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ASPECTS OF CATEGORICAL ALGEBRA IN INITIALSTRUCTURE CATEGORIES ★

by Manfred Bernd WISCHNEWSKY

Initialstructure functors $F: K \to L$, the categorical generalization of BOURBAKI's notion of an «initial object» [3], equivalent to Kennison's pullback stripping functors, which Wyler calls Top-functors, reflect almost all categorical properties from the base category L to the initial structure category K briefly called INS-category [1, 4, 5, 6, 8, 11, 12, 13, 15, 16, 18, 19, 20, 21, 22, 37]. So for instance if L is complete, cocomplete, wellpowered, cowellpowered, if L has generators, cogenerators, projectives, injectives, or (coequalizer, mono)-bicategory structures ($\Longrightarrow$ homomorphism theorem), then the same is valid for any INS-category over L. Finally all important theorems of equationally defined universal algebra (e.g. existence of free K-algebras, adjointness of algebraic functors...) can be proved for algebras in INS-categories, if they hold for algebras in L. The most well known INS-categories over *Ens* are the categories of topological, measurable, limit, locally path-connected, uniform, compactly generated, or zero dimensional spaces.

This survey article deals with the following three types of algebraic categories over INS-categories, namely with

- algebraic categories of Σ-continuous functors [29, 30, 33],
- monoidal algebraic categories over monoidal base categories in the sense of PFENDER [35],
- algebraic categories in the sense of THIEBAUD [36] resp. in the sense of EILENBERG-MOORE.

These three types of «algebraic categories» include comma-categories, algebraic categories in the sense of LAWVERE, categories of finite algebras, locally presentable categories, EILENBERG-MOORE categories, categories of monoids in monoidal categories.

★ Conférence donnée au Colloque d'Amiens (1973)

In this paper it is shown that each of the above algebraic categories over an INS-category is again an INS-category. Hence the whole theory of INS-categories, presented here in the first part, can again be applied to algebraic categories over INS-categories. In particular it is shown that this implies that adjointness of «algebraic» functors over L induces adjointness of «algebraic» functors over an INS-category K. Since furthermore together with K also K^{op} is an INS-category, and since the algebras in K^{op} are just the coalgebras in K, the theory is also valid for coalgebras in INS-categories. Moreover I will show that all the basic results on algebraic categories even hold for reflective or coreflective subcategories of INS-categories if they hold for the corresponding INS-categories. So for instance let U be a coreflective or even an epireflective of a coreflective subcategory of the category of topological spaces and continuous mappings like the categories of compactly generated, locally path connected, or T_i-spaces, $i=0, 1, 2, 3$, and let $\Sigma \subset [C, S]$ be a set of functorial morphisms in $[C, S]$. Then the algebraic categories $\Sigma(C, U)$ of all Σ-continuous functors with values in U are complete, cocomplete, wellpowered, and cowellpowered. Furthermore all inclusion functors

$$E : \Sigma(C, U) \longrightarrow [C, U]$$

are adjoint. The same holds for the category $\Sigma(C, U^{op})$ for all Σ-coalgebras in U.

1. INS-functors, INS-categories, and INS-morphisms.

Let $F: K \to L$ be a functor. Let I be an arbitrary not necessarily discrete, small category, and denote by $\Delta_K : K \to [I, K]$ resp. by $\Delta_L : L \to [I, L]$ the corresponding diagonal functors, and for $T \in [I, K]$ the comma categories (Δ_K, T) resp. (Δ_L, FT) of all functorial morphisms

$$\phi : \Delta_K k \to T, \; k \in K, \text{ resp. } \psi : \Delta_L l \to FT, \; l \in L.$$

If there is no confusion, I'll write for Δ_K resp. Δ_L simply Δ. F is called *transportable* if F creates weakly isomorphisms. With this notation we can give the following

1.1. DEFINITION. Let $F: K \to L$ be faithful, fibre-small functor. F is called an INS-*functor,* and K and INS-*category over* L, if for all small categories I, and for all functors $T \in [I, K]$ the functor

$$F_T : (\Delta, T) \to (\Delta, FT) : (\phi : \Delta k \to T) \mapsto (F\phi : \Delta Fk \to FT), k \in K,$$

has an adjoint (=right adjoint) G_T with counit

$$ID : F_T G_T = Id_{(\Delta, FT)} .$$

If $\psi : \Delta l \to FT$ is a cone, then the cone $G_T \psi : \Delta l^* \to T$ is called an INS-*cone,* and $l^* \in K$ an INS-*object generated by* ψ. If $\phi : l \to Fk$ is a monomorphism, then $G\phi : l^* \to k$ is called an *embedding,* and l^* an INS-*subobject* of k.

Dually one defines *coinitialstructure functor,*.....

1.2. EXAMPLES: The following categories are INS-categories over S the category of sets with obvious INS-functors: the categories of topological, uniform, measurable, or based spaces, of Borel spaces, of principal limit spaces, of spaces with bounded structure, of compactly generated spaces, of limit, completely regular, or zero dimensional spaces....

Let us now recall GROTHENDIECK's construction of a split category. Let $Ord(V)$ be the category of completely ordered sets with suprema preserving mappings. Then there corresponds by GROTHENDIECK ([8]) to each functor $P : L^{op} \to Ord(V)$ a split category $F_P : K_P \to L$: the objects of K_P are the pairs (l, k) with $k \in Pl$ and $l \in L$. The K_P-morphisms $f : (l, k) \to (l', k')$ are exactly those L-morphisms $f : l \to l'$ with $Pfk < k'$. The functor $F_P : K_P \to L$ is the projection $(l, k) \mapsto l$.

Herewith we can give the following

1.3. THEOREM [1, 8, 11, 15, 20, 23]. *Let* $F : K \to L$ *be a faithful, fibre-small functor. Regard the following assertions:*

(i) F *is an INS-functor.*

(ii) F^{op} *is an INS-functor.*

(iii) The category (K, F) *over* L *is* L*-equivalent to a split category* (K_P, F_P).

(iv) F *preserves limits and has a full and faithful adjoint* J.

Then (*i*) $\Longleftrightarrow$ (*ii*) $\Longleftrightarrow$ (*iii*) $\Longrightarrow$ (*iv*). *If moreover* L *and* K *are complete, then* (*iv*) $\Longrightarrow$ (*i*).

The equivalence (i) $\Longleftrightarrow$ (ii) $\Longleftrightarrow$ (iii) was first proved by ANTOINE [1] and ROBERTS [15], but follows straightforward from GROTHENDIECK's paper on fibred categories [8]. The characterization (iv) $\Longrightarrow$ (i) was first given by HOFFMANN [11]. A proof of (iv) $\Longrightarrow$ (i), in some way simpler than the original one, can immediately be obtained from both the following statements:

1) A cone $(l^*, \psi : \Delta l^* \to T)$, $T \in [I, K]$, is an INS-cone if and only if the induced cone
$$(l^*, \psi = (\psi_i) : l^* \to \prod_{i \in I} T_i)$$
is an INS-cone,

2) Let K, L be categories with pullbacks. Let $F : K \to L$ be a pullback preserving functor with an adjoint right inverse. Then F is a fibration (cf. [14] Corollaire 3.7) (cf. Theorem 1.9a).

Hence, if F fulfills (iv), F is a fibration by 2 and thus together with 1 an INS-functor provided K and L are complete.

The full and faithful right adjoint of an INS-functor is obtained from definition 1.1 by regarding the void category as index category. The objects of the image category of this functor are called *codiscrete K-objects.*

1.4. PROPOSITION [18, 20, 22]. *Let* $F : K \to L$ *be an INS-functor. Then we have the following assertions:*

1) K-limits (K-colimits) are limits (colimits) in L *supplied with the «initialstructure» (coinitialstructure) generated by the projections (injections).*

2) F preserves and reflects monos and epis.

3) K is wellpowered (cowellpowered) if and only if L *has this property.*

4) K has generators, cogenerators, projectives, or injectives if and only if K has such objects.

5) K is a (coequalizer, mono)-bicategory if and only if L is such a category.

Let now $F: K \to L$ and $F': K' \to L'$ be INS-functors, and let $M: K \to K'$ as well as $N: L \to L'$ be arbitrary functors.

1.5. DEFINITION [22, 23, 37]. The pair $(M, N): (K, F) \to (K', F')$ is called an INS-*morphism* if both of the following conditions hold:

1) $NF = F'M$,

2) for all small categories I and all functors $T: I \to K$ the adjoint right inverses G'_{MT} resp. G_T of F'_{MT} resp. F_T make the following diagram commutative up to an isomorphism

$$\begin{tikzcd}
(\Delta, MT) & (\Delta, F'MT) \arrow[l, "G'_{MT}"'] \\
(\Delta, T) \arrow[u, "M_T"] & (\Delta, FT) \arrow[l, "G_T"] \arrow[u, "N_{FT}"']
\end{tikzcd}$$

In this case we say that the pair (M, N) preserves INS-cones.
In WYLER's language of Top-categories, i.e. reduced INS-categories [19] the INS-morphisms correspond to his taut liftings [22, 23].

1.6. REMARKS.

1) The category *Initial* of all INS-functors and INS-morphisms is a double category in the sense of EHRESMANN.

2) The category *Initial(L)* of all INS-categories and INS-morphisms over a constant base category L is canonically isomorphic to the full functor category $[L^{op}, Ord(V)]$ (cf. [19]).

1.7. THEOREM [22, 23]. *Suppose that $(M, N): (K, F) \to (K', F')$ be an INS-morphism. Then we have the following equivalent statements:*

(i) M is adjoint.

(ii) N is adjoint.

The implication (i) $\Longrightarrow$ (ii) is trivial by theorem 1.3 (iv). In case that all categories involved are complete, it suffices to find a solution set for M, since M preserves obviously limits by proposition 1.4 and definition 1.5. If $R: L' \to L$ denotes a coadjoint of N, then for all

$k' \in K'$ the fibre $F^{-1}(RF'(k'))$ is a solution set for k', as one sees immediately by factorizing $F'f : F'k' \to NFk$, $f : k' \to Mk \in K'$, through the unit of $(R \dashv N)$, and then by supplying $RF'k'$ with the INS structure... In the general case one takes the infimum of all objects in this fibre appearing in any of the above factorizations.

For the rest of this paper assume now that all base categories of INS-categories are complete.

1.8. DEFINITION. Let $F : K \to L$, resp. $F' : K' \to L'$, be INS-functors. Denote by J, resp. J', an adjoint right inverse functor of F, resp. F', and by $\Phi : Id \to JF$, resp. $\Phi' : Id \to J'F'$, the corresponding units. Let furthermore $M : K \to K'$ and $N : L \to L'$ be a pair of functors with $F'M = NF$. The pair (M, N) *preserves codiscrete objects* if

$$J'N = MJ \quad \text{and} \quad M\Phi = \Phi' M.$$

1.9. THEOREM. *Let $(M, N) : (K, F) \to (K', F')$ be a pair of functors between INS-functors. If (M, N) preserves codiscrete objects, the following statements are equivalent:*

(i) $(M, N) : (K, F) \to (K', F')$ is an INS-morphism.

(ii) M and N preserve limits.

The implication (i) $\Longrightarrow$ (ii) is obvious, since M preserves INS-cones and hence in particular limit-cones. Thus one has only to prove (ii) $\Longrightarrow$ (i). From the remark 1 of theorem 1.3 it follows that one has only to show that (M, N) preserves INS-cones of the form $(l^*, \psi : l^* \to k)$ generated by $F\psi : l \to Fk$, i.e. (M, N) is a morphism of fibration in the sense of GROTHENDIECK. But this follows immediately from the fact, that

a) the INS-cone $\psi : l^* \to k$ is a projection of a pullback, namely

$$\begin{array}{ccc} l^* & \xrightarrow{\ \psi\ } & k \\ {\scriptstyle id^*}\downarrow & & \downarrow{\scriptstyle \Phi k} \\ Jl & \xrightarrow[J(F\psi)]{} & JFk \end{array}$$

where $\Phi : Id \to JF$ denotes the unit of the adjoint functor pair (F, J), and

id^* denotes the K-morphism induced by $id_I : I \to I$.

b) (M, N) preserves codiscrete objects, and M preserves pullbacks.

1.10. EXAMPLE [18, 20, cp. 22, 36, 37]. Let A be an algebraic theory in the sense of LAWVERE and let $F: K \to L$ be an INS-functor over a complete category L. Then we obtain the following commutative diagram of limit preserving functors between complete categories

$$\begin{tikzcd} Alg(A,K) \arrow[r, "Alg(A,F)"] \arrow[d, "V_K"'] \arrow[dr, phantom, "="] & Alg(A,L) \arrow[d, "V_L"] \\ K \arrow[r, "F"] & L \end{tikzcd} .$$

V_K resp. V_L denote the corresponding forgetful functors. (V_K, V_L) fulfills obviously all assumptions of theorem 1.9. Hence we obtain in particular:

V_K is adjoint if and only if V_L is adjoint.

1.11. COROLLARY. *Each morphism in Initial(L) is adjoint.*

1.12. EXAMPLE. Let A be an algebraic theory in the sense of LAWVERE. Then the functor $Alg(A, -)$ defines in an obvious way a functor [19]:

$$Alg(A, -): Initial(L) \to Initial(Alg(A, L))$$

by
$$\begin{tikzcd} K \arrow[rr, "H"] \arrow[dr, "F"'] & {} \arrow[d, phantom, "="] & K' \arrow[dl, "F'"] \\ & L & \end{tikzcd} \longmapsto \begin{tikzcd} Alg(A,K) \arrow[rr, "Alg(A,H)"] \arrow[dr, "Alg(A,F)"'] & {} \arrow[d, phantom, "="] & Alg(A,K') \arrow[dl, "Alg(A,F')"] \\ & Alg(A,L) & \end{tikzcd}$$

Since $Alg(A, H)$ is an INS-morphism, it is coadjoint by the above Corollary. A typical example for this situation is the functor

uniform groups $\longrightarrow$ topological groups

induced by the INS-morphism

uniform spaces $\longrightarrow$ topological spaces.

1.13. DEFINITION [19, 23]. Let $F: K \to L$ be an INS-functor, and let U be a strictly full subcategory of K. U is called an *INS-subcategory* if U is closed under INS-objects, i.e. if for all small categories I and all functors $T \in [I, U]$ the INS-object l^* generated by a cone $(\psi : \Delta l \to FT)$ lies again in U.

INS-subcategories can be characterized in the following way:

1.14. THEOREM [19]. *Let $F: K \to L$ be an INS-functor. A strictly full subcategory U of K is an INS-subcategory of K if and only if*

1) U is closed under products and INS-subobjects,

2) U contains all codiscrete K-objects.

1.15. EXAMPLES:

1) Let K be an INS-category over S. Since K is complete, wellpowered, and cowellpowered, a stricly full subcategory U of K is epireflective in K if and only if U is closed under products and INS-subobjects [9, 18, 19]. Hence we get the

COROLLARY [19]. *Let K be an INS-category over S, and let U be a strictly full subcategory of K containing all codiscrete K-objects. Then there are equivalent:*

(i) U is an INS-subcategory of K.

(ii) U is epireflective in K.

2) Since all but one discrete K-object of an INS-category over S are generators in K, we obtain by dualizing and applying the results of HERRLICH-STRECKER [10] the following

COROLLARY [19, 23]. *Let K be an INS-category over S, and let U be a strictly full subcategory of K containing all discrete K-objects. Then there are equivalent:*

(i) U is a COINS-subcategory of K.

(ii) U is coreflective in K.

In case of $K = Top$ one can find a whole host of examples for these corollaries in [9].

Finally we will need the following

1.16. DEFINITION [18]. Let $F: K \to L$ be an INS-functor. A strictly full subcategory U of K is called a *PIS-subcategory* of K, if U is closed under products and INS-subobjects.

Recall that each extremal K-monomorphism in an INS-category K is an embedding [18, 19, 20]. Furthermore if K is a complete, wellpowered and cowellpowered category, then a strictly full subcategory is epireflective in K if and only if it is closed under products and extremal subobjects [9]. Hence we obtain the

1.17. PROPOSITION. *Let $F: K \to L$ be an INS-functor over a complete, wellpowered, and cowellpowered category L. Then each PIS-subcategory is epireflective in K. If moreover in L each monomorphism is a kernel, then the notions PIS-subcategory and epireflective subcategory are equivalent* [18].

So for instance a strictly full subcategory of the category of locally convex spaces is epireflective if and only if it is a PIS-subcategory, i.e. closed under products and subspaces, since the category of locally convex spaces is an INS-category over the category of complex-valued vector spaces.

2. Algebraic categories of Σ-continuous functors with values in INS-categories.

Let us briefly recall some standard notions on Σ-continuous functors [28, 29, 30, 31]. Let K be a category, and $\Sigma \subset Mor K$ a class of K-morphisms. A K-object k is called *Σ-bijective*, or *Σ-continuous*, if for all $\sigma: a \to b$ the mapping

$$K(\sigma, k): K(b, k) \to K(a, k): f \to f\sigma$$

is bijective. ΣK shall denote the full subcategory of all Σ-bijective K-objects in K. If C is a small category and $\Sigma \subset Mor\,[C, S]$ is a class of functorial morphisms, then the pair (C, Σ) is called a *theory*. A functor $A: C \to K$, where K is an arbitrary category, is called a *Σ-algebra* if all functors

$$K(k, A\,\cdot): C \to S, \quad k \in K,$$

are Σ-bijective. A theory (C, Σ) is called *algebraic* if the inclusion functor $\Sigma(C, S) \to [C, S]$ is adjoint. Let Σ^* denote an arbitrary class of theories. A category K is called *Σ^*-algebraic* if for all theories (C, Σ)

in Σ^* the inclusion functor $\Sigma(C,K) \to [C,K]$ is adjoint, where $\Sigma(C,K)$ denotes the full subcategory of all Σ-algebras in K.

Let now (C,Σ) be a theory, and let $F: K \to L$ be an INS-functor with coadjoint right inverse D. Let furthermore

$$A \in \Sigma(C,K), \; l \in L \text{ and } \sigma: G \to H \in \Sigma.$$

Herewith we obtain the following commutative diagram [37] :

$$\begin{array}{ccc} [H,[l,F^C A\cdot]] & \xrightarrow{\cong} & [H,[Dl,A\cdot]] \\ {\scriptstyle [\sigma,[l,F^C A\cdot]]}\downarrow & = & \downarrow{\scriptstyle [\sigma,[Dl,A\cdot]]} \\ [G,[l,F^C A\cdot]] & \xrightarrow{\cong} & [G,[Dl,A\cdot]] \end{array}$$

Hence if A is a Σ-algebra in K, then $F^C A$ is a Σ-algebra in L. By the same method one shows that the functor $F^\Sigma = F^C|_{\Sigma(C,K)}$ has an adjoint right inverse induced pointwise by that of F. For the rest of this chapter assume that the base category L is complete, in order to be able to apply characterizations of INS-functors and INS-morphisms given in the preceeding chapter, although the following theorems are valid without any restriction.

2.1. THEOREM (cp. [37]). *Let $F: K \to L$ be an INS-functor over a complete category L. Then we obtain the following assertions:*

1) The functor $F: \Sigma(C,K) \to \Sigma(C,L): A \to FA$ is an INS-functor.

Denote by

$$v_c^K: \Sigma(C,K) \to K, \text{ resp. } v_c^L: \Sigma(C,L) \to L,$$

the evaluation functors for $c \in C$, and by

$$E: \Sigma(C,K) \to [C,K], \text{ resp. } E: \Sigma(C,L) \to [C,L],$$

the corresponding inclusion functors. Let furthermore (C,Σ) and (D,Ψ) be theories. A functor $f: D \to C$ is called a *morphism of theories* if the induced functor $[f,S]: [C,S] \to [D,S]$ preserves algebras. If $f: (D,\Psi) \to (C,\Sigma)$ is a theory morphism, and K an arbitrary category, then the canonical functor $[f,K]: [C,K] \to [D,K]$ preserves algebras

[37]. The restriction of $[f, K]$ on $\Sigma(C, K)$ is called an *algebraic functor* [37] and denoted by f^K if there is no misunderstanding. With this notation we obtain

2) The pair (v_c^K, v_c^L) *is an INS-morphism. In particular* v_c^K *is adjoint if and only if* v_c^L *is adjoint.*

3) The pair (E, E) *of inclusion functors is an INS-morphism. In particular* K *is* Σ^**-algebraic if and only if* L *is* Σ^**-algebraic for any class* Σ^* *of theories.*

4) Let $f:(D, \Psi) \to (C, \Sigma)$ *be a morphism of theories. Then the pair* (f^K, f^L) *of algebraic functors is an INS-morphism. In particular an algebraic functor over* K *is adjoint if and only if the corresponding algebraic functor over* L *is adjoint.*

As an immediate application we obtain the following

2.2. THEOREM. *Let* P^* *be the class of all theories* (C, Σ), *where* Σ *is a set, and let* $F: K \to L$ *be an INS-functor over a locally presentable category* L. *Then*

1) K *is* P^**-algebraic.*

2) Each P^**-algebraic functor over* K *is adjoint.*

3) Each evaluation functor $v_c : \Sigma(C, K) \to K$ *is adjoint.*

4) $\Sigma(C, K)$ *is complete, cocomplete, wellpowered and cowellpowered.*

5) $\Sigma(C, K)$ *is again an INS-category over a locally presentable category.*

6) Each theory (C, Σ) *defines a functor*

$$\Sigma(C, -): Initial(L) \to Initial(\Sigma(C, L))$$

by

$$\begin{tikzcd} K \arrow[rr, "H"] \arrow[dr, "F"'] & & K' \arrow[dl, "F'"] \\ & L & \end{tikzcd} \longmapsto \begin{tikzcd} \Sigma(C, K) \arrow[rr, "\Sigma(C, H)"] \arrow[dr, "\Sigma(C, F)"'] & & \Sigma(C, K') \arrow[dl, "\Sigma(C, F')"] \\ & \Sigma(C, L) & \end{tikzcd}$$

In particular $\Sigma(C, H)$ is adjoint for all INS-morphisms H.

2.3. REMARK. Theorem 2.2 allows us to construct in a simple way a lot of INS-categories over locally presentable categories. So for instance start with an INS-category over the locally presentable category S, the category of sets: $F: K \to S$. Then take any locally presentable theory (C, Σ), as e.g. an algebraic theory in the sense of LAWVERE, a GROTHENDIECK-topology, or more general a limit-cone bearing category in the sense of BASTIANI-EHRESMANN [24]. Then the functor

$$F^{\Sigma}: \Sigma(C, K) \to \Sigma(C, S)$$

is an INS-functor over the locally presentable category $\Sigma(C, S)$. Now one can continue with this procedure applying theorem 2.2.5. Thus one obtains that the categories of topological, measurable, compactly generated, locally convex, bornological or zero dimensional spaces, groups, rings, sheaves...are P^*-algebraic, bicomplete, biwellpowered...

Since with K also K^{op} is an INS-category, and since each dual of a locally presentable category is P^*-algebraic (BASTIANI unpublished), we get the following

2.4. THEOREM. *Let K be an INS-category over a locally presentable category. Then we obtain the following statements:*

1) K^{op} is P^*-algebraic.

2) Each P^*-algebraic functor over K^{op} is adjoint.

3) Each evaluation functor $v_c: \Sigma(C, K^{op}) \to K^{op}$ is adjoint.

Let us now regard algebraic categories of Σ-continuous functors over subcategories of INS-categories.

2.5. THEOREM [37]. *Let $F: K \to L$ be an INS-functor and $U \subset K$ be a PIS-subcategory. Then the following assertions are valid:*

1) If (C, Σ) is a theory, then $\Sigma(C, U)$ is a PIS-subcategory of $\Sigma(C, K)$. In particular if $\Sigma(C, K)$ is complete and biwellpowered then $\Sigma(C, U)$ is an epireflective subcategory of $\Sigma(C, K)$.

2) If L is (C, Σ)-algebraic and if $\Sigma(C, L)$ is cowellpowered, then U is again (C, Σ)-algebraic.

From this theorem follows for instance that each epireflective subcategory of an INS-category over S is P^*-algebraic, or that each epireflective subcategory of the category of locally convex spaces is P^*-algebraic.

3. Monoidal algebraic categories over monoidal INS-categories.

The theory of monoidal universal algebra over monoidal categories, or more exactly over S-monoidal categories, is in some way a generalization of the equationally defined universal algebra in the classical sense. The S-monoidal categories, defined by M. PFENDER [35] using ideas of BUDACH-HOEHNKE, are a generalization of monoidal or symmetric monoidal categories in the sense of EILENBERG-KELLY. A S-monoidal theory $(C, \square, can)$ is a small category C equipped with a S-monoidal structure. The S-monoidal algebras are functors from a S-monoidal theory into a S-monoidal category preserving the given S-monoidal structure. Standard examples for this procedure are the monoids in monoidal categories as e.g. the monads over a fixed category, Hopf-algebras, resp. coalgebras in the sense of SWEEDLER, or monoids in the classical sense. In order not to complicate the presentation here by lengthy technical details, I will regard here only monoids over monoidal categories in the usual sense. Everything, which is stated here in the following for these special monoidal algebraic categories, is also valid for arbitrary S-monoidal algebraic categories.

By a monoidal functor I always mean a strict monoidal functor. Let now $F: K \to L$ be an INS-functor over a monoidal category. Since F has an adjoint right-inverse, we get the

3.1. LEMMA. *Let* $F: K \to L$ *be an INS-functor over a monoidal category* $L = (L, \square, can)$. *Then there exists at least one monoidal structure on* K, *such that*

$$F: K = (K, \square, can) \to L = (L, \square, can)$$

is a monoidal functor.

In general there are a lot of monoidal structures on K, such that F becomes a monoidal functor, as the following examples show.

EXAMPLES. Denote by $(S, \times, can)$ the category of sets with the cartesian closed structure defined by the product $\times$. Let now $F: K \to S$ be an INS-functor over S. Then the most important monoidal structures on K are the following:

1) $K = (K, \Pi, can)$ with the product-monoidal structure lifted by the INS-functor F. In general (K, Π, can) is no more cartesian closed as the cases $K = Top$ or $K = Unif$ show.(*)

2) Denote by $K = (K, D\times, can)$ the category K with the monoidal structure defined by the product-monoidal structure on S, and the discrete K-object functor $D: S \to K$, i.e.

$$D\times(k, k') := D(Fk \times Fk').$$

Then each functor

$$D\times k(\cdot) := D(Fk \times F\cdot): K \to K$$

has an adjoint, but $(K, D\times, can)$ is in general not closed monoidal.

3) Denote by $K = (K, \square, can)$ the category K together with the «inductive» cartesian product-structure, i.e. $k \square k'$ is the cartesian product on S supplied with the finest K-structure, such that $id_{Fk\times Fk'}$ is an F-morphism (continuous, uniformly continuous, measurable...([20])) in each argument. The canonical functorial morphisms «can» are defined as in S. Then $(K, \square, can)$ is a closed monoidal category. So for instance each coreflective subcategory of Top or $Unif$ is closed monoidal.

In general it is not known if closed monoidality is an INS-hereditary property, i.e. if an INS-category over a closed monoidal category is again closed monoidal. Furthermore there do not exist internal characterizations of those INS-categories over cartesian closed categories which are again cartesian closed. The most well-known cartesian closed INS-categories over S are the categories of compactly generated and of quasi-topological spaces.

In the following we assume that K carries any monoidal structure, such that

(*) $K = (K, \Pi, can)$ is cartesian closed iff $k\,\Pi$-, $k \in K$, preserves colimits. (Apply special adjoint functor theorem.)

$$F : \mathbf{K} = (K, \square, can) \to \mathbf{L} = (L, \square, can)$$

becomes a monoidal functor. Denote by $Mon\,\mathbf{K}$ resp. $Mon\,\mathbf{L}$ the categories of monoids over $\mathbf{K}$ resp. over $\mathbf{L}$. With this notation we obtain the following

3.3. THEOREM. *Let $F : (K, \square, can) \to (L, \square, can)$ be a monoidal INS-functor. Then the following assertions are valid:*

1) The induced functor $Mon\,F : Mon\,\mathbf{K} \to Mon\,\mathbf{L}$ is again an INS-functor.

2) The pair of forgetful functors $V_K : Mon\,\mathbf{K} \to K$ and $V_L : Mon\,\mathbf{L} \to L$ defines an INS-morphism:

$$(V_K, V_L) : (Mon\,F, Mon\,K) \to (F, K).$$

In particular V_K is adjoint if and only if V_L is adjoint.

In particular the adjoint right-inverse of $Mon\,F$ is given by

$$Mon\,J : Mon\,\mathbf{L} \to Mon\,\mathbf{K} : \langle l, \mu, \eta \rangle \mapsto \langle J\,l, \mu_J, \eta_J \rangle$$

with

$$\mu_J : J\,l \square J\,l \to J\,l, \quad F\mu_J = \mu, \quad \text{and} \quad \eta_J : e_K \to J\,l, \quad F\eta_J = \eta.$$

In the same way as for equationally defined algebraic categories one can prove the following

3.4. LEMMA. *Let $\mathbf{K}$ be an arbitrary monoidal category, and $V : Mon\,\mathbf{K} \to K$ be the corresponding forgetful functor. Then*

1) V creates limits.

2) V creates absolute coequalizers.

Since an adjoint functor is monadic if and only if it creates absolute coequalizers, we obtain the following

3.5. COROLLARY. *Let $F : (K, \square, can) \to (L, \square, can)$ be a monoidal INS-functor. Then the following assertions are equivalent:*

(i) $V : Mon\,\mathbf{K} \to K$ is monadic.

(ii) $V : Mon\,\mathbf{L} \to L$ is monadic.

3.6. COROLLARY. *Let $F : (K, \square, can) \to (L, \square, can)$ be a monoidal INS-functor over a monoidal category $(L, \square, can)$ with countable coproducts. Assume that $l \square_L \cdot$ and $\cdot \square_L l$ preserve these coproducts. Then V:*

$Mon\, K \to K$ is monadic.

3.7. EXAMPLE. The forgetful functor $V : Top(R\text{-}mod) \to R\text{-}mod$ from the category of topological R-modules over a topological ring R into the category of R-modules is a monoidal INS-functor. The monoidal structure on $R\text{-}mod$ is defined by the tensor product and on $Top(R\text{-}mod)$ by the inductive topology on the tensor product. Since $R\text{-}mod$ is closed monoidal, all assumptions of the corollary 3.6 are fulfilled. Hence the forgetful functor

$$V : Top(R\text{-}Alg) = Mon(Top(R\text{-}mod)) \to Top(R\text{-}mod)$$

from the category of topological R-algebras into $Top(R\text{-}mod)$ is monadic. The coadjoint is the functor «topological tensor algebra».

Let now $(k, \mu, \varepsilon) \in Mon\, K$. Denote by $Lact(k, K)$ the category of K-objects, on which k acts on the left. Let $F : (K, \Box, can) \to (L, \Box, can)$ be a monoidal INS-functor. Then F induces a functor

$$Lact\, F : Lact(k, K) \to Lact(Fk, L) : (k, f) \mapsto (Fk, Ff).$$

With this notation we obtain the

3.8. THEOREM. *Let $F : (K, can) \to (L, can)$ be a monoidal INS-functor. Then the induced functor $Lact\, F : Lact(k, K) \to Lact(Fk, L)$ is again an INS-functor.*

3.9. EXAMPLE. Let K be an INS-category over S, e.g. the category of topological, measurable, compactly generated, zero dimensional or uniform spaces. The category $K(Ab)$ of all abelian groups in K is a closed monoidal INS-category over Ab, the category of abelian groups. The monoidal structure is given by the «inductive tensorproduct». The monoids in in $K(Ab)$ are just the rings in K. The category $Lact(r, K(Ab))$ is the category of all K-modules over the K-ring r. Hence the forgetful functor $K(r\text{-}mod) \to r\text{-}mod$ into the category of all r-modules in Ens is an INS-functor. Hence the category $K(r\text{-}mod)$ is complete, cocomplete, wellpowered, cowellpowered, has generators, cogenerators, projectives, injectives and a canonically defined (coequalizer, mono) bicategory structure. But $K(r\text{-}mod)$ is in general not abelian, since bimorphisms need not to be isomorphisms.

4. Algebraic Categories (in the sense of THIEBAUD) over INS-categories.

THIEBAUD's notion of an algebraic category [36] includes EILENBERG-MOORE categories, categories of finite algebras, comma-categories... It is defined in a completely natural way by BENABOU's profunctors. I regard here algebraic categories over arbitrary base categories, but restrict myself in the case of the underlying algebraic types to types which are induced by *Ens*-valued algebraic functors.

4.1. ALGEBRAIC FUNCTORS AND CATEGORIES [36]. THIEBAUD has defined in his thesis (unpublished) a pair of adjoint functors

$$(Cat, A) \underset{Sem_A}{\overset{Str_A}{\rightleftarrows}} Comon(A)$$

the structure Str_A and the semantics Sem_A from the category of all categories over an arbitrary category A to the category of comonoids $Comon(A)$ in the monoidal category $Dist(A)$ of all distributors (=profunctors in BENABOU's terminology) over A. This pair of adjoint functors induces a monad on (Cat, A). The algebras in the corresponding EILENBERG-MOORE category are called algebraic functors, resp. algebraic categories. In the presence of an adjoint (or a coadjoint) the notions of categories algebraic over A and categories monadic (resp. comonadic) over A coincide. In other words this pair of adjoint functors allows us to associate a category of algebras to an arbitrary functor, in such a way that, if this functor has an adjoint or a coadjoint, then we obtain the category of algebras or coalgebras in the sense of EILENBERG-MOORE.

Let us now briefly recall the basic definitions of algebraic functors as well as some of their properties, in particular the stability under pullbacks. We assume that the reader is familiar with bicategories in the sense of BENABOU [25].

4.1.1. DEFINITION [25]. The bicategory *Dist* of all distributors (over *Ens*) is given by the following data:

- the set of objects of *Dist* is the category *Cat* of «all» categories (small rel. to *Ens*);

- for $A, B \in Cat$, the category $Dist(A, B)$ of all distributors $A \dashrightarrow B$ is defined to be the functor category $[A^{op} \times B, Ens]$;

- let $\Phi : A \dashrightarrow B$ and $\Psi : B \dashrightarrow C$ be two distributors. The composition $\Phi \otimes \Psi : A \dashrightarrow C$ is defined pointwise as coequalizer in Ens of the following pair (d_0, d_1) of morphisms :

$$
\begin{tikzcd}
(\phi, f, \psi) \arrow[r, mapsto] & (\phi, \Psi(f,c)\psi) \\
\Phi(a,b)\times B(b,b')\times\Psi(b',c) \arrow[r] \arrow[d, "incl"'] & \Phi(a,b)\times\Psi(b,c) \arrow[d, "incl"] \\
\coprod_{b,b'\in B} \Phi(a,b)\times B(b,b')\times\Psi(b',c) \arrow[r, shift left, "d_0"] \arrow[r, shift right, "d_1"'] & \coprod_{b\in B} \Phi(a,b)\times\Psi(b,c) \\
\Phi(a,b)\times B(b,b')\times\Psi(b',c) \arrow[r] \arrow[u, "incl"] & \Phi(a,b)\times\Psi(b,c) \arrow[u, "incl"'] \\
(\phi, f, \psi) \arrow[r, mapsto] & (\Phi(a,f)\phi, \psi)
\end{tikzcd}
$$

These attachments define a bifunctor

$$\otimes : Dist(A, B) \times Dist(B, C) \to Dist(A, C);$$

- for $A \in Cat$, the Hom-functor $A(-,-)$ is defined to be the identity-distributor $1_A : A \dashrightarrow A$.

- The coherent natural isomorphisms are defined in an obvious way.

As abbreviation we write $\phi \underset{B}{\otimes} \psi$ for the equivalence class in $\Phi \otimes \Psi(a,c)$ generated by $(\phi, 1_b, \psi)$ with $\phi \in \Phi(a,b)$, $\psi \in \Psi(b,c)$.

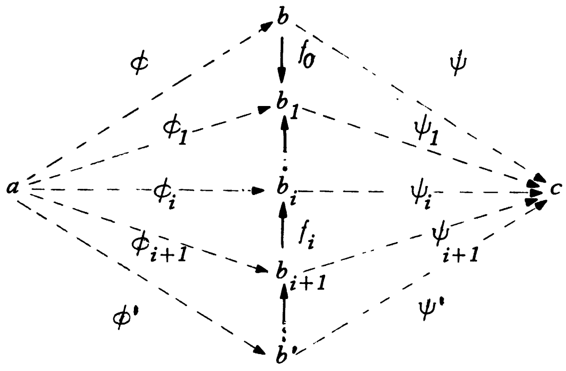
4.1.2. REMARK [36]. $\phi \underset{B}{\otimes} \psi = \phi' \otimes \psi'$ in $\Phi \underset{B}{\otimes} \Psi(a,c)$ iff there exists a finite chain

$$b \xrightarrow{f_0} b_1 \longleftarrow \dots b_i \longrightarrow \dots b_k \xleftarrow{f_k} b'$$

in B, and elements $\phi_i \in \Phi(a, b_i)$ and $\psi_i \in \Psi(b_i, c)$, $i = 1, \dots, k$, such that for all i,

$$\Phi(a, f_i)(\phi_{i+1}) = \phi_i \quad \text{and} \quad \Psi(f_i, c)(\psi_i) = \psi_{i+1},$$

visualized by the following commutative diagram :



The category $Dist(A, A)$, $A \in Cat$, is a monoidal category with the above defined functor «$\otimes$» as multiplication. Denote by $Comon(A)$ the category of all comonoids in $Dist(A, A)$. We define now the pair of adjoint functors (Str_A, Sem_A).

Let (G, ε, δ) be a comonoid on A. In particular

$$\varepsilon : G \to 1_A = A(-,-) \quad \text{and} \quad \delta : G \to G \otimes G$$

are functorial morphisms. A *G-algebra* is a pair (a, x), where $a \in A$ and $x \in G(a, a)$, such that

$$\varepsilon(x) = 1_a \quad \text{and} \quad \delta(x) = x \underset{A}{\otimes} x.$$

A *morphism* $f : (a, x) \to (a', x')$ *of G-algebras* is a A-morphism $f : a \to a'$, such that

$$\begin{tikzcd} a \arrow[r, dashed, "x"] \arrow[d, "f"'] & a \arrow[d, "f"] \\ a' \arrow[r, dashed, "x'"'] & a' \end{tikzcd}$$

is commutative, i.e. $G(f, a')x' = G(a, f)x$.

We shall denote by $Alg(A, G)$ the category of G-algebras. The underlying forgetful functor is denoted by $U(G) : Alg(A, G) \to A$. Furthermore each comonoid-morphism $\phi : (G, \varepsilon, \delta) \to (G', \varepsilon', \delta')$ defines in a canonical way a functor

$$Alg(A, \phi) : Alg(A, G) \to Alg(A, G')$$

over A. The assignment

$$G \mapsto U(G): Alg(A, G) \to A \text{ and } \phi \mapsto Alg(A, \phi)$$

defines a functor $Sem_A : Comon(A) \to (Cat, A)$.

The functor

$$Str_A : (Cat, A) \to Comon(A)$$

is defined in the following way: Let $U : B \to A$ be a functor. Then U defines in an obvious way two distributors:

$$\Phi_U : B \nrightarrow A \text{ by } \Phi_U(b, a) := A(Ub, a),$$

$$\Phi^U : A \nrightarrow B \text{ by } \Phi^U(a, b) := A(a, Ub).$$

Let $\Phi \in Dist(A, B)$ and $\Psi \in Dist(B, A)$ be two arbitrary 1-cells, i.e. distributors of $Dist$. Recall that Φ is $Dist$-coadjoint (left-adjoint) to Ψ, if there exist 2-cells (functorial morphisms)

$$\eta : 1_A \to \Phi \otimes \Psi \text{ and } \varepsilon : \Psi \otimes \Phi \to 1_B$$

satisfying the relations

$$\eta \otimes \Phi . \Phi \otimes \varepsilon = 1_\Phi \text{ and } \Psi \otimes \eta . \varepsilon \otimes \Psi = 1_\Psi .$$

One of the most important properties of the bicategory $Dist$ is the fact that in $Dist$ each functor has a coadjoint, i.e. for any functor $U : B \to A$ the distributor $\Phi_U : B \nrightarrow A$ is $Dist$-caoadjoint to $\Phi^U : B \nrightarrow A$. Since each pair of $Dist$-adjoint functors defines a monad resp. a comonad, we can define $Str_A(U)$ as the comonoid $(\Phi^U \otimes \Phi_U, \varepsilon_U, \delta_U)$ on A generated by the pair (Φ^U, Φ_U) of $Dist$-adjoint functors.

4.1.3. PROPOSITION (THIEBAUD). *Structure is adjoint to semantics.*

The adjoint pair of functors (Str_A, Sem_A) induces on (Cat, A) a monad denoted by Alg_A.

4.1.4. DEFINITION. *A category algebraic over A* is an Alg_A-algebra. *Morphisms of algebraic categories over A* are the Alg_A-morphisms.

4.1.5. EXAMPLES.

1) Let G be a comonoid on A. Then the category $Alg(A, G)$ of G-algebras is algebraic over A.

2) Let $F: A \to C$ and $G: B \to C$ be functors. Then the comma category (F, G) is algebraic over $A \times B$. In particular the category (A, a) of objects over a and (a, A) of objects under a, $a \in A$, are algebraic over A.

4.1.6. PROPOSITION [36]. *Let $U: B \to A$ be a category over A. Then U is monadic if and only if U is algebraic and has a coadjoint.*

4.1.7. PROPOSITION [36]. *Let*

$$\begin{tikzcd} B' \arrow[r] \arrow[d, "U'"'] & B \arrow[d, "U"] \\ A' \arrow[r] & A \end{tikzcd}$$

be a pullback-diagram in Cat. Then if U is algebraic so is U'.

4.1.8. PROPOSITION [36]. *Let $U: B \to A$ be algebraic. Then U creates those limits and colimits which are absolute.*

4.2. ALGEBRAIC CATEGORIES OVER INS-CATEGORIES.

4.2.1. DEFINITION (cf. [17]). Let $T: A \to Ens$ be a functor. The category of A-objects or *T-objects T-obj(K) in an arbitrary category K* is defined as a category over K by the pullback in *Cat*:

$$\begin{tikzcd} T\text{-}obj(K) \arrow[r] \arrow[d] & {[K^{op}, A]} \arrow[d, "{[K^{op}, T]}"] \\ K \arrow[r, "Y"] & {[K^{op}, Ens]} \end{tikzcd}$$

where Y is the Yoneda-embedding.

Since with T also $[K^{op}, T]$ is algebraic, we get from 4.1.7:

4.2.2. PROPOSITION. *If A is algebraic over Ens, then T-obj(K) is algebraic over K.*

4.2.3. COROLLARY. *Let $T: A \to Ens$ be monadic. Then T-obj(K) is a category algebraic over K, and in particular again monadic, if and only if T-obj$(K) \to K$ is adjoint.*

One can easily prove the following

4.2.4. PROPOSITION. *Let* $T: A \to Ens$ *be an algebraic functor. Then*

1) $T\text{-}obj(K)$ *is complete, if* K *is complete.*

2) The forgetful functor $T\text{-}obj(K) \to K$ *creates all colimits which are preserved by the Yoneda-embedding* $Y: K \to [K^{op}, Ens]$.

Let now $F: K \to L$ be an INS-functor with coadjoint D. Then F induces a functor

$$T\text{-}obj\,F : T\text{-}obj(K) \to T\text{-}obj(L) \quad \text{by} \quad (k, A_k) \mapsto (Fk, A_k D^{op}).$$

4.2.5. THEOREM. *Let* $F: K \to L$ *be an INS-functor. Then*

1) $T\text{-}obj\,F : T\text{-}obj(K) \to T\text{-}obj(L)$ *is an INS-functor iff* $T\text{-}obj\,F$ *is fibresmall. This is equivalent to the condition, that there exists up to isomorphisms only a set of structures* A_k, $k \in K$, *such that* (k, A_k) *is a* $T\text{-}obj(K)$*-algebra. This for instance is always the case, when* K *is an INS-category over* L *and* $T\text{-}obj(L) \to L$ *is monadic. In the following we assume that* $T\text{-}obj\,F$ *is always fibresmall.*

2) The pair of forgetful functors

$$U_K : T\text{-}obj(K) \to K \quad \text{and} \quad U_L : T\text{-}obj(L) \to L$$

forms an INS-morphism

$$(U_K, U_L) : (T\text{-}obj(K), T\text{-}obj\,F) \to (K, F)$$

visualized by

$$\begin{tikzcd} T\text{-}obj(K) \arrow[r, "T\text{-}obj\,F"] \arrow[d, "U_K"'] & T\text{-}obj(L) \arrow[d, "U_L"] \\ K \arrow[r, "F"] & L \end{tikzcd}$$

In particular U_K *is monadic if and only if* U_L *is monadic.*

4.2.6. COROLLARY (ERTEL-SHUBERT [6]). *Let* K *be an INS-category over* Ens, *and let* $T: A \to Ens$ *be monadic. Then* $T\text{-}obj(K) \to K$ *is again monadic.*

4.2.7. COROLLARY [18]. *Let* K *be an INS-category over an arbitrary category* L, *and let* A *be an algebraic category in the sense of* LAWVERE. *Then the forgetful functor* $U_K : Alg(A, K) \to K$ *is monadic if and only if the forgetful functor* $U_L : Alg(A, L) \to L$ *is monadic.*

4.3. CONNECTION BETWEEN THE CATEGORIES OF T-OBJECTS IN K AND PRE-T-OBJECTS IN K.

4.3.1. DEFINITION. Let $F : K \to L$ be an INS-functor, and let $T : A \to Ens$ be an algebraic functor. The category *Pre-T-obj(K)* is defined by the following pullback in *Cat*:

$$\begin{array}{ccc} Pre\text{-}T\text{-}obj(K) & \longrightarrow & T\text{-}obj(L) \\ {\scriptstyle U_K^p}\downarrow & & \downarrow{\scriptstyle U_L} \\ K & \xrightarrow{F} & L \end{array}$$

i.e. the objects of *Pre-T-obj(K)* are the objects of *T-obj(L)* with an arbitrary «K-structure» on it.

The following propositions are categorical routine.

4.3.2. PROPOSITION. *The forgetful functor*

$$Pre\text{-}T\text{-}obj(K) \to T\text{-}obj(L)$$

is an INS-functor.

4.3.3. PROPOSITION. $U_K^P : Pre\text{-}T\text{-}obj(K) \to K$ *is algebraic.* U_K^P *is monadic if and only if* U_L *is monadic. The pair* (U_K^P, U_L) *defines an INS-morphism.*

4.3.4. PROPOSITION. *Assume that* $T : A \to Ens$ *is monadic. Then T-obj(K) is up to isomorphisms a* BIRKHOFF*-subcategory of Pre-T-obj(K), i.e.*

1) *T-obj(K) is closed under products in Pre-T-obj(K).*
2) *T-obj(K) is closed under extremal monos.*
3) *T-obj(K) is closed under retracts in Pre-T-obj(K).*

4.3.5. PROPOSITION. *Assumption as above. T-obj(K) is an INS-subcategory of Pre-T-obj(K). In particular T-obj(K) is an epireflective subcategory of Pre-T-obj(K) for any category K.*

4.4. CATEGORIES OF FINITE ALGEBRAS OVER INS-CATEGORIES.

In this paragraph we have to restrict ourselves to INS-categories over *Ens*, since otherwise the following notions give no sense. Denote by *Fin(Ens)* the category of finite sets in *Ens*.

4.4.1. DEFINITION. Let $F : K \to Ens$ be an INS-functor. The category $Fin(K)$ of *finite K-objects* is defined by the following pullback in Cat:

$$(*)\qquad \begin{array}{ccc} Fin(K) & \longrightarrow & Fin(Ens) \\ \downarrow & & \downarrow \\ K & \xrightarrow{\ F\ } & Ens \end{array} .$$

4.4.2. PROPOSITION.

1) $Fin(K) \to Fin(Ens)$ is an INS-functor.

2) $Fin(K) \to K$ is algebraic.

The proof of assertion 1 is trivial, whereas 2 follows from the fact that $Fin(Ens) \to Ens$ is algebraic [36] and that $(*)$ is a pullback diagram.

4.4.3. DEFINITION. *Let* $T : A \to Ens$ be an algebraic functor. The category $T\text{-}obj(Fin(K))$ is called the *category of finite T-objects in K* and is denoted by $Fin(T\text{-}obj(K))$. In particular $Fin(T\text{-}obj(K)) \to Fin(K)$ is an algebraic functor.

4.4.4. THEOREM. *Let $T : A \to Ens$ be an algebraic functor and $F : K \to Ens$ be an INS-functor. Then the forgetful functor*

$$Fin(T\text{-}obj(K)) \to Fin(T\text{-}obj(Ens))$$

is again an INS-functor.

4.4.5. THEOREM. *Let $T : A \to Ens$ be an algebraic functor. Then the assignement*

$$Fin : Initial(Ens) \to Initial(Fin(T\text{-}obj(Ens)))$$

$$K \to Ens \longmapsto Fin(T\text{-}obj(K)) \to Fin(T\text{-}obj(Ens))$$

defines a functor.

4.4.6. EXAMPLE. Let $H : Unif \to Top$ be the canonical functor from the category of uniform spaces to the category of topological spaces. Since H is an INS-morphism, the induced functor

$$Fin(T\text{-}obj(Unif)) \to Fin(T\text{-}obj(Top))$$

is again an INS-morphism, and hence in particular adjoint.

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