

GROUPE DE TRAVAIL D'ANALYSE ULTRAMÉTRIQUE

ELKEDAGMAR HEINRICH

On the class group of affinoid spaces

Groupe de travail d'analyse ultramétrique, tome 9, n° 3 (1981-1982), exp. n° J10, p. J1-J2

http://www.numdam.org/item?id=GAU_1981-1982__9_3_A11_0

© Groupe de travail d'analyse ultramétrique
(Secrétariat mathématique, Paris), 1981-1982, tous droits réservés.

L'accès aux archives de la collection « Groupe de travail d'analyse ultramétrique » implique l'accord avec les conditions générales d'utilisation (<http://www.numdam.org/conditions>). Toute utilisation commerciale ou impression systématique est constitutive d'une infraction pénale. Toute copie ou impression de ce fichier doit contenir la présente mention de copyright.

NUMDAM

Article numérisé dans le cadre du programme
Numérisation de documents anciens mathématiques
<http://www.numdam.org/>

ON THE CLASS GROUP OF AFFINOID SPACES

by Elkedagmar HEINRICH (*)

[Universität Bochum]

The field k is supposed to be complete with respect to a non-archimedean valuation and to be stable (i. e. $|k^*| = \sqrt{|k^*|}$), and for every finite field extension ℓ of k one has $[\ell : k] = [\bar{\ell} : \bar{k}]$. By \dot{k} and \bar{k} , we denote the valuation ring and the residue field of k .

The affinoid k -algebra A is supposed to be reduced and to be provided with its spectral norm $\|\cdot\|$. Furthermore we assume that $\|\cdot\|$ takes its value in $|k^*|$. By \bar{A} , we denote the affine \bar{k} -algebra $\dot{A} \otimes \bar{k}$, where $\dot{A} := \{x \in A; \|x\| \leq 1\}$. The group of isomorphism classes of projective, rank 1, A -modules is called the class group $Cl(A)$ of A .

THEOREM.

(i) There is a natural injective map

$$\alpha : Cl(\bar{A}) \hookrightarrow Cl(A) .$$

(ii) If \bar{A} is regular, α is also surjective.

Definition of α .

(a) $\beta : Cl(\dot{A}) \rightarrow Cl(\bar{A})$

$$\beta([M]) := [M \otimes_A \bar{A}]$$

It is easy to show that β is a bijective homomorphism.

(b) $\gamma : Cl(\dot{A}) \rightarrow Cl(A)$

$$\gamma([M]) := [M \otimes_A A] .$$

The homomorphism γ is injective, and we define

$$\alpha := \gamma \circ \beta^{-1} .$$

Several examples show that in general α is not surjective.

(*) Elkedagmar HEINRICH, Institut für Mathematik, Universität Bochum, Postfach 102148, D-4630 BOCHUM 1 (Allemagne fédérale).

These examples are constructed geometrically : every 1-dimensional, normal, connected affinoid space $\gamma = \text{Sp}(A)$ is an affinoid subspace of a non-singular complete curve. We consider special affinoid subspaces of Tak-curves, of an elliptic curve with good reduction, of a Mumford-curve of genus 2 with stable reduction



We also consider the product of two of these subspaces with the A -dimensional unit disk.

A detailed paper "Über die Picardgruppen affinoider Algebren", by M. van der PUT and E. HEINRICH is to appear.
