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CORRECTION TO "TOPOLOGICAL BALLS"

By Michael BARR and Heinrich KLEISLI

Résumé

Cette note corrige une erreur dans notre article [Barr & Kleisli, 1999] dans ces Cahiers.

Abstract

This note is to correct an error in [Barr & Kleisli, 1999] in these Cahiers.

We have discovered an error in the paper [Barr & Kleisli, 1999]. Proposition 3.12 should have read as follows, since that is what was actually proved:

Every seminorm on the ball τB *has the form* $\sup_{\varphi \in C} |\varphi b|$ *for a weakly compact subball* $C \subseteq B'$.

It follows that the proofs of Corollary 3.14 and Corollary 3.15 are incorrect; both should be omitted. Corollary 3.15 was used to prove the first assertion of Proposition 5.3:

If B *is a* ζ*-ball, then the associated Mackey ball is also a* ζ*-ball.*

However that assertion can be seen to be true without using Corollary 3.15 as follows.

Proof. Let B be a ζ-ball, C_0 a dense subball of a compact ball C and $\rho_0 : C_0 \longrightarrow \tau B$ a continuous morphism. Since B is a ζ-ball, there is a continuous morphism $\rho : C \longrightarrow B$ for which the diagram

$$\begin{array}{ccc} C_0 & \longrightarrow & C \\ {\scriptstyle \rho_0}\downarrow & & \downarrow{\scriptstyle \rho} \\ \tau B & \longrightarrow & B \end{array}$$

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commutes. We have to show that f considered as a morphism $C \longrightarrow \tau B$ is continuous. By hypothesis, for every weakly compact subball $K \subseteq (\tau B)'$ there is a continuous seminorm φ on C_0 and a positive constant c such that $\sup_{y\in K} |y(\rho_0 x)| \leq c\varphi x$ for all $x \in C_0$. Let ψ be the unique continuous seminorm on C that extends the seminorm φ on C_0. Since C_0 is dense in C, for every $x \in C$ there is a Cauchy net (x_α) in C_0 that converges to x. Hence $\psi x = \lim \varphi x_\alpha$ and, since f is weakly continuous, we have $|y(fx)| = \lim |y(\rho_0 x_\alpha)|$ for all $y \in B'$. In other words, for every $\epsilon > 0$, there are indices α_0 and $\alpha(y)$ such that $|\varphi x_\alpha - \psi x| < \epsilon$ for all $\alpha > \alpha_0$ and $|y(fx)| - |y(f_0 x_\alpha)| < \epsilon$ for all $\alpha > \alpha(y)$. Hence

$$|y(fx)| < |y(f_0 x_\alpha)| + \epsilon \leq c\varphi(x_\alpha) + \epsilon \leq c\psi x + 2\epsilon$$

for all $\alpha \geq \max(\alpha_0, \alpha(y))$. Therefore, $|y(fx)| < c\psi x$ for all all $y \in K$ and $x \in C$, so that $\sup_{y\in K} |y(fx)| \leq c\psi x$. This means that $f : C \longrightarrow \tau B$ is continuous. □

We observe that the results of Section 5 concerning the $*$-autonomous category $\mathcal{R}$ are not affected by the omission of Corollary 3.15. On the other hand, the example in Section 6 of a Mackey ball that is not ζ-complete is lost. Hence the question of the existence of such a ball is still open.

Reference

M. Barr and Heinrich Kleisli (1999), Topological balls. Cahiers de Topologie et Géométrie Différentielle Catégorique, **40**, 3–20.