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Correction : “On a class of starlike functions”

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Correction

On a class of starlike functions

by

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Compositio Mathematica 19 (1968), 78–82

Introduction

In a recent paper (Comp. Math., 19 (1968), 78–82) with the above title we investigated the class $\bar{S}$ of functions

$$f(z) = z + a_2 z^2 + \cdots$$

that are analytic and starlike in $|z| < 1$ and satisfy the condition $|\{zf'(z)/f(z)\} - 1| \leqq 1$ in $|z| < 1$. Theorem 4 of that paper, giving the radius of convexity of the class $\bar{S}$, contains an error and as such the result obtained is not sharp. Below we give the correct proof of that theorem.

THEOREM. *Each function $f(z) \in \bar{S}$ maps*

$$|z| < \frac{(3-\sqrt{5})}{2}$$

onto a convex domain.

PROOF. If $f(z) \in \bar{S}$, we can write

$$z\frac{f'(z)}{f(z)} = 1 + \psi(z), \tag{1}$$

when $\psi(z)$ is analytic in $|z| < 1$ and satisfies $|\psi(z)| \leqq 1$ and $\psi(0) = 0$. Logarithmic differentiation of (1) yields:

$$z\frac{f''(z)}{f'(z)} = \frac{z\psi'(z)}{1+\psi(z)} + \psi(z).$$

Making use of the fact that $|\psi(z)| \leqq |z|$ and

$$|\psi'(z) \leqq \frac{1-|\psi(z)|^2}{(1-|z|^2)},$$

we find that

$$\left| z \frac{f''(z)}{f'(z)} \right| \leqq \frac{|z|(1-|\psi(z)|^2)}{(1-|\psi(z)|)(1-|z|^2)} + |\psi(z)|$$

$$= \frac{|z|(1+|\psi(z)|)}{(1-|z|^2)} + |\psi(z)|$$

$$\leqq \frac{2|z|-|z|^2}{(1-|z|)}.$$

Therefore, $f(z)$ is convex if

$$\frac{2|z|-|z|^2}{1-|z|} < 1$$

or

$$|z| < \frac{(3-\sqrt{5})}{2}.$$

To show that $(3-\sqrt{5})/2$ is the exact radius of convexity, we consider the function

$$f_0(z) = z \exp(z).$$

A little calculation shows that for this function

$$1+ \frac{zf_0''(z)}{f_0'(z)} = \frac{z^2+3z+1}{1+z}$$

vanishes when $z = (\sqrt{5}-3)/2$.

(Oblatum 30–1–69)

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