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INDUCTIVE ČECH COMPLETENESS AND DIMENSION

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Abstract

We give an example of a Lindelöf space X so that (a) any closed subspace of X has arbitrarily small neighborhoods with locally compact boundaries, and (b) if Y is a Čech complete space containing X then $\dim(Y - X) = \operatorname{ind}(Y - X) = \infty$.

0. Definitions and notation

All spaces under discussion are completely regular and T_1.

A *Čech complete space* is a space which is a G_δ in some (equivalently, in any) of its compactifications.

A space X has *strong inductive completeness degree* -1, $\operatorname{Icd} X = -1$, if X is Čech-complete. If for any two disjoint closed subsets F and G of a space X there exists an open set U such that $F \subset U \subset X - G$ and $\operatorname{Icd} B(U) \leq n - 1$ ($B(U)$ denotes the boundary of U), then X has strong inductive completeness degree $\leq n$, $\operatorname{Icd} X \leq n$. $\operatorname{Icd} X = n$ if $\operatorname{Icd} X \leq n$ and $\operatorname{Icd} X \nleq n - 1$. If $\operatorname{Icd} X \nleq n$ for all n, then $\operatorname{Icd} X = \infty$.

An *extension* of a space X is a space Y containing X as a dense subset.

If X is a space then $\mathscr{C} - \operatorname{Def} X = \min\{n \geq -1 : \exists$ Čech complete extension Y of X with $\dim(Y - X) \leq n\}$ (we allow n to be ∞ of course) and similarly $\mathscr{C} - \operatorname{def} X = \min\{n \geq -1 : \exists$ Čech complete extension Y of X with $\operatorname{ind}(Y - X) \leq n\}$.

ω is the smallest infinite ordinal. Therefore, if $k < \omega$, then $k = \{0, 1, \ldots, k - 1\}$.

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1. Introduction

In [1], J.M. Aarts found an interesting characterization of those metrizable spaces X having a completion Y with $\dim(Y-X)\le n$. The characterization is in terms of the strong inductive completeness degree. To be more precise, a metrizable space X has a completion Y with $\dim(Y-X)\le n$ iff $\operatorname{Icd} X \le n$, or, in the terminology of section 0, $\operatorname{Icd} X = \mathscr{C} - \operatorname{Def} X$.

The aim of this paper is to show that the assumption on metrizability in the above result is essential. We construct an example of a separable Lindelöf space X of weight ω_1 with the following properties:

(a) if F and G are disjoint closed subsets of X then there is an open neighborhood U of F with locally compact boundary and $F \subset U \subset X - G$ (as a consequence, $\operatorname{Icd} X \le 0$);

(b) $\mathscr{C} - \operatorname{Def} X = \mathscr{C} - \operatorname{def} X = \infty$.

As is well-known, any rimcompact space X has a compactification γX with $\operatorname{ind}(\gamma X - X) \le 0$ (Freudenthal [6]). In view of this result it is natural to ask whether every "rimlocally compact" space X has a compactification γX with $\operatorname{ind}(\gamma X - X)\le 1$. Our example shows that this is not true. See section 4 for additional remarks.

I am indebted to Jan Aarts for some helpful comments.

2. Independent families

A collection of sets $\mathscr{C}$ is called an *independent family* if for each pair of disjoint finite subsets $\mathscr{F}$ and $\mathscr{H}$ of $\mathscr{C}$ the set $\bigcap \mathscr{F} - \bigcup \mathscr{H}$ is infinite. It is well-known that there is an uncountable independent family of subsets of ω. This can be shown as follows. Let $S \subset 2^{\omega_1}$ be a countable dense set. It is trivial to verify that the family $\mathscr{A} = \{A_\alpha : \alpha < \omega_1\}$ where, for each $\alpha < \omega_1$,

$$A_\alpha = \{f \in S : f(\alpha) = 0\}$$

is an uncountable independent family of subsets of the countable set S.

Let $\mathscr{A} = \{A_\alpha : \alpha \in I\}$ and $\mathscr{B} = \{B_\alpha : \alpha \in I\}$ be two collections of sets such that $A_\alpha \subset B_\alpha$ for all $\alpha \in I$. The family $\mathscr{A}$ is called *independent over* $\mathscr{B}$ if for each pair of disjoint finite subsets F and G of I the set $\bigcap_{\alpha\in F} A_\alpha - \bigcup_{\alpha \in G} B_\alpha$ is infinite. Notice that if $\mathscr{A}$ is independent then $\mathscr{A}$ is independent over $\mathscr{A}$.

Our construction heavily relies on the following combinatorial result in Bell and van Mill [4]

2.1. THEOREM: *Let* $n \geq 1$. *Let* $\{A_{\alpha i}: \alpha < \omega_1, i < n\}$ *and* $\{B_\alpha: \alpha < \omega_1\}$ *be two collections of subsets of* ω *such that for each* $\alpha < \omega_1$ *we have that* $\bigcup_{i<n} A_{\alpha i} \subset B_\alpha$ *and* $\{\bigcup_{i<n} A_{\alpha i}: \alpha < \omega_1\}$ *is independent over* $\{B_\alpha: \alpha < \omega_1\}$. *Then there exists an uncountable subset* M *of* ω_1 *and for each* $\alpha \in M$ *an* $n_\alpha < n$ *such that* $\{A_{\alpha n_\alpha}: \alpha \in M\}$ *is independent over* $\{B_\alpha: \alpha \in M\}$.

3. The example

Let I denote the closed unit segment $[0, 1]$. The projection onto the α^{th} coordinate of I^{ω_1} will be denoted by π_α. As usual, 2^{ω_1} denotes $\{0, 1\}^{\omega_1}$.

The underlying set of X is

$$2^{\omega_1} \cup (\omega \times (I^{\omega_1} - 2^{\omega_1})).$$

A basic neighborhood of a point of type $\langle n, f\rangle \in \omega \times (I^{\omega_1} - 2^{\omega_1})$ has the form $\{n\} \times U$, where U is any neighborhood of f in I^{ω_1} disjoint from 2^{ω_1}. In addition, a basic neighborhood of a point $f \in 2^{\omega_1}$ has the form $E(U)$, where U is any neighborhood of f in I^{ω_1} and

$$E(U) = (U \cap 2^{\omega_1}) \cup (\omega \times (U - 2^{\omega_1})).$$

It is clear that X is a countable union of copies of I^{ω_1} sewn together along 2^{ω_1}. Obviously, X is regular, Lindelöf and separable. We will proceed to prove that X has all properties listed in the introduction.

FACT 1: *If* F *and* G *are disjoint closed subsets of* X *then there is an open neighborhood* U *of* F *with locally compact boundary so that* $F \subset U \subset X - G$.

Let F and G be disjoint closed subsets of X. It clearly suffices to show that there is an open neighborhood U of F with $F \subset U \subset X - G$ and $B(U) \cap 2^{\omega_1} = \emptyset$. Let $C_0 \subset 2^{\omega_1}$ be clopen so that $F \cap 2^{\omega_1} \subset C_0 \subset 2^{\omega_1} - G$. In addition, let $C_1 = 2^{\omega_1} - C_0$. By normality, there are disjoint open sets V_0, $V_1 \subset X$ with $V_i \cap 2^{\omega_1} = C_i$ $(i < 2)$. We may assume that $V_0 \cap G = \emptyset = F \cap V_1$. There is a neighborhood U_0 of $F - V_0$ not intersecting G whose closure does not intersect 2^{ω_1}. Let $U = U_0 \cup V_0$. It is clear that U is as required. □

For each $\alpha < \omega_1$ let

$$A_\alpha^0 = E(\pi_\alpha^{-1}[0, \tfrac{1}{4}))$$

and

$$A_\alpha^1 = E(\pi_\alpha^{-1}(\tfrac{3}{4}, 1]).$$

Let γX be a compactification of X, and, for each $\alpha < \omega_1$ and $i < 2$ choose an open $U_\alpha^i \subset \gamma X$ with

$$U_\alpha^i \cap X = A_\alpha^i.$$

For each $\alpha < \omega_1$ and $i < 2$ let $\mathcal{U}_\alpha^i$ be a finite cover of $\pi_\alpha^{-1}(\{i\}) \cap 2^{\omega_1}$ consisting of sets of type $E(U)$, where U is a linearly convex open neighborhood of some point $f \in \pi_\alpha^{-1}(\{i\}) \cap 2^{\omega_1}$, while moreover $(\cup \mathcal{U}_\alpha^i)^- \subset U_\alpha^i$ (the closure is taken in γX!). This can be done of course since $\pi_\alpha^{-1}(\{i\}) \cap 2^{\omega_1}$ is compact and $\pi_\alpha^{-1}(\{i\}) \cap 2^{\omega_1} \subset U_\alpha^i$. Let $E \subset \omega_1$ be uncountable so that $|\mathcal{U}_\alpha^0| = |\mathcal{U}_\beta^0|$ for all $\alpha, \beta \in E$. As in section 2, let S be countable dense subset of 2^{ω_1}. The family $\mathcal{A} = \{A_\alpha : \alpha \in E\}$ where $A_\alpha = \{f \in S : f(\alpha) = 0\}$ is independent. Therefore, by Theorem 2.1, there is an uncountable subset $E_0 \subset E$ and for each $\alpha \in E_0$ an element $A'_\alpha \in \{U \cap S : U \in \mathcal{U}_\alpha^0\}$ so that the family

$$\mathcal{A}' = \{A'_\alpha : \alpha \in E_0\}.$$

is independent over $\{A_\alpha : \alpha \in E_0\}$.

For each $\alpha \in E_0$ put $B_\alpha = S - A_\alpha$ and $C_\alpha = S - A'_\alpha$. Notice that

$$B_\alpha = \{f \in S : f(\alpha) = 1\}$$

and that the family $\mathcal{B} = \{B_\alpha : \alpha \in E_0\}$ is independent over $\mathcal{B}' = \{B'_\alpha : \alpha \in E_0\}$. Let $E_1 \subset E_0$ be uncountable so that $|\mathcal{U}_\alpha^1| = |\mathcal{U}_\beta^1|$ for all $\alpha, \beta \in E_1$. Again, by Theorem 2.1, we can find an uncountable subset $E_2 \subset E_1$ and for each $\alpha \in E_2$ an element $B'_\alpha \in \{U \cap S : U \in \mathcal{U}_\alpha^1\}$ so that the family

$$\mathcal{B}' = \{B'_\alpha : \alpha \in E_2\}$$

is independent over $\mathcal{C} = \{C_\alpha : \alpha \in E_2\}$.

These observations show that there exists an uncountable subset $E \subset \omega_1$, namely the set E_2 constructed above, and for each $\alpha \in E$ and

$i<2$ an element $V^i_\alpha \in \mathcal{U}^i_\alpha$ so that for any finite subset $F \subset E$ and for any function $f: F \to 2$ we have that

$$\bigcap_{\alpha\in F} V^{f(\alpha)}_\alpha \cap 2^{\omega_1} \neq \emptyset.$$

We will now show that $\gamma X - X$ is infinite dimensional. To this end, we construct for each $n \geq 0$ a compact subspace $Z_n \subset \gamma X - X$ with $\dim Z_n \geq n$. From this it is easy to derive that $\dim(\gamma X - X) = \operatorname{ind}(\gamma X - X) = \infty$ (see Engelking [5], chapter 7).

FACT 2: *For each* $n \geq 0$ *there is a compact subspace* $Z_n \subset \gamma X - X$ *with* $\dim Z_n \geq n$.

For $n = 0$ there is nothing to prove, so assume that $n \geq 1$. We will only give the proof of the Fact for $n = 2$. The general case can be shown by similar arguments.

Take two distinct indices $\alpha, \beta \in E$, where E is defined as above. In addition, take $f_{ij} \in V^i_\alpha \cap V^j_\beta \cap 2^{\omega_1}$ for $i, j < 2$ and put $F = \{f_{00}, f_{10}, f_{11}, f_{01}\}$. Let $I(f_{ij}, f_{kl})$ denote the convex segment (in I^{ω_1}) connecting f_{ij} and f_{kl}. Notice that

$$S = I(f_{00}, f_{10}) \cup I(f_{10}, f_{11}) \cup I(f_{11}, f_{01}) \cup I(f_{01}, f_{00})$$

is homeomorphic to the 1-sphere S^1. Let $D \subset I^{\omega_1}$ be a disc whose topological boundary is S. We may assume that $D \cap 2^{\omega_1} = F$ and that there is a homeomorphism $\varphi: D \to I^2$ with $\varphi(I(f_{00}, f_{10})) = I \times \{0\}$, $\varphi(I(f_{10}, f_{11})) = \{1\} \times I$, $\varphi(I(f_{11}, f_{01})) = I \times \{1\}$ and $\varphi(I(f_{01}, f_{00})) = \{0\} \times I$. Define

$$Y = F \cup (\omega \times (D - F)).$$

Notice that Y is a countable union of discs sewn together along F. We claim that $\bar{Y} - Y$ contains a compactum of dimension ≥ 2. Put $Z = (\bar{Y} - Y) \cup F$. We will first show that Z is ≥ 2 dimensional. This is simple. Define

$$C_0 = (\{f_{00}, f_{10}\} \cup (\omega \times (I(f_{00}, f_{10}) - \{f_{00}, f_{10}\})))^- \cap Z,$$

$$C_1 = (\{f_{01}, f_{11}\} \cup (\omega \times (I(f_{01}, f_{11}) - \{f_{01}, f_{11}\})))^- \cap Z,$$

$$D_0 = (\{f_{00}, f_{01}\} \cup (\omega \times (I(f_{00}, f_{01}) - (f_{00}, f_{01}\})))^- \cap Z,$$

and

$$D_1 = (\{f_{10}, f_{11}\} \cup (\omega \times (I(f_{10}, f_{11}) - \{f_{10}, f_{11}\})))^- \cap Z.$$

We claim that $C_0 \cap C_1 = \emptyset$. This is obvious, since $\{f_{00}, f_{01}\} \subset V_\alpha^0$ and by the special choice of V_α^0 it follows that $C_0 \subset \overline{V_\alpha^0}$. Similarly it follows that $C_1 \subset \overline{V_\alpha^1}$. Since

$$\overline{V_\alpha^0} \cap \overline{V_\alpha^1} \subset U_\alpha^0 \cap U_\alpha^1 = \emptyset,$$

we conclude that $C_0 \cap C_1 = \emptyset$. In precisely the same way we conclude that $D_0 \cap D_1 = \emptyset$.

Now suppose that $\dim Z \leq 1$. Then, by Engelking [5, 7.2.15] we can find closed sets H_0, H_1, K_0, $K_1 \subset Z$ with $C_0 \subset H_0 - H_1$, $C_1 \subset H_1 - H_0$, $D_0 \subset K_0 - K_1$, $D_1 \subset K_1 - K_0$ and $H_0 \cup H_1 = K_0 \cup K_1 = Z$ while moreover $H_0 \cap H_1 \cap K_0 \cap K_1 = \emptyset$. By normality of $\bar{Y}$ we can find open neighborhoods G_0, G_1, G_2 and G_3 of, respectively, H_0, H_1, K_0 and K_1 so that

(a) $\bigcap_{i<4} \bar{G}_i = \emptyset$, and

(b) $\bar{G}_0 \cap H_1 = \bar{G}_1 \cap H_0 = \bar{G}_2 \cap K_1 = \bar{G}_3 \cap K_0 = \emptyset$.

The closure of $\bar{Y} - (\bar{G}_0 \cup \bar{G}_1 \cup \bar{G}_2 \cup \bar{G}_3)$ in $\bar{Y}$ does not intersect Z and therefore, by compactness, must be contained in $k \times (D - F)$ for some $k < \omega$.

Choose $k_0 > k$ so that

$$\{f_{00}, f_{10}\} \cup (\{k_0\} \times (I(f_{00}, f_{10}) - \{f_{00}, f_{10}\}) \subset G_0 - \bar{G}_1,$$

$$\{f_{01}, f_{11}\} \cup (\{k_0\} \times (I(f_{01}, f_{11}) - \{f_{01}, f_{11}\}) \subset G_1 - \bar{G}_0,$$

$$\{f_{00}, f_{01}\} \cup (\{k_0\} \times (I(f_{00}, f_{01}) - \{f_{00}, f_{01}\}) \subset G_2 - \bar{G}_3,$$

and

$$\{f_{10}, f_{11}\} \cup (\{k_0\} \times (I(f_{10}, f_{11}) - \{f_{10}, f_{11}\}) \subset G_3 - \bar{G}_2.$$

Since $F \cup (\{k_0\} \times (D - F)) \subset \bar{G}_0 \cup \bar{G}_1 \cup \bar{G}_2 \cup \bar{G}_3$ this obviously contradicts D being two dimensional ([5, 7.2.15]). This proves that $\dim Z \geq 2$.

Since Y is separable metric, and hence first countable, $\bar{Y}$ is first countable at each point of Y. Since F is finite, this implies that F is a G_δ of Z. We therefore conclude that $Z - F$ is a countable union of compacta. Since Z is normal, being compact, by the Countable Sum Theorem, [5, 7.2.1], one of these compacta must be at least two dimensional. This shows that $\gamma X - X$ contains a two dimensional compact subspace. □

We will now show that for any Čech complete extension cX of X we have that $\dim(cX - X) = \operatorname{ind}(cX - X) = \infty$.

FACT 3: *Let cX be a Čech complete extension of X. Then for each $n \geq 0$ there is a compact subspace $S_n \subset cX - X$ with* $\dim S_n \geq n$.

It is clear that we must show that for any compactification γX of X and for any σ-compact subspace $F \subset \gamma X - X$ we have that $\gamma X - (X \cup F)$ contains compacta of arbitrarily large dimension. Therefore, let γX be a compactification of X and let $F \subset \gamma X - X$ be σ-compact. Write $F = \bigcup_{n<\omega} F_n$ where the F_n's are compact. Take $f \in 2^{\omega_1}$ arbitrarily.

For each $n < \omega$ let U_n be an open neighborhood of f in I^{ω_1} so that

$$E(U_n)^- \cap F_n = \emptyset$$

(the closure is taken in γX). There is a countable set $E \subset \omega_1$ so that the set

$$P = \{g \in I^{\omega_1}: g(\alpha) = f(\alpha) \text{ for each } \alpha \in E\}$$

is contained in $\bigcap_{n<\omega} U_n$. Clearly $P \approx I^{\omega_1}$ which implies that

$$T = (P \cap 2^{\omega_1}) \cup (\omega \times (P - 2^{\omega_1}))$$

is homeomorphic to X. Also, T is closed in X and

$$\bar{T} \cap \bigcup_{n<\omega} F_n = \emptyset.$$

Therefore $\bar{T} - T \subset \gamma X - (X \cup F)$ and consequently, by Fact 2, we conclude that $\gamma X - (X \cup F)$ contains compacta of arbitrarily large dimension. □

Since our Example is infinite dimensional one might ask whether for finite dimensional X it is true that $\operatorname{Icd} X = \mathscr{C} - \operatorname{Def} X$. This is not true however. It is easy to modify our Example to get a one dimensional space Y which is again Lindelöf and separable so that $\operatorname{Icd} Y = 0$ (in fact, any two disjoint closed sets in Y can be separated by a locally compact closed set) and $\mathscr{C} - \operatorname{Def} Y = \mathscr{C} - \operatorname{def} Y = 1$.

4. Remarks

In [7], J. de Groot tried to find for each $n = 0, 1, 2, \ldots$ an internal characterization of those separable metric spaces X having a separable metric compactification γX with $\dim(\gamma X - X) \leq n$. He showed that the rimcompact separable metric spaces are precisely those spaces having a separable metric compactification with zero-dimensional remainder. Generalizing the notion of rimcompactness he defined the *compactness degree* cmp X of a space X in the same way the small inductive dimension is defined but with the empty set replaced by the class of compact spaces. Here is the definition of the *compactness degree* of X:

$\operatorname{cmp} X = -1$	iff X is compact,
$\operatorname{cmp} X \leq n+1$	iff each point $x \in X$ has arbitrarily small open neighborhoods U with $\operatorname{cmp} B(U) \leq n$,
$\operatorname{cmp} X = n$	iff $\operatorname{cmp} X \leq n$ and $\operatorname{cmp} X \not\leq n-1$,
$\operatorname{cmp} X = \infty$	iff $\operatorname{cmp} X \neq n$ for all n.

In addition, the *compactness deficiency* $\operatorname{def} X = \min\{n\colon \exists$ compactification γX of X with $\dim(\gamma X - X) \leq n\}$ (we allow n to be ∞ of course). J. de Groot [7], and later de Groot and Nishiura [8], conjectured that for separable metric X, $\operatorname{cmp} X \leq n$ is a sufficient condition for X to have a separable metric compactification γX with $\dim(\gamma X - X) \leq n$. This conjecture, posed almost forty years ago, is still unresolved. The above result of de Groot, which is also due to Freudenthal, shows that the conjecture is true for separable metric spaces of compactness degree 0. The Example of section 3 has compactness degree 1 and compactness deficiency ∞. This shows that the restriction to separable metric spaces is essential in de Groot's conjecture.

Let us notice that for separable metric spaces $\operatorname{cmp} X \leq n$ is a necessary condition for $\operatorname{def} X \leq n$. De Groot [7] and de Groot and Nishiura [8] show that if X has a metric compactification γX with $\dim(\gamma X - X) \leq n$ then $\operatorname{cmp} X \leq n$. Therefore, to show that $\operatorname{cmp} X \leq \operatorname{def} X$ for separable metric spaces it suffices to prove that if $\operatorname{def} X \leq n$ then there is actually a metric compactification γX of X with $\dim(\gamma X - X) \leq n$. This can be done by the technique used in the proof of the Mardešic Factorization Theorem [10]. Let us also notice that in general $\operatorname{cmp} X$ need not always be less than or equal to $\operatorname{def} X$. Smirnov [11] constructed an example of a nonrimcompact space X (i.e. $\operatorname{cmp} X > 0$) whose Čech–Stone remainder $\beta X - X$ is zero-dimensional (i.e. $\operatorname{def} X = 0$) (this information was brought to my attention by Eric van Douwen). Eric van Douwen has informed me that he thinks that Smirnov's example can be modified to obtain a space Y with $\operatorname{cmp} Y = \infty$ and $\dim(\beta Y - Y) = 0$. This is interesting in its own right but not if one wishes to obtain necessary internal conditions on a space X which guarantee that X has a compactification with zero-dimensional, or finite dimensional, remainder. Our example shows that a natural candidate for such a condition does not work.

As noted in the introduction, if compactness is replaced by completeness then, for metric spaces, a generalized form of de Groot's conjecture can be proved. Interestingly, compactness cannot be replaced by σ-compactness. Aarts and Nishiura [2] give an example of a separable metric space X so that any nonempty closed subspace of X has arbitrarily small open neighborhoods with σ-compact boundary (i.e. X has inductive σ-compactness degree 0) but if Y is any σ-compact separable metric space containing X as a dense subspace then $\dim(Y - X) \geq 1$.

For more information concerning the conjecture of de Groot and related topics, see Isbell [9].

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