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This volume contains the contributions to the congress Complex Analytic Methods in Dynamical Systems held at Instituto de Matemática Pura e Aplicada, IMPA, in January 1992.

We hope this book will provide an extensive overview on the subject.

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Rio de Janeiro, September 10, 1993

C. Camacho
A. Lins Neto
R. Moussu
P. Sad,

The Editors

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ABSTRACTS

C. BONATTI and X. GOMEZ-MONT. *The index of holomorphic vector fields on singular varieties I*

Given an analytic space V with an isolated singularity p, a Poincaré-Hopf type of index, $\mathrm{Ind}(X, V, p)$ is associated to every holomorphic vector field X tangent to V for which p is an isolated zero.

In this paper this topological index is related to the algebraic multiplicity $\mu_V(X, p)$. In particular, it is shown that the set of indices $\mathrm{Ind}(X, V, p)$, where X is tangent to V with an isolated zero at p, admits a minimum which is reached for X in the open dense subset of vector fields of smallest V-multiplicity.

M. BRUNELLA. *Vanishing holonomy and monodromy of certain centres and foci*

In this paper we study a particular class of germs of analytic differential equations on the real plane, which present a singularity of the type centre - focus. For these singularities it is defined a monodromy map, which is a germ of analytic diffeomorphism on the real line. A complexification of these germs allows to introduce (following R. Moussu and D. Cerveau) their vanishing holonomy. We study the relation existing between monodromy and vanishing holonomy; corollaries about normal forms are obtained.

D. CERVEAU. *Théorèmes de type fuchs pour les tissus feuilletés*

Après avoir rappelé des résultats pour certains bien anciens - et souvent méconnus - concernant les d-tissus sur un ouvert de $\mathbf{C}^n$ on s'intéresse à la dynamique des 3-tissus feuilletés hexagonaux globaux. Bien souvent - c'est le cas sur les espaces projectifs - un tel objet va présenter des singularités. On se propose, moyennant des hypothèses de type Fuchs, de donner une description des feuilles comme niveaux de fonctions multivaluées de type Liouville ($\Sigma\lambda_i \log f_i + H$, f_i et H holomorphes). Ce travail est motivé par la description de la variété des feuilletages algébriques de codimension un sur des espaces projectifs $\mathbf{CP}(n)$.

D. CERVEAU and A. LINS NETO. *Codimension one foliations in $\mathbf{C}P^n$, $n \geq 3$, with Kupka components*

We consider holomorphic foliations of codimension one in $\mathbf{C}P(n)$, $n \geq 3$, with a Kupka component. We prove that if the Kupka component is a complete intersection, then the foliation has a first integral of the type f^p/g^q, where p, q are positive integers with $(p, q) = 1$, f and g are homogeneous polynomials in $\mathbf{C}^{n+1}$ such that p degree $(f) = q$ degree (g) and the Kupka component is $\{f = g = 0\}$ in homogeneous coordinates.

J. ECALLE. *Compensation of small denominators and ramified linearisation of local objects*

We show, on typical examples, how local objects (i.e. germs of analytic vector fields or diffeomorphisms of $\mathbf{C}^\nu$) which, due to resonance or small denominators, fail to possess an analytic linearisation, may still be reduced to their linear part by means of ramified changes of coordinates. The latter are not merely formal, but canonically resummable in spiral-like neighbourhoods of the ramified origin $\underset{\bullet}{0}$ of $\underset{\bullet}{\mathbf{C}}^\nu$. Apart from its obvious bearing on local dynamics, ramified linearisation leads to an extension of the concept of holonomy.

J.E. FORNAESS and N. SIBONY. *Complex dynamics in higher dimension I*

We study global questions of iteration for holomorphic self maps of $\mathbf{P}^k$. After discussing some basic properties of holomophic and meromorphic maps in $\mathbf{P}^k$, we describe the maps f in $\mathbf{P}^2$ for which there exists a variety V satisfying $f^{-1}(V) = V$. We show that for a Zariski dense set of holomorphic maps in $\mathbf{P}^2$ the complement of the critical orbit is Kobayashi hyperbolic. We then study expansive properties of the maps in the interior of the complement of the critical orbit, under suitable hyperbolicity assumptions. We finally classify maps in $\mathbf{P}^2$ such that the orbit of the critical set is a variety.

Y. ILYASHENKO. *Normal forms for local families and nonlocal bifurcations*

The study of nonlocal bifurcations from the topological point of view requires not only topological, but smooth normal forms of the families of differential equations near singular points. In the first part of the paper a survey of these normal forms is presented. In the second part these normal forms are applied to the study of the bifurcations of planar vector fields. A complete list of polycycles appearing in generic two or three parameter families (Zoo of Kotova) is presented.

The proof of the finite cyclicity of elementary polycycles occuring in typical finite parameter families of planar vector fields is outlined.

V.P. KOSTOV. *Regular linear systems on* $\mathbf{CP}^1$ *and their monodromy groups*

In this paper we prove that the $p+1$ Jordan normal forms of the monodromy operators of a regular linear system on $\mathbf{C}P^1$ with $p+1$ poles and the possible reducibility of the monodromy group define an analytic stratification of $(GL(n,\mathbf{C}))^p$ - the space of monodromy groups of such systems.

J.F. MATTEI and M. NICOLAU. *Equisingular unfoldings of foliations by curves*

We prove the existence of a versal equisingular unfolding of a given holomorphic foliation F with isolated singularities on a compact complex surface. Under suitable cohomological assumptions the parameter space is isomorphic to the product of the spaces parametrizing the (local) versal equisingular unfoldings of the germ of F at its singular points. As an application it is shown that any equisingular unfolding of a germ of polynomial foliation on $(\mathbf{C}^2, 0)$ is still polynomial.

M. EL MORSALANI, A. MOURTADA and R. ROUSSARIE. *Quasi-regularity property for unfoldings of hyperbolic polycycles*

Some years ago Yu. Ilyashenko proved that the return map of any planar analytic hyperbolic polycycle has a quasi-regularity property. This implies that the polycycle is isolated among limit cycles, a key step in the proof that any polynomial planar vector field has just a finite number of limit cycles.

Here one proves a similar property for analytic one-parameter unfoldings of hyperbolic polycycles. As a consequence one deduces that some special unfoldings, with an unbrocken connection and a fixed product of again value ratios, have a finite cyclicity i.e., that the number of created limit cycles is bounded. Such unfoldings arrives for instance in quadratic vector fields, so that the result solves some of the cases in a general program formulated else where about the Hilbert's 16^{th} Problem for quadratic vector fields.

I. NAKAI. *A rigidity theorem for transverse dynamics of real analytic foliations of codimension one*

We prove a topological rigidity theorem for transverse dynamics of real analytic foliations of codimension one (Theorem 1) as well as for pseudo-groups of real analytic diffeomorphisms of open neighbourhoods of 0 in the real line $\mathbf{R}$ (Theorem 3). We apply those results to prove the topological rigidity of analytic actions of the surface group Γ^g on the circle S^1 (Corollary 4) and also the topological invariance of the Godbillon-Vey class (Corollary 2).

R. PÉREZ-MARCO et J-C. YOCCOZ. *Germes de feuilletages holomorphes à holonomie prescrite*

Les germes de singularités irreductibles de feuilletages holomorphes de type Siegel sont définis par un champ de vecteurs holomorphe avec valeurs propres λ_1, λ_2 en $0 \in \mathbf{C}$, tels que $\lambda_1 \cdot \lambda_2 \neq 0$ et $\alpha = -\lambda_2/\lambda_1$ soit réel positif. L'holomie d'une des séparatrices determine le feuilletage.

Étant donné un germe holomorphe $f(z) = e^{2\pi i\alpha} + z + \mathcal{O}(z^2)$, on construit un tel feuilletage avec holonomie f. On obtient alors l'équivalence entre la classification analytique de ce type de germes de singularités et la classification analytique de germes de difféomorphismes holomorphes de $(\mathbf{C}, 0)$. La condition optimale arithmétique pour la linéarisation est obtenue.

C. ROCHE. *Densities for certain leaves of real analytic foliations*

Khovanskii's theory applies for non spiralating leaves of real analytic foliations as was shown in a joint work with R. Moussu. This theory proves that these leaves behave much like subanalytic subsets at least for the finiteness properties. Here it is shown that Kurdyka-Raby's technique can be applied to prove the existence of densities in each boundary point of a non spiralating leaf. This result doesn't use regularity assumptions on the boundary set of a very few results are known on these sets.

M. SHISHIKURA. *The boundary of the Mandelbrot set has Hausdorff dimension two*

The boundary of the Mandelbrot set M has Hausdorff dimension two and for a generic $c \in \partial M$, the Julia set of $z \mapsto z^2 + c$ also has Hausdorff dimension two. The proof of these statements is based on the analysis of the bifurcation of parabolic fixed points. This paper is an attempt to explain the main point of the proof, using the notion of geometric limit of rational maps.

D. TISCHLER. *Perturbations of critical fixed points of analytic maps*

We consider perturbations of a locally defined analytic function which has a critical point which is also a fixed point. The second derivative at the fixed point determines an inequality for the positions of the critical point, critical value and fixed point of the perturbed analytic function relative to some reference point. We apply this to the case of critical points of polynomials where the reference point is another fixed point. We also use the topological description of polynomials, all of whose critical points are fixed, to examine some inequalities relating the positions of critical points and critical values for polynomials which depend on the branching of the polynomial.