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NON-ARCHIMEDEAN (UNIFORMLY) CONTINUOUS MEASURES ON HOMOGENEOUS SPACES

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I. Introduction

Let \mathcal{K} be a complete non-archimedean valued field. Let G be a locally compact zerodimensional group. If G has an open compact q -free subgroup, where q is the characteristic of the residue class field of \mathcal{K} , then for every closed subgroup H of G the homogeneous space of all left cosets of H in G has a \mathcal{K} -valued quasi-invariant measure μ (see [4]). Let $M^\infty(G/H)$ be the space of all measures on G/H . In this paper we will prove the following result: an element λ of $M^\infty(G/H)$ translates continuously for the strong (resp. weak) topology on $M^\infty(G/H)$ if and only if $\lambda = f\mu$ where f is a bounded uniformly continuous (resp. continuous) function on G/H .

II. Functions and measures

Let \mathcal{K} be a complete non-archimedean valued field. Let X be a locally compact zerodimensional space. Let $B_c(X)$ be the ring of all open compact subsets of X . If A is an element of $B_c(X)$ then $\xi(A)$ will be the \mathcal{K} -valued characteristic function of A . Let $BC(X)$ be the space of all bounded continuous \mathcal{K} -valued functions on X . With the norm $\|f\| = \sup_{x \in X} |f(x)|$, ($f \in BC(X)$), $BC(X)$ becomes a non-archimedean Banach space over \mathcal{K} . With the seminorms $p_x(f) = |f(x)|$, ($f \in BC(X)$; $x \in X$), $BC(X)$ becomes a non-archimedean locally convex space over \mathcal{K} . The topology (resp. uniformity) on $BC(X)$ induced by the norm $\|\cdot\|$ will be called the uniform topology (resp. uniformity). The topology (resp. uniformity) on $BC(X)$ induced by the seminorms p_x ($x \in X$) will be called the pointwise topology (resp. uniformity). Let $BUC(X)$ be the space of all bounded \mathcal{U} -uniformly continuous \mathcal{K} -valued functions on X where \mathcal{U} is a non-archimedean uniformity on X compatible with the topology on X . Let $C_\infty(X)$ be the space of all continuous \mathcal{K} -valued functions on X which vanish at infinity. Both $BUC(X)$ and $C_\infty(X)$ are subspaces of $BC(X)$. In the same way as $BC(X)$, $BUC(X)$ and $C_\infty(X)$ can be made into a non-archimedean Banach space and a locally convex space over \mathcal{K} . Let $M^\infty(X)$ be

the space of all *bounded additive* \mathcal{X} -valued functions on $B_c(X)$. The elements of $M^\infty(X)$ will be called *measures* on X . With the norm $\|\lambda\| = \sup_{A \in B_c(X)} |\lambda(A)|$ ($\lambda \in M^\infty(X)$), $M^\infty(X)$ becomes a non-archimedean Banach space over \mathcal{X} . $(M^\infty(X), \|\cdot\|)$ is the *dual space* of $(C_\infty(X), \|\cdot\|)$. The duality is given by

$$(f, \lambda) \rightarrow \int_X f d\lambda = \int_X f(x) d\lambda(x) \quad (f \in C_\infty(X), \lambda \in M^\infty(X)).$$

Let λ be an element of $M^\infty(X)$. If for every x in X we define $N_\lambda(x) = \inf_{A \ni x} \sup_{B \subset A} |\lambda(B)|$ then for every A in $B_c(X)$ we have $\sup_{B \subset A} |\lambda(B)| = \sup_{x \in A} N_\lambda(x)$. In particular we have $\|\lambda\| = \sup_{x \in X} N_\lambda(x)$. With the seminorms $q_x(\lambda) = N_\lambda(x)$ ($x \in X, \lambda \in M^\infty(X)$), $M^\infty(X)$ becomes a non-archimedean locally convex space over \mathcal{X} . The topology (resp. uniformity) on $M^\infty(X)$ induced by the norm $\|\cdot\|$ will be called the *strong* topology (resp. uniformity). The topology (resp. uniformity) on $M^\infty(X)$ induced by the seminorms q_x ($x \in X$) will be called the *weak* topology (resp. uniformity).

If λ is an element of $M^\infty(X)$ and g is an element of $BC(X)$ we can define an element $g\lambda$ of $M^\infty(X)$ by setting:

$$\int_X f d g\lambda = \int_X f g d\lambda \quad (f \in C_\infty(X)).$$

It is not difficult to see that $N_{g\lambda}(x) = |g(x)|N_\lambda(x)$.

III. PROPOSITION (1): *Let λ be an element of $M^\infty(X)$ with $\inf_{x \in X} N_\lambda(x) \geq m > 0$. The map $g \rightarrow g\lambda$ is a linear homeomorphism from $BC(X)$ with the uniform (resp. pointwise) topology onto a closed subspace of $M^\infty(X)$ with the strong (resp. weak) topology.*

PROOF: It is clear that the map $g \rightarrow g\lambda$ from $BC(X)$ with the uniform (resp. pointwise) topology to $M^\infty(X)$ with the strong (resp. weak) topology is a linear homeomorphism. We only need to prove that its image is closed.

For the the uniform topology on $BC(X)$ and the strong topology on $M^\infty(X)$ this is trivial.

For the pointwise topology on $BC(X)$ and the weak topology on $M^\infty(X)$ this runs as follows: let $(f_\alpha \lambda)_{\alpha \in I}$ be a net in $M^\infty(X)$ converging weakly to μ . It is clear that the net $(f_\alpha)_{\alpha \in I}$ in $BC(X)$ converges pointwise to a function f . We have

$$N_{f(x)\lambda - \mu}(x) \leq \max [N_{f(x)\lambda - f_\alpha \lambda}(x), N_{f_\alpha \lambda - \mu}(x)] \quad (\alpha \in I, x \in X).$$

Therefore

$$\begin{aligned}
 N_{f(x)\lambda-\mu}(x) &= 0 \quad (x \in X). \\
 |f(x)| &\leq \frac{1}{m} |f(x)| N_\lambda(x) = \frac{1}{m} N_{f(x)\lambda}(x) \\
 &\leq \frac{1}{m} \max[N_{f(x)\lambda-\mu}(x), N_\mu(x)] \leq \frac{N_\mu(x)}{m} \tag{1}
 \end{aligned}$$

$$\begin{aligned}
 |f(x) - f(y)| &\leq \frac{1}{m} |f(x) - f(y)| N_\lambda(y) = \frac{1}{m} N_{f(x)\lambda-f(y)\lambda}(y) \\
 &\leq \frac{1}{m} \max[N_{f(x)\lambda-\mu}(y), N_{f(y)\lambda-\mu}(y)] \\
 &= \frac{1}{m} N_{f(x)\lambda-\mu}(y). \tag{2}
 \end{aligned}$$

From the inequalities (1) and (2) it is easy to conclude that $f \in BC(X)$. (Notice that the function $N_{f(x)\lambda-\mu}$ is uppersemicontinuous and $N_{f(x)\lambda-\mu}(x) = 0$). Now

$$N_{f\lambda-\mu}(x) \leq \max[N_{f\lambda-f(x)\lambda}(x), N_{f(x)\lambda-\mu}(x)] = 0$$

and we may conclude that $\mu = f\lambda$ and we are done.

IV. Groups and homogeneous spaces

Let G be a *locally compact zerodimensional group*. For every *open compact subgroup* K of G we can define an equivalence relation on G by setting:

$$s \sim t \Leftrightarrow sK = tK \quad (\text{resp. } s \sim t \Leftrightarrow Ks = Kt) \quad (s, t \in G).$$

Using those equivalence relations we can define the so called *left* (resp. *right*) *group uniformity* on G which is compatible with the topology on G .

Let $BLUC(G)$ (resp. $BRUC(G)$) be the space of all *bounded left* (resp. *right*) *uniformly continuous* \mathcal{X} -valued functions on G . If f is an element of $BC(G)$ and s is an element of G , we can define an element $R_s f$ (resp. $L_s f$) of $BC(G)$ by setting:

$$R_s f(t) = f(ts) \quad (\text{resp. } L_s f(t) = f(st)) \quad (t \in G).$$

If f is an element of $BC(G)$ then the functions $s \rightarrow L_s f$ and $s \rightarrow R_s f$ from G to $BC(G)$ are continuous for the pointwise topology on $BC(G)$.

An element f of $BC(G)$ is an element of $BLUC(G)$ (resp. $BRUC(G)$)

if and only if the function $s \rightarrow R_s f$ (resp. $s \rightarrow L_s f$) from G to $BC(G)$ is continuous for the uniform topology on $BC(G)$. It is important to notice that $C_\infty(G)$ is a subspace of $BLUC(G)$ (resp. $BRUC(G)$).

Let H be a closed subgroup of G . Let G/H be the set of all *left cosets* of H in G . Let $\pi: G \rightarrow G/H$ be the *natural quotient map* from G to G/H . With the quotient topology G/H also becomes a locally compact zero-dimensional space. G has an *action* on G/H by setting:

$$s(\pi(t)) = \pi(st) \quad (s \in G, \pi(t) \in G/H).$$

For every open compact subgroup K of G we can define an equivalence relation on G/H by setting:

$$\pi(s) \sim \pi(t) \Leftrightarrow \pi(Ks) = \pi(Kt) \quad (\pi(s), \pi(t) \in G/H).$$

Using those equivalence relations we can define the so called *homogeneous uniformity* on G/H which is compatible with the topology on G/H . Let $BUC(G/H)$ be the space of all *bounded uniformly continuous* \mathcal{X} -valued functions on G/H . If f is an element of $BC(G/H)$ and s is an element of G we can define an element $L_s f$ of $BC(G/H)$ by setting:

$$L_s f(\pi(t)) = f(\pi(st)) \quad (\pi(t) \in G/H).$$

If f is an element of $BC(G/H)$ then the function $s \rightarrow L_s f$ from G to $BC(G/H)$ is continuous for the pointwise topology on $BC(G/H)$. An element f of $BC(G/H)$ is an element of $BUC(G/H)$ if and only if the function $s \rightarrow L_s f$ from G to $BC(G/H)$ is continuous for the uniform topology on $BC(G/H)$. It is important to notice that $C_\infty(G/H)$ is a subspace of $BUC(G/H)$.

V. Quasi-invariant measures on homogeneous spaces

Let G be a *locally compact zero-dimensional group*. Let H be a *closed subgroup* of G . Let G/H be the *homogeneous space* of all left cosets of H in G . We suppose that G has an open compact q -free subgroup where q is the characteristic of the *residue class field* of \mathcal{X} . In that case G has an *invariant measure* m . H also has an *invariant measure* n . Let Δ be the *modular function* on G . Let δ be the *modular function* on H . Let f be an element of $C_\infty(G)$. We can define an element f^b of $C_\infty(G/H)$ by setting:

$$f^b(\pi(s)) = \int_H f(st) dt \quad (\pi(s) \in G/H).$$

The map $f \rightarrow f^b$ from $C_\infty(G)$ to $C_\infty(G/H)$ is linear and continuous and using duality we can define a linear and continuous map $\lambda \rightarrow \lambda^\#$ from

$M^\infty(G/H)$ to $M^\infty(G)$. We have:

$$\int_G f d\lambda^\# = \int_{G/H} f^b d\lambda.$$

For every s in G we have $N_\lambda(\pi(s)) = N_{\lambda^\#}(s)$.

A quasi-invariant measure on G/H is a measure μ such that $\mu^\# = \rho\mu$, where ρ is an invertible element of $BRUC(G)$. Such a measure does always exist and it is unique up to an invertible element of $BUC(G/H)$. We can even say more: for every open compact subgroup K of G there exists a quasi-invariant measure μ on G/H with $\mu^\# = \rho\mu$ where $|\rho| \equiv 1$ and ρ is constant on the right cosets of K (see [4]).

VI. (Uniformly) continuous measures

Let λ be an element of $M^\infty(G/H)$ and s an element of G . We can define an element λ_s of $M^\infty(G/H)$ by setting:

$$\int_{G/H} f d\lambda_s = \int_{G/H} L_s f d\lambda \quad (f \in C_\infty(G/H)).$$

An element λ of $M^\infty(G/H)$ is called a *continuous* (resp. *uniformly continuous*) *measure* if the function $s \rightarrow \lambda_s$ from G to $M^\infty(G/H)$ is weakly (resp. strongly) continuous.

It is clear that the function $s \rightarrow \lambda_s$ from G to $M^\infty(G/H)$ is weakly (resp. strongly) continuous if and only if the function $s \rightarrow (\lambda^\#)_s$ from G to $M^\infty(G)$ is weakly (resp. strongly) continuous. In order to find the continuous (resp. uniformly continuous) measures on G/H it suffices to find the continuous (resp. uniformly continuous) measures on G .

Let $(K_\alpha)_{\alpha \in I}$ be a fundamental system of open compact q -free subgroups of G . We can define functions $(u(K_\alpha))_{\alpha \in I}$ from G to \mathcal{X} by setting

$$u(K_\alpha) = \frac{1}{m(K_\alpha)} \xi(K_\alpha).$$

VII. PROPOSITION (2): *Let λ be an element of $M^\infty(G)$. Let m be an invariant measure on G . λ is a uniformly continuous (resp. continuous) measure if and only if $\lambda = fm$ for some element f of $BRUC(G)$ (resp. $BC(G)$).*

PROOF: (1) If $\lambda = fm$ for some element f of $BRUC(G)$ (resp. $BC(G)$) then the function $s \rightarrow \lambda_s$ from G to $M^\infty(G)$ is strongly (resp. weakly) continuous as can easily be verified.

(2) Suppose now that the function $s \rightarrow \lambda_s$ from G to $M^\infty(G)$ is strongly (resp. weakly) continuous. For every f in $C_\infty(G)$ we can define an element

$f * \lambda$ of $M^\infty(G)$ by setting:

$$f * \lambda = \int_G f(s) \lambda_s ds$$

$$\left(\text{resp. } \int_G g df * \lambda = \int_G f(s) \left[\int_G g d\lambda_s \right] ds \quad g \in C_\infty(G) \right).$$

We will prove that $u(K_\alpha) * \lambda$ converges strongly (resp. weakly) to λ . (*)

From this it follows (using Proposition 1) that it suffices to prove that for every f in $C_\infty(G)$ there exists an element F of $BRUC(G)$ (resp. $BC(G)$) with $f * \lambda = Fm$. Define

$$F(s) = \int_G \Delta(t)^{-1} f(st^{-1}) d\lambda(t)$$

$$\|F\| = \sup_{s \in G} |F(s)| = \sup_{s \in G} \left| \int_G \Delta(t)^{-1} f(st^{-1}) d\lambda(t) \right|$$

$$\leq \|\lambda\| \sup_{s \in G} \sup_{t \in G} |\Delta(t)^{-1} f(st^{-1})| \leq \|\lambda\| \|f\|.$$

$$\|L_t F - F\| = \sup_{s \in G} |F(ts) - F(s)|$$

$$= \sup_{s \in G} \left| \int_G \Delta(q)^{-1} [f(ts q^{-1}) - f(s q^{-1})] d\lambda(q) \right|$$

$$\leq \|\lambda\| \sup_{s \in G} \sup_{q \in G} |\Delta(q)^{-1} [f(ts q^{-1}) - f(s q^{-1})]|$$

$$\leq \|\lambda\| \|L_t f - f\|.$$

It is clear that F is an element of $BRUC(G)$ (in particular it is an element of $BC(G)$). Let g be an element of $C_\infty(G)$.

$$\int_G g df * \lambda = \int_G \int_G f(s) g(t) d\lambda_s(t) ds$$

$$= \int_G \int_G f(s) g(st) d\lambda(t) ds$$

$$= \int_G \int_G f(s) g(st) ds d\lambda(t)$$

$$= \int_G \int_G \Delta(t)^{-1} f(st^{-1}) g(s) ds d\lambda(t)$$

$$\begin{aligned}
&= \int_G g(s) \left[\int_G \Delta(t)^{-1} f(st^{-1}) d\lambda(t) \right] ds \\
&= \int_G g(s) F(s) ds \\
&= \int_G g dFm
\end{aligned}$$

and we may conclude that $f * \lambda = Fm$. We still need to prove (*):

– For the strong topology this runs as follows:

$$\forall \epsilon > 0 \exists K_\alpha \vdash \forall K_\beta \subset K_\alpha: \sup_{s \in K_\beta} \|\lambda_s - \lambda\| \leq \epsilon.$$

Using the inequality

$$\|u(K_\beta) * \lambda - \lambda\| = \left\| \frac{1}{m(K_\beta)} \int_{K_\beta} (\lambda_s - \lambda) ds \right\| \leq \sup_{s \in K_\beta} \|\lambda_s - \lambda\|$$

we see that

$$\forall \epsilon > 0 \exists K_\alpha \vdash \forall K_\beta \subset K_\alpha: \|u(K_\beta) * \lambda - \lambda\| \leq \epsilon \text{ and we are done.}$$

– For the weak topology this runs as follows:

$$\forall t \in G \forall \epsilon > 0 \exists K_\alpha \vdash \forall K_\beta \subset K_\alpha: \sup_{s \in K_\beta} N_{\lambda_s, -\lambda}(t) \leq \epsilon.$$

If $p \in K_\beta t$ (say $p = q^{-1}t$, $q \in K_\beta$) then

$$\begin{aligned}
N_{\lambda_s, -\lambda}(p) &= N_{\lambda_s, -\lambda}(q^{-1}t) \\
&= N_{\lambda_{sq}, -\lambda_q}(t) \leq \max(N_{\lambda_{sq}, -\lambda}(t), N_{\lambda_q, -\lambda}(t))
\end{aligned}$$

and we see that

$$\forall t \in G \forall \epsilon > 0 \exists K_\alpha \vdash \forall K_\beta \subset K_\alpha: \inf_{A \ni t} \sup_{s \in K_\beta} \sup_{p \in A} N_{\lambda_s, -\lambda}(p) \leq \epsilon$$

thus

$$\forall t \in G \forall \epsilon > 0 \exists K_\alpha \vdash \forall K_\beta \subset K_\alpha: \inf_{A \ni t} \sup_{s \in K_\beta} \sup_{B \subset A} |\lambda_s(B) - \lambda(B)| \leq \epsilon.$$

Using the inequality

$$N_{u(K_\beta) * \lambda, -\lambda}(t) = \inf_{A \ni t} \sup_{B \subset A} |(u(K_\beta) * \lambda - \lambda)(B)|$$

$$\leq \inf_{A \ni t} \sup_{B \subset A} \sup_{s \in K_\beta} |\lambda_s(B) - \lambda(B)|$$

we may conclude that

$$\forall t \in G \forall \epsilon > 0 \exists K_\alpha \vdash \forall K_\beta \subset K_\alpha: N_{u(K_\beta) * \lambda - \lambda}(t) \leq \epsilon$$

and we are done.

VIII. REMARK: *The function f of the foregoing proposition is the uniform (resp. pointwise) limit of the functions $(F_\alpha)_{\alpha \in I}$ defined by:*

$$F_\alpha(s) = \frac{\lambda(K_\alpha s)}{m(K_\alpha s)} \quad (s \in G, \alpha \in I).$$

PROOF: $\lambda = \lim_{\alpha \in I} u(K_\alpha) * \lambda = \lim_{\alpha \in I} F_\alpha m = (\lim_{\alpha \in I} F_\alpha) m$ with

$$\begin{aligned} F_\alpha(s) &= \int_G \Delta(t)^{-1} u(K_\alpha)(st^{-1}) d\lambda(t) \\ &= \frac{1}{m(K_\alpha)} \int_G \Delta(t)^{-1} \xi(K_\alpha s)(t) d\lambda(t) \\ &= \frac{1}{m(K_\alpha)} \int_G \Delta(s)^{-1} \xi(K_\alpha s)(t) d\lambda(t) \\ &= \frac{\lambda(K_\alpha s)}{m(K_\alpha) \Delta(s)} = \frac{\lambda(K_\alpha s)}{m(K_\alpha s)}. \end{aligned}$$

IX. THEOREM: *Let λ be an element of $M^\infty(G/H)$. Let μ be a quasi-invariant measure on G/H . λ is a uniformly continuous (resp. continuous) measure if and only if $\lambda = f\mu$ for some element f of $BUC(G/H)$ (resp. $BC(G/H)$).*

PROOF: Let ρ be the invertible function of $BRUC(G)$ with $\mu^\# = \rho m$. The function $s \rightarrow \lambda_s$ is strongly (resp. weakly) continuous if and only if the function $s \rightarrow (\lambda^\#)_s$ is strongly (resp. weakly) continuous.

(a) If the function $s \rightarrow (\lambda^\#)_s$ from G to $M^\infty(G)$ is strongly (resp. weakly) continuous there exists an element g of $BRUC(G)$ (resp. $BC(G)$) with $\lambda^\# = gm$. It is easy to see that $\lambda = f\mu$ where $f(\pi(s)) = [g(s)/\rho(s)]$ is an element of $BUC(G/H)$ (resp. $BC(G/H)$).

(b) If there exists an element f of $BUC(G/H)$ (resp. $BC(G/H)$) with $\lambda = f\mu$ then it is easy to see that $\lambda^\# = gm$ where $g(s) = \rho(s)f(\pi(s))$ is an element of $BRUC(G)$ (resp. $BC(G)$) and therefore we see that the function $s \rightarrow (\lambda^\#)_s$ from G to $M^\infty(G)$ is strongly (resp. weakly) continuous.

X. REMARK: The function f of the foregoing theorem is the uniform (resp. pointwise) limit of the functions $(F_\alpha)_{\alpha \in I}$ defined by:

$$F_\alpha(\pi(s)) = \frac{\lambda(\pi(K_\alpha s))}{\mu(\pi(K_\alpha s))} (\pi(s) \in G/H, \alpha \in I).$$

PROOF: Let ν be a quasi-invariant measure on G/H with $\nu^\# = \rho m$ where $|\rho| \equiv 1$ and ρ is constant on every right coset of every K_α ($\alpha \in I$). If $\lambda = g\nu$ then g is the uniform (resp. pointwise) limit of the functions $(G_\alpha)_{\alpha \in I}$ where

$$G_\alpha(\pi(s)) = \frac{\lambda^\#(K_\alpha s)}{m(K_\alpha s)\rho(s)}.$$

Now

$$\begin{aligned} \lambda^\#(K_\alpha s) &= \int_{G/H} \xi(K_\alpha s)^b d\lambda = \int_{G/H} \int_H \xi(K_\alpha s)(tr) dr d\lambda(\pi(t)) \\ &= n(K_\alpha s \cap H) \int_{G/H} \xi(\pi(K_\alpha s))(\pi(t)) d\lambda(\pi(t)) \\ &= n(K_\alpha s \cap H) \lambda(\pi(K_\alpha s)) \end{aligned}$$

and

$$\begin{aligned} m(K_\alpha s)\rho(s) &= \int_G \xi(K_\alpha s)(t)\rho(t) dt \\ &= \int_{G/H} \int_H \xi(K_\alpha s)(tr) dr d\pi(t) \\ &= n(K_\alpha s \cap H) \int_{G/H} \xi(\pi(K_\alpha s))(\pi(t)) d\pi(t) \\ &= n(K_\alpha s \cap H) \nu(\pi(K_\alpha s)) \end{aligned}$$

so we may conclude that

$$G_\alpha(\pi(s)) = \frac{\lambda(\pi(K_\alpha s))}{\nu(\pi(K_\alpha s))}.$$

If $\mu = h\nu$ for some invertible element h of $BUC(G/H)$ then, in the same way, h is the uniform limit of the functions $(H_\alpha)_{\alpha \in I}$ where

$$H_\alpha(\pi(s)) = \frac{\mu(\pi(K_\alpha s))}{\nu(\pi(K_\alpha s))}.$$

If $\lambda = f\mu = (g/h)\mu$ then we see that f is the uniform (resp. pointwise) limit of the functions $(F_\alpha)_{\alpha \in I}$ where

$$F_\alpha(\pi(s)) = \frac{G_\alpha(\pi(s))}{H_\alpha(\pi(s))} = \frac{\lambda(\pi(K_\alpha s))}{\mu(\pi(K_\alpha s))}$$

and were are finished.

XI. Final remark

The most important results of this paper can be reformulated as follows:

Let μ be a quasi-invariant measure on G/H . The map $f \rightarrow f\mu$ from $BUC(G/H)$ with the uniform topology (resp. $BC(G/H)$ with the pointwise topology) to $M^\infty(G/H)$ with the strong (resp. weak) topology is a linear homeomorphism onto a closed subspace of $M^\infty(G/H)$. This subspace consists exactly of those measures wick translate continuously for the strong (resp. weak) toplogy on $M^\infty(G/H)$.

We can always find a quasi-invariant measure μ on G/H with $N_\mu \equiv 1$. In that case the map $f \rightarrow f_\mu$ from $BUC(G/H)$ to $M^\infty(G/H)$ is a linear isometry.

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