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p-ADIC TEICHMÜLLER SPACE FOR GENUS 2

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Over an algebraically closed complete nonarchimedean field k, like over the complex numbers, one has, for every integer $g \geq 2$, an analytic manifold $\mathcal{T}_g$ and a group Ψ_g of analytic automorphisms of $\mathcal{T}_g$ acting discontinuously on $\mathcal{T}_g$ such that the quotient space is isomorphic to the space $\mathfrak{M}_g$ of Mumford curves of genus g. $\mathcal{T}_g$ is called the p-adic Teichmüller space, Ψ the p-adic Teichmüller modular group (see [2]).

In this paper, we shall mainly consider the case $g = 2$. Here we have the result that $\mathcal{T}_2$ is a Stein domain. The proof relies on an effective algorithm to decide whether or not a given pair of hyperbolic transformations generates a Schottky group. It seems not very likely that a similar algorithm can be found for higher genus, although $\mathcal{T}_g$ is probably a Stein domain for arbitrary g.

We begin with the study of treelike metric spaces, a generalization of the trees used in graph theory which possibly has some interest in itself.

In the second part of the paper, we construct for any ultrametric field a treelike metric space which for discrete fields coincides with the Bruhat-Tits-tree. Investigation of the action of hyperbolic linear transformations on this space is the main tool in proving that $\mathcal{T}_2$ is a Stein domain.

1. Treelike metric spaces.

Let (X, d) be a metric space. For $x, y \in X$ define the section $S(x, y)$ to be

$$S(x, y) := \{z \in X ;\ d(x, y) = d(x, z) + d(y, z)\} .$$

Definition. - A metric space (X, d) is called treelike if, for any $x, y, z \in X$,

(T1) $S(x, y) \cap S(x, z) \cap S(y, z) \neq \emptyset$.

(T2) If $z \in S(x, y)$, then $S(x, z) \cup S(z, y) = S(x, y)$.

This definition is justified by the following property :

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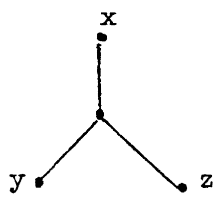
Let G be a connected graph without loops and multiple edges, G_0 the set of vertices of G, and d the metric on G_0 defined by the minimal number of edges between two vertices. Then we have :

PROPOSITION 1. – (G_0 , d) <u>is treelike if, and only if,</u> G <u>is a tree.</u>

<u>Proof.</u> – If G is a tree, for any $x , y \in G_0$, the section $S(x , y)$ consists of the unique simple path in G joining x and y, whence (T2). The first property results from the fact that the subtree of G spanned by x, y and z is isomorphic to

or



Conversely suppose (G_0 , d) is treelike, and assume there is a circle C of minimal length $n \geq 3$ in G.

If $n = 2m$ is even, let $x , y \in C$, such that $d(x , y) = m$ (this is possible due to the minimality of C). Since $m \geq 2$ there are $z_1 \neq z_2 \in C$, such that $d(z_1 , x) = d(z_2 , x)$ and $z_1 \neq y \neq z_2$. Obviously, $z_2 \notin S(x , z_1) \cup S(z_1 , y)$.

If $n = 2m + 1$ is odd, choose $x , y \in C$ with $d(x , y) = 1$, and let $z \in C$ be the unique vertex such that $d(x , z) = d(y , z) = m$. Then $S(x,y) \cap S(x,z) \cap S(y,z) = \emptyset$ and the proposition is proved. A trivial example for a non discrete treelike metric space are the real numbers with the usual metric ; further, any subset of a treelike metric space is itself treelike.

Next we list some formal properties of the sections in a treelike metric space :

LEMMA 1. – <u>Let</u> (X , d) <u>be a treelike metric space and</u> $x , y , z \in X$. <u>Then</u>

(i) <u>if</u> $z \in S(x , y)$, <u>then</u>

$$S(x , y) = \{v \in S(x , y) \; ; \; d(x , y) \leq d(x , z)\}$$

<u>and</u>

$$S(x , z) \cap S(z , y) = \{z\} ,$$

(ii) <u>there is</u> $u \in X$ <u>such that</u>

$$S(x , y) \cap S(x , z) \cap S(y , z) = \{u\}$$

<u>and</u>

$$S(x , y) \cap S(x , z) = S(x , u) .$$

<u>Proof.</u>

(i) is a straight forward application of (T2).

(ii) Let $S := S(x, y) \cap S(x, z)$. For $v \in S$, we have

$$d(y, z) \leqslant d(v, y) + d(v, z) = d(x, y) + d(x, z) - 2d(x, v) .$$

Because of (T1) and (i) of this lemma, there exists a unique $u \in S$ such that $d(x, u) = \sup_{v \in S} d(x, v)$. This u is the only element of S such that $d(y, z) = d(u, y) + d(u, z)$, i. e. the only element of $S \cap S(z, y)$. The last identity follows from (i).

A subset Y of a treelike metric space (X, d) is called <u>connected</u> if $y_1, y_2 \in Y$ implies $S(y_1, y_2) \subset Y$. Note that the intersection of connected subsets of X is again connected.

For a connected subset $Y \subset X$, let

$$\operatorname{diam}(Y) := \sup\{d(y_1, y_2) ; y_1, y_2 \in Y\} .$$

A <u>ray</u> $R \subset X$ is a connected subset with $\operatorname{diam}(R) = \infty$ such that there exists a sequence $(x_i)_{i \geqslant 0}$ in R with $x_i \in S(x_0, x_{i+1})$ for all i and $\bigcup_{i=0}^{\infty} S(x_0, x_i) = R$. Two rays R , R' are called equivalent if $\operatorname{diam}(R \cap R') = \infty$. (This is indeed an equivalence relation since $\operatorname{diam}(R \cap R') = \infty$ implies that $R \cap R'$ is again a ray.) An equivalence class of rays in X is called an <u>end</u> of X .

An <u>axis</u> in X is a connected subset $A \subset X$ such that there are two rays R_1 , R_2 in X with $R_1 \cup R_2 = A$ and $R_1 \cap R_2$ is a single point. Thus the axes in X are in 1-1 correspondence with those pairs (E_1, E_2) of ends of X for which $E_1 \neq E_2$.

The isometries of a treelike metric space can be characterized very much like the automorphisms of a tree (see [3]) because of the following observation :

If (X, d) is a treelike metric space and Φ an isometry of (X, d) there always exists a treelike extension space $(\tilde{X}, \tilde{d})$ of (X, d) (i. e. there is a distance-preserving injection $X \hookrightarrow \tilde{X}$) and a continuation $\tilde{\Phi}$ of Φ such that

$$\inf_{x \in \tilde{X}} \tilde{d}(x, \tilde{\Phi}(x)) = \inf_{x \in X} d(x, \Phi(x))$$

is attained in $\tilde{X}$.

LEMMA 2. - <u>If</u> Φ <u>is an isometry of</u> (X, d) <u>such that there is</u> $y \in X$ <u>with</u> $d(y, \Phi(y)) = \inf_{x \in X} d(x, \Phi(x))$ <u>then</u> Φ <u>has exactly one of the following properties</u> :

(a) Φ <u>has a fixed point in</u> X ,

(b) <u>there is</u> $y \in X$ <u>with</u> $\Phi(y) \neq y$, $S(y, \Phi(y)) = \{y, \Phi(y)\} = \Phi(S(y, \Phi(y)))$,

(c) <u>there is an axis</u> $A \subset X$ <u>on which</u> Φ <u>acts by nontrivial translation.</u>

In (b) and (c) the pair $(y, \Phi(y))$ (resp. A) are unique. Of course (b) is impossible if X is everywhere dense, i. e. for any $x \neq y \in X$ exists $z \in S(x, y)$, $x \neq z \neq y$.

Proof. - If Φ has neither property (a) nor (b), then it is easily checked that $\bigcup_{n=0}^{\infty} \Phi^n(S(y , \Phi(y)))$ and $\bigcup_{n=-1}^{-\infty} \Phi^n(S(y , \Phi(y)))$ are rays defining an axis on which Φ acts by translation by $d(y , \Phi(y)) > 0$.

2. Generalization of the Bruhat-Tits-tree.

For a field k with a non archimedean valuation $|.|$, let

$$\mathcal{K}(k) := \{B(a , r) ; \quad a \in k ; \quad r \in |k^*|\} ,$$

where $B(a , r) = \{z \in k ; \quad |z - a| \leq r\}$. For $B_i = B(a_i , r_i) \in \mathcal{K}(k)$, $i = 1 , 2$, define

$$d(B_1 , B_2) := \log \frac{r_{12}^2}{r_1 \, r_2}$$

where $r_{12} := \max\{|b_1 - b_2| ; \quad b_1 \in B_1 ; \quad b_2 \in B_2\}$.

PROPOSITION 2. - For any nonarchimedean valued field k , $(\mathcal{K}(k) , d)$ is a tree-like metric space.

Proof.

(i) Since $r_{12} \geq \max(r_1 , r_2)$, for B_1 and B_2 as above we have $d(B_1 , B_2) \geq 0$, and $d(B_1 , B_2) = 0$ if, and only if, $r_{12} = r_1 = r_2$, i. e. $B_1 = B_2$. For any $B_1 , B_2 , B_3 \in \mathcal{K}(k)$, we have $r_{12} \leq \max(r_{13} , r_{23})$ which implies

$$\frac{r_{12}^2}{r_1 \, r_2} \leq \frac{r_{13}^2}{r_1 \, r_3} \times \frac{r_{23}^2}{r_2 \, r_3} .$$

and thus proves the triangle inequality.

(ii) To prove (T1) note that for $B_1 , B_2 \in \mathcal{K}(k)$, we have

$$S(B_1 , B_2) = \{B_3 \in \mathcal{K}(k) ; \quad r_3 \, r_{12} = r_{13} \, r_{23}\}$$

$$= \{B_3 \in \mathcal{K}(k) ; \quad r_{12} = \max(r_{13} , r_{23}) ; \quad r_3 = \min(r_{13} , r_{23})\} .$$

Thus if $B_3 , B_4 \in S(B_1 , B_2)$ and $r_{13} < \max(r_{14} , r_{34})$, then $r_{14} = r_{34}$, and $r_{12} = \max(r_{34} , r_{24}) = r_{23}$ and $r_4 = \min(r_{14} , r_{24}) = \min(r_{34} , r_{24})$, so $B_4 \in S(B_2 , B_3)$.

If $B_1 , B_2 , B_3 \in \mathcal{K}(k)$, choose the indices so that $r_{12} \leq \min(r_{13} , r_{23})$. Then it is easily verified that $B_4 := B(a_1 , r_{12}) \in S(B_1 , B_2) \cap S(B_1 , B_3) \cap S(B_2 , B_3)$ so (T2) also holds.

If the valuation of k is discrete, the Bruhat-Tits-tree for k can be reconstructed from $\mathcal{K}(k)$ by letting the points of $\mathcal{K}(k)$ be the vertices of a graph and by drawing edges between points on minimal distance.

If on the other hand k is algebraically closed $\mathcal{K}(k)$ is everywhere dense (but not complete).

An extension $k' : k$ of ultrametric fields gives a natural distance preserving embedding $\mathcal{K}(k) \hookrightarrow \mathcal{K}(k')$. Thus if k is algebraically closed, we may view $\mathcal{K}(k)$ as the direct limit of the Bruhat-Tits-trees for the discrete subfields of k.

For the completion $\hat{k}$ of k, we always have $\mathcal{K}(\hat{k}) = \mathcal{K}(k)$. If R is a ray in $\mathcal{K}(k)$, and $B_0, B_1, B_2, \ldots$ is a sequence of points on R with $d(B_0, B_n) \longrightarrow \infty$ as $n \longrightarrow \infty$ then either $r_n \longrightarrow \infty$ or $r_{Cn} = r_{On+1}$ for $n \geq n_0$ and $r_n \longrightarrow 0$. Therefore the ends of $\mathcal{K}(k)$ correspond to the points of $\underset{\sim}{P}^1(k)$.

$PGL_2(k)$ acts isometrically on $\mathcal{K}(k)$ if we make the following convention : if $\gamma^{-1}(\infty) \in B$ for a $B \in \mathcal{K}(k)$ and a $\gamma \in PGL_2(k)$, let γB be the affinoid hull (= geometric closure) of the "open" disk $\underset{\sim}{P}^1(k) - \gamma(B)$. This action commutes with field extensions.

$\gamma \in PGL_2(k)$ is hyperbolic if, and only if, it is of type (c) of lemma 2 ; the axis A_γ is determined by the fixed points of γ in $\underset{\sim}{P}^1(k)$, the shift v_γ on A_γ is given by $v_\gamma = -\log |t_\gamma|$, where t_γ is the multiplier of γ.

Let $\pi_\gamma : \mathcal{K}(k) \longrightarrow A_\gamma$ denote the projection ; this is meaningful also for ends of $\mathcal{K}(k)$ different from the fixed points of γ. γ defines an orientation $<_\gamma$ on A_γ such that $B <_\gamma \gamma B$ for all $B \in A$.

Let $\gamma_1, \gamma_2 \in PGL_2(k)$ be hyperbolic with mutually different fixed points x_1, x_{-1}, x_2, x_{-2} such that the translation of γ_i on $A_i := A_{\gamma_i}$ is towards x_i. Let

$$v_i := v_{\gamma_i}, \quad <_i := <_{\gamma_i}, \quad \pi_i := \pi_{\gamma_i} ;$$

let

$$B_{12} := \pi_1(x_{-2}), \quad B'_{12} := \pi_1(x_2) \quad \text{and} \quad d_{12} := d(B_{12}, B'_{12}) .$$

Call γ_1, γ_2 parallel if $B_{12} <_1 B'_{12}$, otherwise antiparallel. Finally assume $v_1 \leq v_2$.

LEMMA 3. - <u>With the above notations and assumptions, we have</u> :

(i) $\gamma_1 \gamma_2$ <u>is not hyperbolic if</u> γ_1, γ_2 <u>are antiparallel, and</u> $v_1 = v_2 \leq d_{12}$.

(ii) $\gamma_1 \gamma_2$ <u>is possibly not hyperbolic if</u> γ_1, γ_2 <u>are antiparallel and</u> $v_1 = d_{12} < v_2$

(iii) $\gamma_1 \gamma_2$ <u>is hyperbolic in all other cases.</u>

Proof. - In the first case, B_{12} is fixed point of $\gamma_1 \gamma_2$, in (ii) $\gamma_1 \gamma_2$ may have fixed points on $S(B_{12}, \gamma_2^{-1} B'_{12})$, in all other cases one easily sees that

$$S \cap \gamma_1 \gamma_2 S = \{\gamma_1 \gamma_2 B_{12}\} \quad \text{with} \quad S = S(B_{12}, \gamma_1 \gamma_2 B_{12}) .$$

In view of the proof of lemma 2, this shows that $\gamma_1 \gamma_2$ is hyperbolic.

3. p-adic Teichmüller space $\mathcal{T}_2$

In this section, k is assumed to be algebraically closed and complete. We briefly recall from [2] the definition of the p-adic Teichmüller space $\mathcal{T}_g$, $g \geq 2$ an integer.

Let $G := PGL_2(k)$; for $\zeta = (\gamma_1, \dots, \gamma_g) \in G^g$, let $\Gamma(\zeta)$ be the subgroup of G generated by $\gamma_1, \dots, \gamma_g$. Then

$$\mathcal{T}_g := \tilde{\mathcal{T}}_g \bmod G$$

where $\tilde{\mathcal{T}}_g := \{\zeta = (\gamma_1, \dots, \gamma_g) \in G^g ; \ \Gamma(\zeta) \text{ is Schottky group of rank } g\}$, and G acts on $\mathcal{T}_g$ by componentwise conjugaition. For the Teichmüller modular group and the connection with the space of Mumford curves, we refer to [2].

Recall that a subgroup $\Gamma \subset G$ is a Schottky group if, and only if, every element $\gamma \in \Gamma$, $\gamma \neq \mathrm{id}$, is hyperbolic. As coordinates on $\tilde{\mathcal{T}}_g$, we use the multipliers t_i, the attracting and repelling fixed points x_i and x_{-i} of the hyperbolic transformation γ_i. For $\mathcal{T}_g$, we take the set of representatives normalized by the conditions $x_1 = 0$, $x_{-1} = \infty$, $x_2 = 1$.

In order to replace the condition that $\Gamma(\zeta)$ be a Schottky group by inequalities involving rational functions of the coordinates on $\mathcal{T}_g$, we introduce the following notations :

Let F_g be a nonabelian free group of rank g, $e_1, \dots, e_g$ a fixed base of F_g, and $\alpha_\zeta : F_g \to \Gamma(\zeta)$, $e_i \to \gamma_i$, the canonical homomorphism for any $\zeta \in G^g$. If $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL_2(k)$, let $T(\gamma) := (a+d)^2/ad - bc$. Obviously $T(\lambda\gamma) = T(\gamma)$, so T is a rational function on G. $\gamma \in G$ is hyperbolic if, and only if, $|T(\gamma)| > 1$.

Therefore

$$\mathcal{T}_g = \{(t_1, \dots, t_g ; \ x_{-2}, x_3, x_{-3}, \dots, x_g, x_{-g}) \in k^{3g-3} ;$$

$$0 < |t_i| < 1 ; \ i = 1, \dots, g ; \ |T(\alpha(w))| > 1 \ \text{for all } w \in F_g ; \ w \neq 1\} .$$

Note that since γ_i can be represented by the matrix

$$\begin{pmatrix} x_i - t_i x_{-i} & (t_i - 1) x_i x_{-i} \\ 1 - t_i & t_i x_i - x_{-i} \end{pmatrix} \quad (i \geq 2) ,$$

$T(\alpha(w))$ is indeed rational in the t_i, x_i and x_{-i}.

Every Schottky group $\Gamma \subset G$ has a Schottky base $\gamma_1, \dots, \gamma_g$, i. e. there are $B_1, B_1', \dots, B_g, B_g' \in \mathfrak{K}(k)$ such that $\gamma_i B_i = B_i'$ and there is an $x \in \underset{\sim}{P}^1(k)$

such that $\pi_i(x) \in S(B_i , B_i')$, $i = 1 , \dots , g$. One sees immediately that hyperbolic transformations $\gamma_1 , \dots , \gamma_g$ form a Schottky base if, and only if, for $i = 1 , \dots , g$,

$$d(\pi_i(x_j) , \pi_i(x_k)) < v_i \quad \text{for all } j , k \neq \pm i .$$

If $g = 2$, this reduces to the following description of the space $\mathcal{B}_2$ of Schottky bases of rank 2 (cf. [1], § 2) :

$$\mathcal{B}_2 = \{(t_1 , t_2 , y) \in k^3 ; \ y \neq 1 ; \ d_{12} < v_i ; \ i = 1 , 2\}$$

(where $d_{12} = d(\pi_1(x_2) , \pi_1(x_{-2}))$ as in lemma 3).

The following lemma is crucial in the proof of the main result :

LEMMA 4. - <u>Let</u> $\zeta = (\gamma_1 , \gamma_2) \in \mathcal{B}_2$ <u>be a normed Schottky base and</u>

$$v := \min\{v_\gamma ; \ \gamma \in \{\gamma_1 , \gamma_2 , \gamma_1 \gamma_2 , \gamma_1 \gamma_2^{-1}\}\} .$$

<u>Then</u> $v_\gamma \geq v$ <u>for any</u> $\gamma \in \Gamma(\zeta)$.

<u>Proof.</u> - By replacing if necessary γ_1 or γ_2 by $\gamma_1 \gamma_2$ or $\gamma_1 \gamma_2^{-1}$ or by taking inverses, we may assume that γ_1 , γ_2 are antiparallel and that $2d_{12} \leq v_1 \leq v_2$, so $v = v_1$. As conjugation doesn't change the multiplier we only have to consider elements of the form

$$\gamma = \gamma_1^{\nu_1} \gamma_2^{\mu_1} \dots \gamma_1^{\nu_r} \gamma_2^{\mu_r} , \quad \nu_i , \mu_i \in \mathbb{Z} \setminus \{0\} , \quad r \geq 1 .$$

By induction on r , one easily verifies

(i) $\pi_1(\gamma B_{12}) \notin S(\gamma_1 B_{12}' , \gamma_1^{-1} B_{12}) \setminus \{\gamma_1 B_{12}' , \gamma_1^{-1} B_{12}\}$,

(ii) $d(\gamma B_{12} , A_1) \geq v_2 - d_{12} \geq v_1 - d_{12}$,

(iii) $B_{12} \in A_\gamma$

which shows that

$$v_\gamma = d(B_{12} , \gamma B_{12}) \geq v_1 - d_{12} + v_1 - d_{12} \geq v_1 .$$

THEOREM. - <u>$\mathcal{T}_2$ is a Stein domain.</u>

<u>More precisely</u> : <u>Let</u> $\varepsilon \in |k^*|$, $0 < |\varepsilon| < 1$. <u>Then the following affinoid domains</u> $\mathcal{T}_2^{(n)} \subset k^3$, $n \geq 1$, <u>exhaust</u> $\mathcal{T}_2$:

$\mathfrak{T}_2^{(n)} := \{(t_1 , t_2 , y) \in k^3 ;$

$\epsilon^n \leq |t_i| \leq \epsilon^{1/n} , \quad i = 1 , 2 ,$

$\epsilon^n \leq |y| \leq \epsilon^{-n} , \quad \epsilon^n \leq |1 - y| ,$

$|t_1 t_2| \leq \epsilon^{1/n} |y|^{\alpha} , \quad \alpha = \pm 1 ,$

$|T(\gamma_i \gamma_j^{\nu})| \geq \epsilon^{-1/n} , \quad i , j = 1 , 2 , \quad i \neq j , \quad \nu = \pm 1 , \dots , \pm n^2 ,$

$|T(\gamma_1(\gamma_i \gamma_j^{\nu})^{\mu}) \geq \epsilon^{-1/n} , \quad i , j , 1 = 1 , 2 ,$

$i \neq j , \quad \nu = \pm 1 , \dots , \pm n^2 , \quad \mu = \pm 1 , \dots , \pm (n^2 - 1)\}$

<u>Proof</u>. - The conditions can be rephrased in terms of v_i , d_{12} , etc. :

(1) $n.\epsilon' \geq v_i \geq \epsilon'/n$, $i = 1 , 2$ (where $\epsilon' := - \log \epsilon$),

(2) $d_{12} \leq n\epsilon'$, $d(A_1 , A_2) \leq n\epsilon'$,

(3) $v_1 + v_2 \leq \epsilon'/n \; d_{12}$,

(4) $v_{\gamma} \leq \epsilon'/n$ for the γ listed above.

Now we divide the proof into several steps :

1° $\mathfrak{T}_2 \subset \bigcup_{n=1}^{\infty} \mathfrak{T}_2^{(n)}$: let $\zeta = (\gamma_1 , \gamma_2) \in \mathfrak{T}_2$. (1), (2) are obviously satisfied for large n .

Lemma 3 shows that $v_1 + v_2 < d_{12}$ is necessary to ensure that $\gamma_1 \gamma_2 \gamma_1^{-1} \gamma_2^{-1}$ and $\gamma_1 \gamma_2^{-1} \gamma_1^{-1} \gamma_2$ are hyperbolic, and (4) results from lemma 4.

2° $\mathfrak{T}_2^{(n)} \subset \mathfrak{T}_2$: Let $\zeta = (\gamma_1 , \gamma_2) \in \mathfrak{T}_2^{(n)}$. We may again assume $v_1 \leq v_2$ and γ_1 , γ_2 antiparallel. Then $d_{12} \leq n^2 v_1$ because of (1), (2), and $d_{12} < 2v_2$ because of (3). Let $m \in \{0 , \dots , n^2\}$ such that $mv_1 \leq d_{12} < (m + 1) v_i$.

We consider the following cases :

(a) $mv_1 \leq d_{12} < v_2$. - Here $\gamma_2' := \gamma_2 \gamma_1^m$ is hyperbolic by lemma 3 (resp. by condition (4)), and $v_2' := v_{\gamma_2'} \leq v_2 - mv_1$, with equality if $mv_1 < d_{12}$.

$$d_{12}' := d(\pi_1(x_2') , \pi_1(x_{-2}')) = d_{12} - mv_1 < \max(v_1 , v_2') ,$$

so $(\gamma_1 , \gamma_2') \in \mathfrak{B}_2$.

(b) $mv_1 \leq v_2 \leq d_{12}$. - Again $\gamma_1' := \gamma_1 \gamma_2^m$ is hyperbolic because of condition (4), and $v_1' \geq \epsilon'/n$. Therefore $d_{12}' = d_{12} - mv_1 < v_1 \leq n\epsilon' \leq n^2 v_1'$, so γ_1' , γ_1 lead to case (a) with an $m' \leq n^2 - 1$; the γ listed in condition (4) are now precisely those needed to show that $\gamma_1' , \gamma_1(\gamma_1')^{m'}$ is a Schottky base.

(c) $v_2 < mv_1 \leq d_{12}$. - There similar to case (b) $\gamma_1' = \gamma_2 \gamma_1^m$ and γ_2 lead to case (a) with an $m' \leq n^2 - 1$, so that $\gamma_1' , \gamma_2(\gamma_1')^{m'}$ is a Schottky base.

3° $\mathcal{T}_2^{(n)} \subset \mathcal{T}_2^{(n+1)}$: Inspection of the construction in 2° shows that any $\zeta \in \mathcal{T}_2^{(n)}$ has a Schottky base satisfying the condition of lemma 4 with $v \gtrsim \epsilon^{1/n}$. So an application of this lemma concludes the proof of the theorem.

REFERENCES

[1] GERRITZEN (L.). - Zur analytischen Beschreibung des Raumes der Schottky-Mumford-Kurven, Math. Annalen, t. 255, 1981, p. 259-271.

[2] GERRITZEN (L.). - p-adic Teichmüller space and Siegel half space, Groupe d'etude d'Analyse ultramétrique, 9e année, 1981/82, fasc. 3, n° J-26, 15 p.

[3] TITS (J.). - Sur le groupe des automorphismes d'un arbre. Essays on topology a and related topics, Mémoires dédiés à G. de Rham. - Berlin, Heidelberg, New York, Springer-Verlag, 1970.