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CAHIERS DE TOPOLOGIE ET GEOMETRIE DIFFERENTIELLE Vol. XVI-1 (1975)

WHEN IS Ω A COGENERATOR IN A TOPOS? (*)

by Francis BORCEUX (**)

Let $\underline{E}$ be a topos such that the subobjects of 1 form a set of generators; then Ω is a cogenerator in $\underline{E}$. This means that the composition map $(A,B) \to ((B,\Omega),(A,\Omega))$ is a monomorphism in the category of sets, for any objects A and B of $\underline{E}$. Let us now consider the composition morphism $B^A \to (\Omega^A)^{(\Omega^B)}$ in $\underline{E}$; this morphism is monic in any topos, proving that Ω is an internal cogenerator in any topos. In particular the functor $\Omega^{(-)}: \underline{E}^* \to \underline{E}$ is faithful for any topos $\underline{E}$.

If the subobjects of 1 form a set of generators in the topos $\underline{E}$, the same property holds in any one of the following topoï: the topoï $\underline{E}/X$, where X is any object of $\underline{E}$; the topoï of sheaves for any topology on $\underline{E}$ and the topoï of $\underline{E}$-valued presheaves over any preordered object of $\underline{E}$. In all these topoï, Ω is thus a cogenerator. We also give an example of a topos in which Ω is not a cogenerator, and another example in which Ω is a cogenerator but the subobjects of 1 do not form a set of generators.

1. Cogenerators in a cartesian closed category.

In this section, $\underline{E}$ will be a cartesian closed category. All the results of this section remain true when $\underline{E}$ is a symmetric monoidal closed category (cf. [1]-§5). We first define the notion of an internal cogenerator.

In the category $\underline{S}$ of sets, an object C is a cogenerator if the composition map

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$$\mathsf{K}_C^{A,B} : B^A \longrightarrow (C^A)^{(C^B)}$$

which sends f to C^f is monic for any sets A and B. If $\underline{E}$ is cartesian closed, such a morphism exists in $\underline{E}$ for any objects A, B, C; we recall its construction (cf. [2]):

$$B^A \times A \times C^B \xrightarrow{ev \times id} B \times C^B \xrightarrow{ev} C$$

$$B^A \times C^B \longrightarrow C^A$$

$$\mathsf{K}_C^{A,B} : B^A \longrightarrow (C^A)^{(C^B)}.$$

DEFINITION 1. *Let $\underline{E}$ be a cartesian closed category. An object $C \in |\underline{E}|$ is called an internal cogenerator if, for any objects A and B of $\underline{E}$, the composition morphism $\mathsf{K}_C^{A,B} : B^A \to (C^A)^{(C^B)}$ is a monomorphism.*

The notion of an internal generator is defined in an analogous way using the left composition morphisms

$$\mathsf{L}_{A,B}^C : B^A \longrightarrow (B^C)^{(A^C)}.$$

PROPOSITION 1. *1 is an internal generator in any cartesian closed category.* ■

If C is an internal cogenerator in the cartesian closed category $\underline{E}$, the maps $(A, B) \to (C^B, C^A)$ which send f to C^f are injective (apply the limit preserving functor $(1, \cdot)$ to the monomorphisms $\mathsf{K}_C^{A,B}$); in other words, the functor $C^{(\cdot)} : \underline{E}^* \to \underline{E}$ is faithful. It is useful to point out that the converse is true.

PROPOSITION 2. *If $\underline{E}$ is a cartesian closed category, the following properties are equivalent:*

(1) $C \in |\underline{E}|$ is an internal cogenerator;

(2) the functor $C^{(\cdot)} : \underline{E}^* \to \underline{E}$ is faithful.

We have already seen that (1) implies (2). Conversely let us assume that (2) is true and let us consider any morphism $\alpha : X \to B^A$ in $\underline{E}$; we denote the corresponding morphism by $\overline{\alpha} : X \times A \to B$. The following composites correspond to each other by the bijections defining the cartesian adjunction :

$$X \xrightarrow{\alpha} B^A \xrightarrow{\mathbf{K}_C^{A,B}} (C^A)^{(C^B)}$$

$$X \times C^B \xrightarrow{\alpha \times id} B^A \times C^B \longrightarrow C^A$$

$$X \times A \times C^B \xrightarrow{\alpha \times id \times id} B^A \times A \times C^B \xrightarrow{ev \times id} B \times C^B \xrightarrow{ev \times id} C$$

$$C^B \xrightarrow{C^{ev}} C^{(B^A \times A)} \xrightarrow{C^{\alpha \times id}} C^{X \times A}$$

$$C^B \xrightarrow{C^{\overline{\alpha}}} C^{X \times A} .$$

If $\alpha, \beta : X \to B^A$ are such that $\mathbf{K}_C^{A,B} \circ \alpha = \mathbf{K}_C^{A,B} \circ \beta$, then $C^{\overline{\alpha}} = C^{\overline{\beta}}$ and thus $\overline{\alpha} = \overline{\beta}$; so $\alpha = \beta$ and $\mathbf{K}_C^{A,B}$ is monic. ■

COROLLARY 1. *If $\underline{E}$ is a cartesian closed category, any cogenerator of $\underline{E}$ is an internal cogenerator.*

The following diagram is commutative :

$$\begin{array}{ccc} E^* & \xrightarrow{C^{(\cdot)}} & \underline{E} \\ & \searrow^{(\cdot, C)} & \downarrow (1, \cdot) \\ & & S \end{array}$$

and thus $C^{(\cdot)}$ is faithful as soon as $(\cdot, C)$ is faithful. ■

COROLLARY 2. *If $\underline{E}$ is a cartesian category such that 1 is a generator, the following conditions are equivalent :*

(1) $C \in |\underline{E}|$ is a cogenerator.

(2) $C \in |\underline{E}|$ is an internal cogenerator.

$(1, \cdot)$ is faithful and thus $C^{(\cdot)}$ is faithful if and only if $(\cdot, C)$ is faithful (cf. diagram of corollary 1). ■

2. Cogenerators in a topos.

In this section, $\underline{E}$ is a topos. We first prove the two properties of Ω announced in the introduction.

THEOREM 1. *If $\underline{E}$ is any topos, the functor $\Omega^{(\cdot)} : E^* \to E$ is faithful and thus Ω is an internal cogenerator.*

If $f: A \to B$ is any morphism of $\underline{E}$, the following diagram is commutative (cf. [4]):

$$\begin{array}{ccc} (\Omega^B)^B & \xrightarrow{(\Omega^B)^f} & (\Omega^B)^A \\ \downarrow & & \downarrow \\ (\Omega^B)^B & \xrightarrow{(\Omega^f)^B} & (\Omega^A)^B . \end{array}$$

So if $f, g: A \to B$ are such that $\Omega^f = \Omega^g$, then $(\Omega^B)^f = (\Omega^B)^g$ and thus

$$(f, \Omega^B) = (1, (\Omega^B)^f) = (1, (\Omega^B)^g) = (g, \Omega^B).$$

In particular, if $\{*\}_B$ denotes the singleton morphism on B:

$$\{*\}_B \circ f = (f, \Omega^B)(\{*\}_B) = (g, \Omega^B)(\{*\}_B) = \{*\}_B \circ g$$

and $f = g$ because $\{*\}_B$ is monic.

We have proved that $\Omega^{(-)}$ is faithful; Ω is an internal cogenerator because of proposition 2. ■

THEOREM 2. *Let $\underline{E}$ be a topos. If the subobjects of 1 form a set of generators, Ω is a cogenerator.*

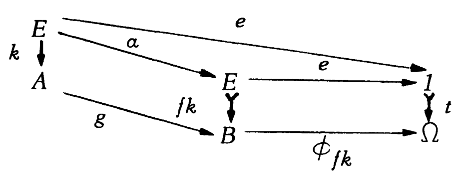
Let $f, g: A \to B$ be two morphisms such that, for any $\phi: B \to \Omega$, $\phi f = \phi g$. For any subobject $e: E \rightarrowtail 1$ of 1 and any morphism $k: E \to B$, we consider the following pullback:

$$\begin{array}{ccc} E & \xrightarrow{e} & 1 \\ {\scriptstyle fk}\downarrow & \text{p.b.} & \downarrow{\scriptstyle t} \\ B & \xrightarrow[\phi_{fk}]{} & \Omega \end{array}$$

(recall that any morphism with domain E is necessarily monic). The following equalities hold

$$\phi_{fk} \circ g \circ k = \phi_{fk} \circ f \circ k = t_E \quad (\text{true on } E)$$

and thus there exists a unique morphism α making the following diagram commutative:



But id_E is the unique morphism from E to E; thus $\alpha = id_E$ and $fk = gk$. Because this is the case for any E and any k and because the subobjects of 1 form a set of generators, $f = g$. So Ω is a cogenerator. ■

The assumption of theorem 2 (the subobjects of 1 form a set of generators) raises two questions:

1° when is this assumption realized? - some partial answers will be given in section 3;

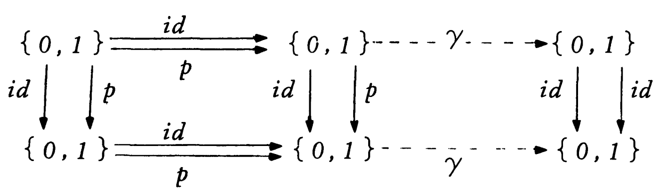
2° is this assumption necessary? - the two following examples show that a non obvious part of the assumption is necessary.

EXAMPLE 1. *Let $\underline{E}$ be the topos of set-valued presheaves over the additive group $\mathbf{Z}_2$. $\underline{E}$ is a boolean topos and its Ω-object is not a cogenerator.*

$\mathbf{Z}_2$ is a groupoid and thus $\underline{E}$ is boolean (cf. [4]). So Ω is the constant functor on $\{0,1\}$. We denote by $p:\{0,1\}\to\{0,1\}$ the map such that $p(0)=1$ and $p(1)=0$. Let $F:\mathbf{Z}_2\to\underline{S}$ be the following functor:

$$\begin{cases} F(*)=\{0,1\}, \\ F(0)=id_{\{0,1\}}, \\ F(1)=p. \end{cases}$$

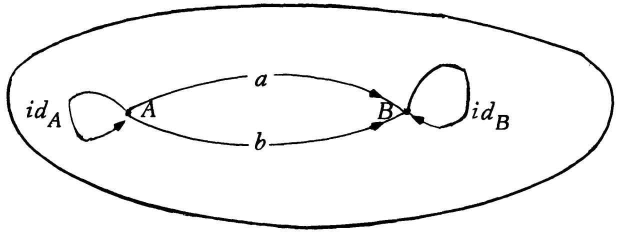
The two maps $id_{\{0,1\}}: F(*)\to F(*)$ and $p: F(*)\to F(*)$ are two different natural transformations from F to itself.



If $\gamma:\{0,1\}\to\{0,1\}$ is any natural transformation from F to Ω, the naturality implies that $\gamma p = \gamma$ and thus no such γ is able to separate $id_{\{0,1\}}$ and p. Therefore Ω is not a cogenerator. ■

EXAMPLE 2. *Let $\underline{E}$ be a topos of set-valued presheaves over the diagram $\underline{A}$ below defining equalizers and coequalizers. The Ω-object of $\underline{E}$ is a cogenerator but the subobjects of 1 do not form a set of generators.*

We denote by $\underline{A}$ the following category:



$A \neq B$
$a \neq b$.

We first prove that the subobjects of 1 do not form a set of generators in $\underline{E}$. Let us denote by $p : \{0,1\} \to \{0,1\}$ the map such that $p(0)=1$ and $p(1)=0$. We define two functors $F, G : \underline{A} \to \underline{S}$ by

$$\begin{cases} FA = \{0,1\} \\ FB = \{0,1\} \\ Fa = id_{\{0,1\}} \\ Fb = p \end{cases} \qquad \begin{cases} GA = \{0,1\} \\ GB = \{0\} \\ Ga = ct_0 \\ Gb = ct_0 \end{cases}$$

and two natural transformations $\alpha, \beta : F \Longrightarrow G$ by:

$$\begin{cases} \alpha_A = id_{\{0,1\}} \\ \alpha_B = ct_0 \end{cases} \qquad \begin{cases} \beta_A = p \\ \beta_B = ct_0 \, . \end{cases}$$

α and β are different and if $E : \underline{A} \to \underline{S}$ and $\gamma : E \Longrightarrow F$ are such that $\alpha\gamma \neq \beta\gamma$

$$\begin{array}{ccccc} E(A) & \xrightarrow{\gamma_A} & \{0,1\} & \overset{id}{\underset{p}{\rightrightarrows}} & \{0,1\} \\ {\scriptstyle E(a)}\downarrow\downarrow{\scriptstyle E(b)} & & {\scriptstyle id}\downarrow\downarrow{\scriptstyle p} & & {\scriptstyle ct_0}\downarrow\downarrow{\scriptstyle ct_0} \\ E(B) & \xrightarrow[\gamma_B]{} & \{0,1\} & \overset{ct_0}{\underset{ct_0}{\rightrightarrows}} & \{0\} \end{array}$$

then $\alpha_A \circ \gamma_A \neq \beta_A \circ \gamma_A$ because $\alpha_B \circ \gamma_B = \beta_B \circ \gamma_B$. Thus $E(A) \neq \emptyset$; we choose $x \in E(A)$. It is clear that $\gamma_A(x) \neq (p \circ \gamma_A)(x)$ and thus, because γ is a natural transformation, we have necessarily $E(a)(x) \neq E(b)(x)$. So $E(B)$ contains at least two different elements and E cannot be a subobject of 1, proving that the subobjects of 1 do not form a set of generators in $\underline{E}$.

We now describe Ω. Recall that $\Omega(X)$ is the set of subfunctors of $(X,-)$ and that $\Omega(x): \Omega(A) \to \Omega(B)$ sends a subfunctor A' of $(A,-)$ to the subobject B' of $(B,-)$ defined by the following pullback (cf. [6]):

$$\begin{array}{ccc} B' & \Longrightarrow & A' \\ \Downarrow & \text{p.b.} & \Downarrow \\ (B,-) & \underset{(x,-)}{\Longrightarrow} & (A,-) \end{array}$$

It is easy to see that Ω is characterized by the following relations:

$$\Omega(A) = \{A_1, A_2, A_3, A_4, A_5\} \quad \text{with}$$

$$\begin{cases} A_1(A) = \emptyset \\ A_1(B) = \emptyset \end{cases} \quad \begin{cases} A_2(A) = \emptyset \\ A_2(B) = \{a\} \end{cases} \quad \begin{cases} A_3(A) = \emptyset \\ A_3(B) = \{b\} \end{cases}$$

$$\begin{cases} A_4(A) = \emptyset \\ A_4(B) = \{a, b\} \end{cases} \quad \begin{cases} A_5(A) = \{id_A\} \\ A_5(B) = \{a, b\} \end{cases} ,$$

$\Omega(B) = \{B_1, B_2\}$ with

$$\begin{cases} B_1(A) = \emptyset \\ B_1(B) = \emptyset \end{cases} \qquad \begin{cases} B_2(A) = \emptyset \\ B_2(B) = \{id_B\} \end{cases} ,$$

$\Omega(a)$ and $\Omega(b)$ are described by:

$$\Omega(a) \begin{cases} A_1 \to B_1 \\ A_2 \to B_2 \\ A_3 \to B_1 \\ A_4 \to B_2 \\ A_5 \to B_2 \end{cases} \qquad \Omega(b) \begin{cases} A_1 \to B_1 \\ A_2 \to B_1 \\ A_3 \to B_2 \\ A_4 \to B_2 \\ A_5 \to B_2 \end{cases}$$

We finally prove that Ω is a cogenerator of $\underline{E}$. We take any two functors $F, G: \underline{A} \to \underline{S}$ and any two natural transformations $\alpha, \beta: F \Longrightarrow G$ such that $\alpha \neq \beta$. We have to build a natural transformation $\gamma: G \Longrightarrow \Omega$ such that $\gamma\alpha \neq \gamma\beta$. We consider two different cases:

$$\begin{array}{ccccc} FA & \overset{\alpha_A}{\underset{\beta_A}{\rightrightarrows}} & GA & \xrightarrow{\gamma_A} & \Omega A \\ Fa \downarrow \downarrow Fb & & Ga \downarrow \downarrow Gb & & \Omega a \downarrow \downarrow \Omega b \\ FB & \overset{\alpha_B}{\underset{\beta_B}{\rightrightarrows}} & GB & \xrightarrow{\gamma_B} & \Omega B \end{array}$$

First case : $\alpha_A \neq \beta_A$.

We denote by $x \in FA$ an element such that $\alpha_A(x) \neq \beta_A(x)$. We define γ by the following relations

$$\begin{cases} \gamma_A(\alpha_A(x)) = A_4 \\ \gamma_A(y) = A_5 \text{ if } y \neq \alpha_A(x) \\ \gamma_B(z) = B_2 \text{ for any } z \in GB . \end{cases}$$

Second case : $\alpha_B \neq \beta_B$.

We denote by $x \in FB$ an element such that $\alpha_B(x) \neq \beta_B(x)$. We define γ by the following relations :

$$\begin{cases} \gamma_B(\alpha_B(x)) = B_1 \\ \gamma_B(z) = B_2 \text{ if } z \neq \alpha_B(x) \\ \gamma_A(y) = A_1 \text{ if } (Ga)(y) = \alpha_B(x) \text{ and } (Gb)(y) = \alpha_B(x) \\ \gamma_A(y) = A_3 \text{ if } (Ga)(y) = \alpha_B(x) \text{ and } (Gb)(y) \neq \alpha_B(x) \\ \gamma_A(y) = A_2 \text{ if } (Ga)(y) \neq \alpha_B(x) \text{ and } (Gb)(y) = \alpha_B(x) \\ \gamma_A(y) = A_4 \text{ if } (Ga)(y) \neq \alpha_B(x) \text{ and } (Gb)(y) \neq \alpha_B(x) \end{cases}$$

It is easy to see that in the two cases, γ is a natural transformation such that $\gamma\alpha \neq \gamma\beta$. Thus Ω is a cogenerator in $\underline{E}$. ■

3. The weak axiom of choice.

By «weak axiom of choice» we mean, for a topos, the fact that the subobjects of 1 form a set of generators; this terminology is due to W. MITCHELL (cf. [6]) and makes sense because of the property we recall in proposition 4 below. In this section we give different conditions under which a topos satisfies the weak axiom of choice. Recall that the weak axiom of choice implies, for a topos, that Ω is a cogenerator (theorem 2).

Proposition 3 generalizes proposition 3.12 of [4] .

PROPOSITION 3. *Let $\underline{E}$ be a boolean topos. The following conditions are equivalent :*

1) the subobjects of 1 form a set of generators;

2) the non-zero subobjects of 1 form a set of generators;

3) an object $X \in |\underline{E}|$ *is non-zero if and only if there exists a non-zero subobject* E *of* 1 *provided with a morphism* $E \to X$;

4) if an object $X \in |\underline{E}|$ *is non-zero, there exists a non-zero subobject of* 1 *provided with a morphism* $E \to X$.

(1) $\Longrightarrow$ (2) is obvious.

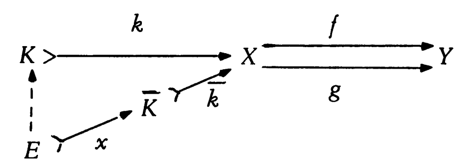
(2) $\Longrightarrow$ (3). If $X \in |\underline{E}|$ is non-zero, the two morphisms t_X (true on X) and f_X (false on X) from X to Ω are different and thus there exists a non-zero subobject E of 1 provided with a morphism $E \xrightarrow{x} X$ such that $f_X \circ x \neq t_X \circ x$:

$$E \xrightarrow{\ x\ } X \overset{f_X}{\underset{t_X}{\rightrightarrows}} \Omega$$

If there exists a non-zero subobject E of 1 provided with a morphism $E \xrightarrow{x} X$, X is a non-zero; indeed, if X were zero, E would also be zero because 0 is initial strict (cf. [4]).

(3) $\Longrightarrow$ (4) is obvious.

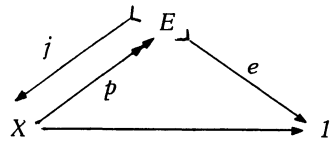
(4) $\Longrightarrow$ (1). Let $f, g : X \to Y$ be two different morphisms. We denote by K their equalizer and by $\overline{K}$ the complement of this equalizer in X. Because $f \neq g$, $K \neq X$; because $K \amalg \overline{K} = X$, $\overline{K} \neq 0$. Thus there exists a non-zero subobject E of 1 provided with a morphism $x : E \to \overline{K}$:



$f \circ (\overline{k} \circ x)$ is different from $g \circ (\overline{k} \circ x)$ because the equality would imply that $\overline{k} \circ x$ factorizes through k and thus $0 \neq E \subset K \cap \overline{K}$; this is impossible because $K \cap \overline{K} = 0$. ∎

PROPOSITION 4. *Let* $\underline{E}$ *be a (boolean) topos. If* $\underline{E}$ *satisfies the axiom of choice, the subobjects of* 1 *form a set of generators.*

Let X be a non-zero object of $\underline{E}$. We denote by E the image of of the morphism from X to 1:



E is non-zero because X is non-zero and 0 is initial strict. The axiom of choice implies the existence of a section $j: E \rightarrowtail X$ of p; the result follows thus from proposition 3. ∎

PROPOSITION 5. *Let $\underline{E}$ be a topos. The following conditions are equivalent:*

1) $\underline{E}$ satisfies the weak axiom of choice;

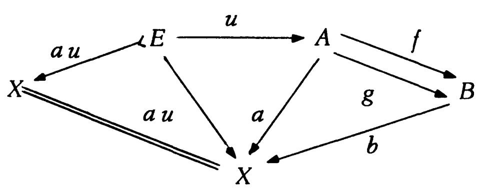
2) for any $X \in |\underline{E}|$, $\underline{E}/X$ satisfies the weak axiom of choice.

(2) $\Longrightarrow$ (1) : choose $X = 1$; $\underline{E}/1 \simeq \underline{E}$.

(1) $\Longrightarrow$ (2). The terminal object of $\underline{E}/X$ is the identity on X. If $f, g: (A, a) \to (B, b)$ are two different morphisms of $\underline{E}/X$, we denote by E a subobject of 1 in $\underline{E}$ and by $u: E \to A$ a morphism of $\underline{E}$, such that $gu \neq fu$. Because any morphism with domain E is monic,

$$au: (E, au) \longrightarrow (X, id_X)$$

is monic in $\underline{E}/X$ and $u: (E, au) \to (A, a)$ is a morphism of $\underline{E}/X$ such that $fu \neq gu$.



∎

PROPOSITION 6. *Let $\underline{E}$ be a topos. The following conditions are equivalent:*

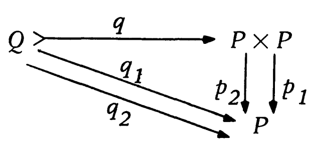
1) $\underline{E}$ satisfies the weak axiom of choice;

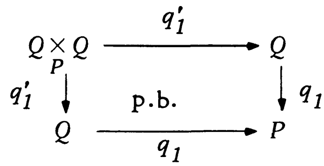
2) for any preordered object P of $\underline{E}$, the topos $\underline{E}^P$ of $\underline{E}$-valued presheaves over P satisfies the weak axiom of choice.

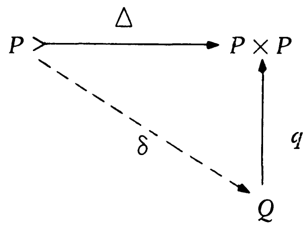
(2) $\Longrightarrow$ (1): choose $P = 1$; $\underline{E}^1 \simeq \underline{E}$.

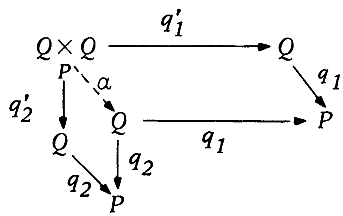
(1) $\Longrightarrow$ (2). First we fix the notations; $q: Q \rightarrowtail P \times P$ denotes the

relation, $\delta : P \rightarrowtail Q$ and $\alpha : Q \times_P Q \to Q$ express the reflexivity and the associativity of the relation.

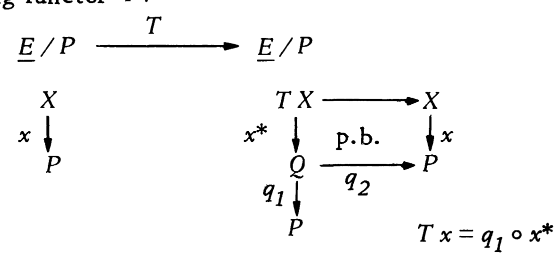






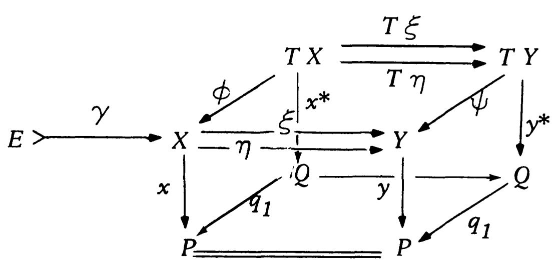


We consider the following functor T:

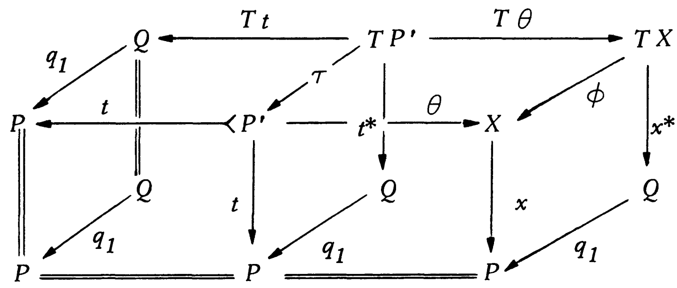


which can be made into a triple (T, ε, μ): $\underline{E}^P$ is the topos of T-algebras (cf. [3] and [6]).

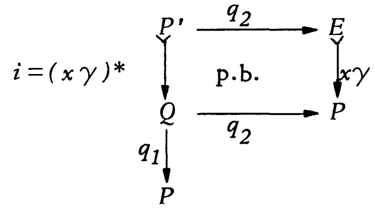
We choose two T-algebras (x, ϕ) and (y, ψ) and two morphisms $\xi, \eta : (x, \phi) \to (y, \psi)$ of T-algebras which are supposed to be different. We denote by $e : E \rightarrowtail 1$ a subobject of 1 and by $\gamma : E \to A$ a morphism such that $\xi \circ \gamma \neq \eta \circ \gamma$:



Recall that the terminal object of $\underline{E}^P$ is the algebra (id_P, q_1). We have to find a subalgebra (t, τ) of (id_P, q_1) and a morphism of algebras $\theta:(t,\tau)\to(x,\phi)$ such that $\xi\circ\theta\neq\eta\circ\theta$.



Recall that any morphism with domain E is monic. (t,τ) is defined as beeing the free T-algebra on $x\gamma$:



$t = q_1 \circ i$

$\tau = \mu_{x\gamma} : T^2(x\gamma) \to T(x\gamma)$

$P' = T(x\gamma)$.

t is monic: indeed if $\alpha, \beta : X \to P'$ are such that $t\alpha = t\beta$, then

$p_1 \circ q \circ i \circ \alpha = p_1 \circ q \circ i \circ \beta$ because $t\alpha = t\beta$,

$p_2 \circ q \circ i \circ \alpha = p_2 \circ q \circ i \circ \beta$ because any two morphisms with target E are equal;

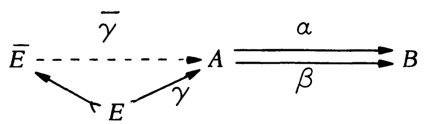
thus $q\circ i\circ\alpha = q\circ i\circ\beta$ and $\alpha=\beta$ because q and i are monic. $\gamma : x\gamma \to x$ is a morphism of $\underline{E}/P$ and thus

$$T\gamma : (t,\tau) = (T(x\gamma), \mu_{x\gamma}) \to (Tx, \mu_x)$$

is a morphism of T-algebras. We define θ to be the composite $\phi\circ T\gamma$; because $\phi : (Tx, \mu_x) \to (x, \phi)$ is a morphism of T-algebras,

$$\theta : (t, \tau) \longrightarrow (x, \phi)$$

is also a morphism of T-algebras and thus γ can be extended to E because A is a sheaf:



Our assumption implies that $\alpha\bar{\gamma} = \beta\bar{\gamma}$ and thus $\alpha\gamma = \beta\gamma$. Because this is true for any E and any γ and because the subobjects of 1 form a set of generators in $\underline{E}$, $\alpha = \beta$. So $Sh_{\underline{E}}(j)$ has the required property. ∎

COROLLARY. *If T is any topological space, the topos of sheaves over T satisfies the weak axiom of choice and thus its Ω-object is a cogenerator.*

It is a consequence of proposition 4 and corollary of proposition 3. ∎

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