector bundle	31
II - Ray-Singer metrics and the de Rham map	33
a) The Ray-Singer metric on $\det H^\bullet(M, F)$	33
b) A quasi-isomorphism of complexes : the de Rham map for smooth triangulations	36
c) A quasi-isomorphism of complexes : the de Rham map for Thom-Smale complexes	37
III - Berezin integrals and Morse functions	39
a) The Berezin integral	40
b) Vector bundles and Berezin integrals : the Mathai-Quillen Thom forms	41
c) Convergence of the Mathai-Quillen currents over E	45
d) A transgressed Euler class	46
e) The Berezin integral formalism over the tangent space	48
f) Berezin integral and gradient vector fields : a symmetry property	49
g) The canonical section of $\Lambda(T^*M) \hat{\otimes} \Lambda(T^*M)$	51
h) A variation formula for forms over M	51
i) The limit as $T \rightarrow +\infty$ of certain currents over M	52
j) An identity of currents over M	55
k) The case where the metric g^{TM} is flat near the critical points	56

IV - Anomaly formulas for Ray-Singer metrics	61
a) A canonical connection on a flat Euclidean vector bundle	62
b) A closed 1– form on M and its cohomology class	63
d) Clifford algebras and exterior algebras	65
e) A Lichnerowicz formula for the Hodge Laplacian	67
f) An infinitesimal variation formula for the Ray-Singer metric	69
g) A Clifford algebra trick	70
h) The small time asymptotics of the supertrace of certain heat kernels . .	72
i) Proof of Theorem 4.7	79
V - A closed 1-form on $\mathbb{R}_+^* \times \mathbb{R}_+$	81
a) A family of smooth metrics on \mathbb{F}	81
b) The Witten Laplacian	83
c) A basic closed 1-form	84
d) A contour integral	86
VI - Some properties of the integral $-\int_M \theta(F, g^F)(\nabla f)^* \psi(TM, \nabla^{TM})$	89
a) Homotopy invariance of the integral	90
b) Variation formulas for the integral $-\int_M \theta(F, g^F)(\nabla f)^* \psi(TM, \nabla^{TM})$	91
c) A cohomological expression for the integral $-\int_M \theta(F, g^F)(\nabla f)^* \psi(TM, \nabla^{TM})$	93
VII - An extension of a theorem of Cheeger and Müller	97
a) An extension of the Cheeger-Müller theorem	98
b) Some simplifying assumptions on the metrics g^{TM}, g^F	98
c) Nine intermediary results	102
d) The asymptotics of the I_k^0 's	104
e) Matching the divergences	116
f) Proof of Theorem 7.1	118
g) Proof of Theorem 0.3	118
VIII - The asymptotic structure of the matrix of the d^F operator on the Helffer-Sjöstrand orthogonal base	121
a) The Agmon metric $ \nabla f ^2 g^{TM}$	122
b) The harmonic oscillator of Witten	123

c) The estimates of Helffer and Sjöstrand for the eigenforms of \tilde{D}_T^2 with Dirichlet boundary conditions	125
d) An orthonormal base for Dirichlet eigenspaces associated to small eigenvalues	127
e) The orthonormal base of Helffer-Sjöstrand of the eigenspaces of the operator \tilde{D}_T^2 associated to small eigenvalues	131
f) The <i>WKB</i> equation for \tilde{D}_T^2	133
g) The transport equation on $W^s(x)$	138
h) The transport equation on $W^u(x)$	139
i) The matrix of d_T^F in the base $\tilde{e}_{T,x,k}$	141
IX - Proof of Theorem 7.6	149
a) A modified scalar product on $\mathbb{F}_T^{[0,1]}$	150
b) The harmonic elements in $\mathbb{F}_T^{[0,1]}$ for the new scalar product	152
c) The asymptotics as $T \rightarrow +\infty$ of the modified scalar product on $H^\bullet(M, F)$	154
d) The asymptotics of the modified metric on $\det H^\bullet(M, F)$	158
e) Proof of Theorem 7.6	160
X - The asymptotics as $T \rightarrow +\infty$ of certain traces associated to the operator D_T^2	161
a) The operator \tilde{D}_T near B	161
b) Proof of Theorem 7.7	163
c) Proof of Theorem 7.8	164
d) Proof of Theorem 7.9	165
XI - The asymptotics of $\text{Tr}_s[N \exp(-tD^2)]$ as $t \rightarrow 0$	167
XII - An asymptotic expansion for $\text{Tr}_s[f \exp(-tD_T^2)]$ as $T \rightarrow +\infty$	169
a) An estimate of the kernel of $\exp(-t\tilde{D}_T^2)$ on $M \setminus \bigcup_{x \in B} B^M(x, \varepsilon)$	169
b) A harmonic oscillator approximation for the kernel of $\exp(-tD_T^2)$ near B	170
c) Proof of Theorem 7.11	177

XIII - An estimate for $\text{Tr}_s[f \exp(-(tD + T\hat{c}(\nabla f))^2)]$ in the range	
$0 < t \leq 1, 0 \leq T \leq \frac{d}{t}$	181
a) Localization of the problem	181
b) An estimate for the kernel of $\exp(-(tD + T\hat{c}(\nabla f))^2)$ in the range $t \in]0, 1], T \in [0, T_0]$	182
c) An estimate for the kernel of $\exp(-(tD + T\hat{c}(\nabla f))^2)$ in the range $t \in]0, 1], T \in [0, \frac{d}{t}]$	189
d) Proof of Theorem 7.12	190
XIV - The asymptotics as $t \rightarrow 0$ of $\text{Tr}_s[f \exp(-(tD + \frac{T}{t}\hat{c}(\nabla f))^2)]$...	197
XV - The asymptotics of $\text{Tr}_s[f \exp(-(tD + \frac{T}{t}\hat{c}(\nabla f))^2)]$ for	
$0 < t \leq 1, T \geq 1$	199
a) An estimate for $S_{t, \frac{T}{t}}(z, z)$ on compact sets of $M \setminus B$	200
b) The kernel $S_{t, \frac{T}{t}}(z, z)$ near B and the harmonic oscillator	200
c) Proof of Theorem 7.14	205
XVI - A direct proof of a formula comparing two Milnor metrics	209
References	215
Appendix. On the Thom-Smale complex, by F. Laudenbach	219
a) Submanifolds with conical singularities	220
b) The main result	221
c) The Thom-Smale complex	223
d) The Thom-Smale complex with local coefficients	225
e) Bifurcation of the Thom-Smale complex in a 1-parameter family ..	225
f) Modification of the Thom-Smale complex near a birth-death point ..	226
g) The Thom-Smale complex near points where (T) is not satisfied ..	228
h) Final comments and complements	230
Abstract (in French)	235