

CAHIERS DE TOPOLOGIE ET GÉOMÉTRIE DIFFÉRENTIELLE CATÉGORIQUES

MANFRED B. WISCHNEWSKY

A lifting theorem for right adjoints

Cahiers de topologie et géométrie différentielle catégoriques, tome 19, n° 2 (1978), p. 155-168

http://www.numdam.org/item?id=CTGDC_1978__19_2_155_0

© Andrée C. Ehresmann et les auteurs, 1978, tous droits réservés.

L'accès aux archives de la revue « Cahiers de topologie et géométrie différentielle catégoriques » implique l'accord avec les conditions générales d'utilisation (<http://www.numdam.org/conditions>). Toute utilisation commerciale ou impression systématique est constitutive d'une infraction pénale. Toute copie ou impression de ce fichier doit contenir la présente mention de copyright.

NUMDAM

Article numérisé dans le cadre du programme
Numérisation de documents anciens mathématiques

<http://www.numdam.org/>

A LIFTING THEOREM FOR RIGHT ADJOINTS

by Manfred B. WISCHNEWSKY *

1. INTRODUCTION.

One of the most important results in categorical topology, resp. topological algebra is Wyler's theorem on taut lifts of adjoint functors along top functors [16]. More explicitly, given a commutative square of functors

$$\begin{array}{ccc}
 \underline{C} & \xrightarrow{Q} & \underline{C}' \\
 \downarrow V & & \downarrow V' \\
 \underline{B} & \xrightarrow{S} & \underline{B}' \\
 & \xleftarrow{R} &
 \end{array}$$

where V and V' are topological (= initially complete) functors and Q «preserves initial cones», then a left adjoint functor R of S can be lifted to a left adjoint of Q . Since the dual of a topological functor is again a topological functor, one can immediately dualize the above theorem and obtains a lifting theorem for right adjoints along topological functors. Recently W. Tholen [12] generalized Wyler's taut lifting theorem in the general categorical context of so-called « M -functors» or equivalently «topologically algebraic functors» (Y.H. Hong [6]). M -functors, or more exactly M -initial functors, generalize (E, M) -topological functors in the sense of Herrlich [4] as well as monadic functors. Although this notion is very general it still allows for instance to prove all propositions and theorems in H. Herrlich's paper on (E, M) -topological functors for this more general notion. Unfortunately the dual of a M -initial functor is no longer a M -initial functor. Hence we don't get in this way a corresponding generalization of Wyler's cotaut lifting theorem. The categorical background of this paper is the theory of semi-final

* I am indebted to W. Tholen for some useful discussions on a preliminary version of this paper.

functors, a generalization of M -initial functors. By this notion we obtain a lifting theorem for right adjoints along M -initial functors which can be applied in topology as well as in algebra, and which generalizes at the same time Wyler's cotaut lifting theorem. As a byproduct of our investigation we will show that every « M -initial category» is a reflective sub-category of a topological category. This result implies for instance that all results in H. Herrlich's paper for (E, M) -topological categories are valid for M -initial categories and trivializes almost all results in Y.H.Hong's fundamental paper. Finally by specializing and dualizing our general lifting theorem for right adjoints, we obtain again Tholen's theorem on lifting left adjoints along M -initial functors.

2. M -INITIAL AND SEMI-FINAL FUNCTORS.

M -initial functors as a common generalization of topological and monadic functors were introduced by Y.H.Hong in her thesis [6] and recently independently by W. Tholen [12].

Let $V: \underline{C} \rightarrow \underline{B}$ be a functor. Denote by $Cone(\underline{C})$ the «class» of all cones $\phi: \Delta C \rightarrow D$ in \underline{C} where $D: \underline{D} \rightarrow \underline{C}$ is a diagram (\underline{D} may be void or large). The class $Mor(V)$ of all V -morphisms is the object class of the comma category (\underline{B}, V) .

$$(b, C) \in Mor(V) \quad \text{and} \quad (\phi, C') \in Cone(\underline{C})$$

are called *orthogonal* $((b, C) \perp (\phi, C'))$ if every commutative diagram

$$\begin{array}{ccc}
 B & \xrightarrow{\Delta b} & VC \\
 \Delta b' \downarrow & \nearrow \Delta V\omega & \downarrow V\alpha \\
 \Delta VC' & \xrightarrow{V\phi} & VD
 \end{array}$$

can be rendered commutative in the following sense by a unique \underline{C} -morphism $\omega: C \rightarrow C'$:

$$V\omega \cdot b = b' \quad \text{and} \quad \phi \cdot \Delta\omega = \alpha.$$

Let $M \subset Cone(\underline{C})$ and $E \subset Mor(V)$ be subclasses. Define

$$M^\perp := \{ (b, C) \in \text{Mor}(V) \mid (b, C)^\perp(\phi, C') \text{ for all } (\phi, C') \in M \},$$

$$E_\perp := \{ (\phi, C') \in \text{Cone}(\underline{C}) \mid (b, C)^\perp(\phi, C') \text{ for all } (b, C) \in E \}.$$

Let $V: \underline{C} \rightarrow \underline{B}$ be a functor and $D: \underline{D} \rightarrow \underline{C}$ be a diagram. Then a cone $\phi: \Delta C \rightarrow D$ from the vertex C to the base D is called a V -initial cone if, for any cone $\xi: \Delta C' \rightarrow D$ from the vertex C' to \check{D} and any \underline{B} -morphism

$$b: VC' \rightarrow VC \text{ with } V\phi \cdot \Delta b = V\xi,$$

there exists exactly one \underline{C} -morphism $b^*: C' \rightarrow C$ with

$$\xi = \phi \cdot \Delta b^* \text{ and } Vb^* = b.$$

DEFINITION 2.1 (Y.H. Hong [6], W. Tholen [12]). Let $V: \underline{C} \rightarrow \underline{B}$ be a functor and let $E \subset \text{Mor}(V)$ and $M \subset \text{Cone}(\underline{C})$ be subclasses. Let M consist of V -initial cones. V is a M -initial functor if:

1° Every cone $\psi: \Delta B \rightarrow VD$, $D: \underline{D} \rightarrow \underline{C}$, has a factorization:

$$\psi = V\phi \cdot \Delta b \text{ with } (b, C) \in E \text{ and } (\phi, C) \in M;$$

2° $M^\perp = E$ and $E_\perp = M$.

EXAMPLES 2.2.

1° Every (E, M) -topological functor is a M -initial functor (H. Herrlich [4]).

2° Every regular functor $V: \underline{C} \rightarrow \underline{B}$ over a regular category \underline{B} is a M -initial functor with respect to the class of all mono-cones of \underline{C} .

3° Every full reflective subcategory $F: \underline{C} \rightarrow \underline{B}$ is a M -initial functor with respect to all cones in \underline{C} .

4° Every monadic functor $V: \underline{C} \rightarrow \underline{B}$ is a M -initial functor with respect to a class $M \subset \text{Cone}(\underline{C})$ provided \underline{C} has a M -factorization in the usual sense of cone factorizations in a category.

DEFINITION 2.3 (cf. [5]). A functor $V: \underline{C} \rightarrow \underline{B}$ is called semifinally complete (or just a semifinal functor) if, for every cone $\psi: VD \rightarrow \Delta B$ in \underline{B} , $D: \underline{D} \rightarrow \underline{C}$, there exist a cone $\phi: D \rightarrow \Delta C$ in \underline{C} and a \underline{B} -morphism $b: B \rightarrow VC$ such that, for every cone $\phi': D \rightarrow \Delta C'$ and every \underline{B} -morphism

$$b': B \rightarrow VC' \text{ with } V\phi' = \Delta b' \cdot \psi,$$

there exists exactly one \underline{C} -morphism $c: C \rightarrow C'$ such that the following diagram commutes:

$$\begin{array}{ccc}
 & & \Delta V C' \\
 & \nearrow V\phi' & \nearrow \Delta b' \\
 V D & \xrightarrow{\psi} & \Delta B = \\
 & \searrow V\phi & \searrow \Delta b \\
 & & \Delta V C
 \end{array}
 \qquad
 \begin{array}{c}
 C' \\
 \uparrow c \\
 C
 \end{array}$$

Every triple (ψ, b, ϕ) with $V\phi = \Delta b \cdot \psi$ is called a *semilift* of ψ . A *semilift* of ψ with the above universal property is called a *V-semifinal lift* of ψ .

Hence a functor $V: \underline{C} \rightarrow \underline{B}$ is semifinally complete iff every cone $\psi: V D \rightarrow \Delta B$ has a *V-semifinal lift*¹⁾.

Denote by D/\underline{C} the comma-category (D, Δ) of all cones from D to \underline{C} . The functor $V: \underline{C} \rightarrow \underline{B}$ induces a functor $D/V: D/\underline{C} \rightarrow V D/\underline{B}$ by the assignment:

$$\psi: D \rightarrow \Delta C \mapsto V\psi: V D \rightarrow \Delta V C.$$

Hence we obtain immediately that $V: \underline{C} \rightarrow \underline{B}$ is a semifinal functor if and only if, for each diagram $D: \underline{D} \rightarrow \underline{C}$, the induced functor D/V has a left adjoint.

$V: \underline{C} \rightarrow \underline{B}$ is called a *D-semifinal functor* if the functor D/V has a left adjoint (if \underline{D} is not small one has to change the universe).

By taking the void diagram $D: \underline{0} \rightarrow \underline{C}$ we obtain the well-known fact that every semifinal functor has a left adjoint [5]. By taking a colimit cone for $V D$ in \underline{B} (if it exists) we obtain a colimit cone of D in \underline{C} as the image of $V D \rightarrow \text{colim } V D$ by applying the left adjoint of D/V . Hence if $V: \underline{C} \rightarrow \underline{B}$ is a semifinal functor and if \underline{B} is cocomplete, \underline{C} is also cocomplete. Conversely, if \underline{B} and \underline{C} are cocomplete and V has a left adjoint then V is a *D-semifinal functor* for all small diagrams $D: \underline{D} \rightarrow \underline{C}$ (Hoffmann [5]).

¹⁾ In contrast to [5] the index categories may be large. This generalization is necessary for our main theorems.

2.4. The construction of an initially complete extension given in the following is due to Herrlich [4], Theorem 9-1, and was used by R. E. Hoffmann and others.

Let $V: \underline{C} \rightarrow \underline{B}$ be a M -initial functor ¹⁾ with respect to (E, M) . We will construct an extension of V :

$$(\underline{C} \xrightarrow{I} \underline{A} \xrightarrow{U} \underline{B}) = (\underline{C} \xrightarrow{V} \underline{B})$$

such that

- (i) $I: \underline{C} \rightarrow \underline{A}$ is a reflective subcategory,
- (ii) $U: \underline{A} \rightarrow \underline{B}$ is a topological (= initially complete) functor.

The construction is given by the following data :

- (1) Objects of \underline{A} are all elements (f, C) in the class E ,
- (2) A morphism $(b, c): (f, C) \rightarrow (f', C')$ is a pair consisting of a \underline{B} -morphism $b: B \rightarrow B'$ and a \underline{C} -morphism $c: C \rightarrow C'$ such that $f'.b = Vc.f$.
- (3) The forgetful functor $U: \underline{A} \rightarrow \underline{B}$ is defined by the assignment :

$$(f: B \rightarrow VC; C) \mapsto B, (b, c) \mapsto b.$$

- (4) The embedding $I: \underline{C} \rightarrow \underline{A}$ is given by the assignments :

$$C \mapsto (id_{VC}, C), (c: C \rightarrow C') \mapsto (Vc, C).$$

Recall that (id_{VC}, C) is in E since M consists of V -initial cones.

- (5) $I: \underline{C} \rightarrow \underline{A}$ is reflective. The reflector $R: \underline{A} \rightarrow \underline{C}$ is given by $(f, C) \mapsto C$ and the unit of the adjunction by

$$\begin{array}{ccc}
 B & \xrightarrow{f} & VC \\
 \downarrow f & & \downarrow id \\
 VC & \xrightarrow{Vid_C} & VC
 \end{array}
 \quad
 \begin{array}{c}
 = \\
 =
 \end{array}$$

THEOREM 2.5. *Every M -initial category is a reflective subcategory of a topological category. (More exactly every M -initial functor has a reflective extension through a topological functor.)*

¹⁾ A M -initial functor is faithful (Y. A. Hong [6], W. Tholen [12]).

PROOF. Let

$$(f_i: B_i \rightarrow VC_i \mid i \in I)$$

be a class of \underline{A} -objects and let

$$g_i: B \rightarrow U(f_i, C_i) = B_i, \quad i \in I$$

be a class of \underline{B} -morphisms $g_i: B \rightarrow B_i$. Take the M -initial factorization of the cone

$$\begin{array}{ccccc} B & \xrightarrow{g_i} & B_i & \xrightarrow{f_i} & VC_i \\ \downarrow b & & & \nearrow V\psi_i & \\ VC & & & & \end{array}$$

where $(\psi_i: C \rightarrow C_i \mid i \in I)$ is an initial cone in M and b is in E . We will show that $(g_i, \psi_i): (b, C) \rightarrow (f_i, C_i)$ is a U -initial cone generated by $g_i, i \in I$. Let

$$(h_i, \phi_i): (b', C') \rightarrow (f_i, C_i), \quad i \in I,$$

be a class of \underline{A} -morphisms and $b'': B' \rightarrow B$ be a \underline{B} -morphism with $g_i \cdot b'' = h_i$. Consider the commutative square

$$\begin{array}{ccc} B' & \xrightarrow{b'} & VC' \\ \downarrow b'' & & \downarrow V\phi_i \\ B & & VC_i \\ \downarrow b & \nearrow Vc & \downarrow V\psi_i \\ VC & & VC_i \end{array}$$

Then there exists a unique \underline{C} -morphism

$$c: C' \rightarrow C \quad \text{with} \quad Vc \cdot b' = b \cdot b''.$$

Then $(b'', c): (b', C') \rightarrow (b, C)$ is the desired \underline{A} -morphism. The rest is clear.

This theorem has a whole host of applications.

COROLLARY 2.6 (Herlich [4]). Every (E, M) -topological functor $V: \underline{C} \rightarrow \underline{B}$

has an «absolute» topological reflective extension.

COROLLARY 2.7. Every regular functor $V: \underline{C} \rightarrow \underline{B}$ over a regular category \underline{B} has a topological reflective extension.

COROLLARY 2.8. Every Eilenberg-Moore category (over Sets) is a reflective subcategory of a topological category.

COROLLARY 2.9 (cf. Hong [6], W. Tholen [12]). Let $V: \underline{C} \rightarrow \underline{B}$ be a M -initial functor. Then:

- (1) If \underline{B} is \underline{D} -complete then \underline{C} is \underline{D} -complete and V preserves limits.
- (2) If \underline{B} is \underline{D} -cocomplete then \underline{C} is \underline{D} -cocomplete.
- (3) $V: \underline{C} \rightarrow \underline{B}$ has a left adjoint.

COROLLARY 2.10. Every M -initial functor $V: \underline{C} \rightarrow \underline{B}$ is semifinal.

PROOF. Obviously every topological functor and every reflective embedding are semifinal functors. Since the composition of semifinal functors is semifinal and since every M -initial functor has a reflective factorization

$$\underline{C} \xrightarrow{I} \underline{A} \xrightarrow{U} \underline{B}$$

through a topological functor $U: \underline{A} \rightarrow \underline{B}$, the functor V is semifinal as composition of two semifinal functors.

Let $\psi: VD \rightarrow \Delta B$ be a cocone over \underline{B} . The semifinal lift is constructed as follows: first take the final (= cointial) cone over \underline{A} generated by ψ . This yields a cone $\psi^*: D \rightarrow \Delta A$. Then compose ψ^* with the unit $A \rightarrow RA$ where R denotes the reflector $\underline{A} \rightarrow \underline{C}$.

COROLLARY 2.11. Every M -initial functor lifts monoidal structures, i. e., if the base category \underline{B} is monoidal then \underline{C} is also monoidal and V is a monoidal functor with respect to these monoidal structures.

(Details appear somewhere else.)

COROLLARY 2.12 (Characterization of M -initial functors, Hong [7]). Let M be a class of initial cones in a category \underline{C} . A functor $V: \underline{C} \rightarrow \underline{B}$ is a M -initial functor iff \underline{C} is a M -category (= has an (E, M) -cone factorization) and V has a left adjoint.

COROLLARY 2.13 (Hong [6]). Let $I: \underline{U} \rightarrow \underline{A}$ be a full subcategory. \underline{U} is

reflective in \underline{A} if and only if I is an M -initial functor.

3. A LIFTING THEOREM FOR RIGHT ADJOINTS.

The lifting theorem proved in the following is extremely general. The usefulness as well as the importance of this theorem follows from the corollaries derived from. Besides a lifting theorem for right adjoints along M -initial functors it contains the lifting theorem proved by O. Wyler [16], W. Tholen [12] and H. Wolff [15].

First we need some technical notions.

Let $\underline{U} \subset \underline{B}$ be a subcategory. Denote by $\underline{B} \downarrow \underline{U}$ the comma category of all \underline{B} -morphisms

$$f: B \rightarrow U, \quad B \in \underline{B}, \quad U \in \underline{U}.$$

Let $M \subset Ob(\underline{B} \downarrow \underline{U})$ be a class of $(\underline{B} \downarrow \underline{U})$ -objects and let E be a class of V -semilifts of a class of cocones $\psi: VD \rightarrow \Delta B$, $D: \underline{D} \rightarrow \underline{C}$ arbitrary. We say that V is (E, M) -cosemi-factorizable relative to \underline{U} if every cocone ψ with $\psi: VD \rightarrow \Delta U$ has a factorization

$$\psi = \Delta m \cdot \psi' \quad \text{with } \psi' \in E \quad \text{and } m \in M.$$

$$(*) \quad \begin{array}{ccc} VD & \xrightarrow{\psi} & \Delta U \\ V\phi \downarrow & \searrow \psi' & \uparrow \Delta m \\ \Delta VC & \xleftarrow{\Delta b} & \Delta B \end{array}$$

where (ψ', b, ϕ) is a semilift of ψ' . ¹⁾

If in particular every triple $(\psi', b, \phi) \in E$ is semifinal, we say that V is semifinal factorizable relative to \underline{U} .

The dual notions are (E, M) -semi-factorizable relative to \underline{U} and semi-initial factorizable relative to \underline{U} .

Given a commutative square of functors (***) as follows, we say that (Q, R) transforms E -semilifts into V' -semifinal lifts if for every V -semi-

1) The factorization is assumed to be orthogonal (two diagonals!). In particular (ψ', b, ϕ) and m are unique up to canonical isomorphisms.

lift $(\psi', b, \phi) \in E$ the triple $(R\psi', Rb, Q\phi)$ is a V' -semifinal lift.

$$(**) \quad \begin{array}{ccc} C & \xrightarrow{Q} & C' \\ V \downarrow & & \downarrow V' \\ B & \xrightarrow{R} & B' \\ & \xleftarrow{S} & \end{array}$$

THEOREM 3.1. Let $(**)$ be a commutative square of functors. Assume

- a) R has a right adjoint S .
- b) V is (E, M) -cosemi-factorizable with respect to $Im S$.
- c) (Q, R) transforms E -semilifts into V' -semifinal lifts.

Then Q has a right adjoint.

PROOF. The comma category $Q \downarrow C', C' \in \underline{C}'$, defines a cocone

$$(f: QC \rightarrow C') \text{ with vertex } C'.$$

Since the functor R has a right adjoint S the cocone

$$(V'f: V'QC = RVC \rightarrow V'C')$$

induces a cocone $(f^*: VC \rightarrow SV'C')$. Since V is (E, M) -cosemi-factorizable with respect to $Im S$ and the vertex of $(f^*: VC \rightarrow SV'C')$ is in $Im S$ we obtain a factorization

$$\begin{array}{ccc} VC & \xrightarrow{f^*} & SV'C' \\ V\phi \downarrow & \searrow \psi & \uparrow m \\ VC^* & \xleftarrow{b} & B \end{array}$$

with $m \in M$ and $(\psi, b, \phi) \in E$ (note that $\psi: VC \rightarrow B$ and $\phi: C \rightarrow C^*$ are components of functorial morphisms). Since (Q, R) transforms E -semilifts into V' -semifinal lifts we obtain the following commutative diagram.

$$\begin{array}{ccccc} & & & & V'C' \\ & & & & \uparrow \text{Can} \\ & & & & V'c \\ & & & & \uparrow \\ V'QC = RVC & \xrightarrow{R\psi} & RB & \xrightarrow{Rb} & RVC^* = V'QC^* \\ & \nearrow V'f & \nearrow Rm & \nearrow R\psi & \\ & & & & \end{array}$$

with unique $c: QC^* \rightarrow C'$, i. e. there exists a unique morphism

$$c: QC^* \rightarrow C' \quad \text{with} \quad f = c \cdot Q\phi \quad \text{and} \quad Rb \cdot V'c = \text{Can} \cdot Rm.$$

Hence Q has a right adjoint

$$P: \underline{C}' \rightarrow \underline{C}: C' \mapsto C^* = PC'.$$

By specializing and/or dualizing the data in (*) and (**) we obtain a whole host of useful corollaries.

Let $V: \underline{C} \rightarrow \underline{B}$ be a semifinal functor. Then V is obviously semifinal factorizable where in (*) m is an isomorphism. Hence we obtain the following

COROLLARY 3.2. *Let (**) be a commutative square of functors. Assume*

- a) V is semifinal;
- b) (Q, R) preserves semifinal lifts;
- c) R has a right adjoint.

Then Q has a right adjoint.

Since every M -initial functor is semifinal the above corollary is the desired lifting theorem for right adjoints along M -initial functors (it is only necessary that V is M -initial!). In particular since a topological functor V has final cones, i. e. the morphisms b in the semifinal lifts (ψ, b, ϕ) are isomorphisms, we obtain:

COROLLARY 3.3 (*Wyler's cotaut lifting theorem [16]*). *Let (**) be a commutative square of functors. Assume*

- a) V is a topological functor;
- b) (Q, R) preserves final cones ((Q, R) is final continuous);
- c) R has a right adjoint.

Then Q has a right adjoint.

V is called *final-factorizable relative to $\underline{U} \subset \underline{B}$* if in each factorization (*) b is an isomorphism and ϕ is a V -final cone. Then we get:

COROLLARY 3.4. *Let (**) be a commutative square of functors. Assume*

- a) V is final-factorizable;
- b) (Q, R) preserves E -final cones;

c) R has a right adjoint.

Then Q has a right adjoint.

By dualizing Corollary 3.4, we obtain the main result in [12] :

COROLLARY 3.5 (Tholen [12]). Let $(**)$ be a commutative square of functors. Assume

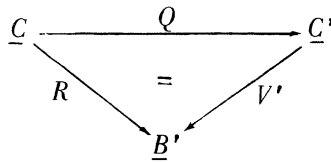
- a) V is a M -initial functor;
- b) (Q, R) preserves M -initial cones;
- c) R has a left adjoint.

Then Q has a left adjoint.

By specializing data in the commutative square of functors we obtain further corollaires. I want to state just two of them. (For a detailed discussion, see Tholen [12].)

Letting $V = Id$, we obtain :

COROLLARY 3.6 (Sandwich Theorem [12]). Let \underline{C} be a category with an (E, M) -cone factorization. Let



be a commutative triangle of functors. If R has a left adjoint and Q transforms M -cones into V' -initial cones then Q has a left adjoint.

By setting $R = Id$ we obtain :

COROLLARY 3.7. Let $V: \underline{C} \rightarrow \underline{B}$ be a M -initial functor and let $V': \underline{C}' \rightarrow \underline{B}$ be an arbitrary functor. Then every M -initial continuous functor $Q: \underline{C} \rightarrow \underline{C}'$ has a left adjoint.

REMARK 3.8. Let V be (E, M) -cosemi-factorizable. If either

- (i) all $m \in M$ or
- (ii) all b in semilifts (ψ', b, ϕ) , $(\psi', b, \phi) \in E$ arbitrary, are isomorphisms,

then the assumption that the factorization fulfills the orthogonal condition is superfluous (in the corresponding lifting theorems for right adjoints).

The dual assumption is valid for the corresponding lifting theorem for left adjoints.

Furthermore we obtain from Corollary 3.5 that the dual of an M -initial functor V is in general not M -initial, because otherwise V would have not only a left adjoint, but also a right adjoint (take $R = Id$ and $V' = Id$). But this is true only for special instances.

The preceding existence theorems for left and right adjoints have numerous applications in various fields. We will just state some of the most important ones appearing in topology as well as in algebra. The obvious deduction of the following propositions from the above lifting theorems is left to the reader.

COROLLARY 3.9. *Let \underline{C} be a strict monoidal M -initial category over a monoidal closed category \underline{B} . Let \underline{B} be cocomplete, well-powered. Then \underline{C} is monoidal closed if (and only if) the functors $- \otimes X, X \in \underline{C}$, preserve colimits. In particular if \underline{B} is cartesian closed then \underline{C} is cartesian closed if (and only if) $- \times X, X \in \underline{C}$, preserve colimits.*

COROLLARY 3.10 (A. and C. Ehresmann [1]). *Notation as in [1]. Let Σ be a cone-bearing category. The category $\Sigma\text{-Alg}(\text{Sets})$ of all Σ -algebras in Sets is cartesian closed if and only if the functor $- \times Y(u)$ commutes with colimits for each $u \in \Sigma$.*

COROLLARY 3.11 (Wischnewsky [13]). *Let (\underline{A}, Σ) be an esquisse in the sense of Ehresmann and \underline{B} be a locally presentable category in the sense of Gabriel-Ulmer. Let \underline{C} be a Top-category over \underline{B} . Then:*

- a) *The inclusion functor $\Sigma\text{-Alg}(\underline{A}, \underline{C}) \rightarrow [\underline{A}, \underline{C}]$ is reflective.*
- b) *The inclusion functor $\Sigma\text{-Coalg}(\underline{A}, \underline{C}) \rightarrow [\underline{A}, \underline{C}^{op}]^{op}$ is coreflective.*

In Herrlich [3] and Porst [9] there were given external characterizations for the MacNeille completion (MNC) of faithful functors, i. e. the smallest injective extension of a faithful functor $V: \underline{C} \rightarrow \underline{B}$. MNC's don't exist in general unless \underline{C} is small [3]. Applying our main results namely Theorem 2.5 and Theorem 3.1, we obtain:

COROLLARY 3.12. (1) *Every M -initial functor has a MNC.*

(2) Let $V: \underline{C} \rightarrow \underline{B}$ be a M -initial functor and $F: (\underline{C}, V) \rightarrow (\underline{A}, U)$ the MNC. Then $F: \underline{C} \rightarrow \underline{A}$ is reflective. (V has a reflective MNC.)

In particular every Eilenberg-Moore category over *Sets* is a reflective subcategory of its MNC. Since every MNC $U: \underline{A} \rightarrow \underline{B}$ is initially complete and since every initially complete category over *Sets* can be embedded into a cartesian closed topological (CCT) category we obtain:

COROLLARY 3.13. *Every algebraic category over Sets is a reflective subcategory of a cartesian closed topological category.*

REFERENCES.

1. BASTIANI, A., EHRESMANN, C., Categories of sketched structures, *Cahiers Topo. et Géo. Diff.* XIII-2 (1972), 105-214.
2. BRÜMMER, G., Topological functors and structure functors, *Lecture Notes in Math.* 540, Springer (1976), 109-136.
3. HERRLICH, H., Initial completions, *Math. Z.* 150 (1976), 101-110.
4. HERRLICH, H., Topological functors, *Gen. Top. Appl.* 4 (1974), 125-142.
5. HOFFMANN, R.-E., Semi-identifying lifts and a generalization of the duality theorem for topological functors, *Math. Nachr.* 74 (1976), 297-307.
6. HONG, Y. H., *Studies on categories of universal topological algebras*, Thesis, Mc Master University, 1974.
7. HONG, S. S., Categories in which every monosource is initial, *Kyngpook Math. J.* 15 (1975), 133-139.
8. KENNISON, I. F., Reflective functors in general topology and elsewhere, *Trans. A.M.S.* 118 (1965), 303-315.
9. PORST, H. E., Characterisation of MacNeille completions and topological functors, *Manuscripta Math.* (to appear) .
10. NEL, L. D., Cartesian closed topological categories, *Lecture Notes in Math.* 540, Springer (1976), 439-451.
11. THOLEN, W., *Relative Bildzerlegungen und algebraische Kategorien*, Dissertation, Universität Münster, 1974.
12. THOLEN, W., *On Wyler's taut lifting theorem* (to appear) .
13. WISCHNEWSKY, M. B., Aspects of categorical algebra in initial structure categories, *Cahiers Topo. et Géo. Diff.* XV-4 (1974), 419-444.
14. WISCHNEWSKY, M. B., On monoidal closed topological categories, I, *Lecture*

Notes in Math. 540, Springer (1976), 676-686.

15. WOLFF, H., *Topological functors and right adjoints*, 1975 (to appear).

16. WYLER, O., On the categories of general topology and topological algebra, *Arch. d. Math.* 22/1 (1971), 7-15.

NOTE ADDED IN PROOF.

In the meantime W. Tholen and the author could generalize Theorem 2.5 and Corollary 3.12 of this paper:

THEOREM (*Tholen-Wischnewsky; Oberwolfach July 1977*). For a functor $V: \underline{C} \rightarrow \underline{B}$ the following assertions are equivalent:

- (i) V is semifinal.
- (ii) V is a full reflective restriction of a topological functor.
- (iii) V has a reflective MNC.

In this context W. Tholen and the author are indebted to R. E. Hoffmann. Without his disbelief and intensive search for a counterexample of the equivalence (i) \longleftrightarrow (ii) we would not have tried to prove this theorem.

Moreover W. Tholen proved that the (E, M) -cosemi-factorizable functors are coreflexive restrictions of semifinal functors. Motivated by Tholen's result the author could just recently give a complete description of arbitrary full coreflexive restrictions of semifinal functors.

Fachbereich Mathematik
 Universität Bremen
 Achterstrass
 D-2800 BREMEN 33. R.F.A.