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ON THE POLES OF THE SCATTERING MATRIX FOR TWO CONVEX OBSTACLES

by Mitsuru IKAWA

§1. Introduction.

Let $\mathcal{O}$ be a bounded open set in $\mathbb{R}^3$ with smooth boundary Γ. We set $\Omega = \mathbb{R}^3 - \overline{\mathcal{O}}$. Suppose that Ω is connected. Consider the following acoustic problem

$$(1.1)\quad \begin{cases} \square u(x,t) = \dfrac{\partial^2 u}{\partial t^2} - \Delta u = 0 & \text{in } \Omega\times(-\infty,\ \infty) \\ u(x,t) = 0 & \text{on } \Gamma\times(-\infty,\ \infty) \end{cases}$$

where $\Delta = \sum_{j=1}^{3} \partial^2/\partial x_j^2$. Denote by $\mathcal{S}(z)$ the scattering matrix for this problem. About the defintion of the scattering matrix, see for example Lax and Phillips[7,page 9]. The result I like to talk about is the following

<u>Theorem 1</u>. Let $\mathcal{O}_j$, j=1,2, be open and strictly convex sets in $\mathbb{R}^3$ with smooth boundary Γ_j, that is, the Gaussian curvature of Γ_j is positive everywhere on Γ_j. Suppose that $\overline{\mathcal{O}_1} \cap \overline{\mathcal{O}_2} = \phi$. Then the scattering matrix $\mathcal{S}(z)$ for

$$\mathcal{O} = \mathcal{O}_1 \cap \mathcal{O}_2$$

satisfies the following:

(1) There exist positive constant c_0 and c_1 such that $\mathcal{S}(z)$ is holomorphic in

$$\{z;\ \mathrm{Im}\ z \leq c_0 + c_1\} - \bigcup_{j=-\infty}^{\infty} D_j$$

where

$$D_j = \{z;\ |z - z_j| \leq C(1+|j|)^{-1/2}\},$$

$$z_j = ic_0 + \frac{\pi}{d} j, \qquad d = \mathrm{dis}(\mathcal{O}_1, \mathcal{O}_2).$$

(2) For large $|j|$, every D_j contains exactly one pole of $\mathcal{S}(z)$.

(3) Denoting the pole in D_j by ζ_j we have an asymptotic expansion

(1.2) $$\zeta_j \sim z_j + \beta_1 j^{-1} + \beta_2 j^{-2} + \cdots\cdots \qquad \text{for } |j| \longrightarrow \infty$$

where $\beta_1, \beta_2, \cdots\cdots$ are complex constants determined by $\mathcal{O}$.

(4) In D_j $\mathcal{S}(z)$ is of the form

(1.3) $$\mathcal{S}(z)f = \frac{1}{z-\zeta_j}(f,\psi_j)m_j + \mathcal{H}_j(z)f$$

for all $f \in L^2(S^2)$, where m_j, $\psi_j \in L^2(S^2)$ such that $m_j \neq 0$, $\psi_j \neq 0$ and $(\cdot,\cdot)$ stands for the scalar product in $L^2(S^2)$, and $\mathcal{H}_j(z)$ is an $\mathcal{L}(L^2(S^2), L^2(S^2))$ [1)] valued holomorphic function in D_j.

Concerning the existence of non-purely imaginary poles of $\mathcal{S}(z)$, Bardos, Guillot and Ralston[1] proved under the same assumption as ours the existence of an infinite number of the poles in

$$\{z;\ \mathrm{Im}\ z \leq \varepsilon \log(1+|z|)\}$$

for any $\varepsilon > 0$. This result is generalized by Petkov[11] and Petkov and Stojanov[12] to a case of many strictly convex obstacles. For non-strictly convex obstacles Ikawa[5] showed an example of two

1) We denote by $\mathcal{L}(E,F)$ the set of all linear bounded mappings from E into F.

convex obstacles whose scattering matrix has a sequence of the poles converging to the real axis. On the other hand Lebeau[9] considered the distribution of poles for one strictly convex obstacle.

§2. Reduction of the problem.

Consider a boundary value problem with a parameter $\mu \in \mathbb{C}$

$$(2.1) \qquad \begin{cases} (\mu^2 - \Delta)u(x) = 0 & \text{in } \Omega \\ u(x) = g(x) & \text{on } \Gamma \end{cases}$$

for $g \in C^\infty(\Gamma)$. For $\mathrm{Re}\mu > 0$ (2.1) has a solution u uniquely in $\bigcap_{m>0} H^m(\Omega)$. We denote the solution by $U(\mu)g$. Then $U(\mu)$ is holomorphic in $\mathrm{Re}\mu > 0$ as an $\mathcal{L}(C^\infty(\Gamma), C^\infty(\overline{\Omega}))$-valued functon. We shall show the following theorems on $U(\mu)$.

<u>Theorem 2.1.</u> (i) $U(\mu)$ is prolonged analytically as an $\mathcal{L}(C^\infty(\Gamma), C^\infty(\overline{\Omega}))$-valued function into

$$\{\mathrm{Re}\mu \geq -c_0 - c_1\} - \bigcup_{j=-\infty}^{\infty} iD_j.$$

(ii) Set for $k \in \mathbb{R}$

$$G_k = \{\mu \in \mathbb{C};\ |\mu + ik| \leq c_0 + c_1,\ \mathrm{Re}\mu \geq -c_0 - (\log(1+|k|))^{-1}\}.$$

Then for large $|k|$, $U(\mu)$ is represented in $G_k \cap \{\mathrm{Re}\mu > 0\}$ as

$$(2.2) \qquad U(\mu) = \frac{\beta(x,k,\mu)}{\mathcal{P}(\mu) - \gamma(k,\mu)} F(k,\mu) + V(k,\mu).$$

Here

(a) $\beta(\bullet,k,\mu)$ is $C^\infty(\overline{\Omega})$-valued holomorphic function in G_k.

(b) $\mathcal{P}(\mu) = 1 - \lambda\tilde{\lambda}e^{-2d\mu}$, $0 < \lambda$, $\tilde{\lambda} < 1$.

(c) For any N positive integer

$$\left| \gamma(k,\mu) - \sum_{1\le \ell<N} \sum_{0\le h<N} \gamma_{\ell,h} k^{-\ell}(\mu+ik)^h \right| \leq C_N|k|^{-N}$$

where $\gamma_{\ell,h}$ are complex constants.

(d) $F(k,\mu)$ is an $\mathcal{L}(L^2(\Gamma),\mathbb{C})$-valued holomorphic function in G_k.

(f) $V(k,\mu)$ is an $\mathcal{L}(C^\infty(\Gamma),C^\infty(\bar{\Omega}))$-valued holomorphic function in G_k.

<u>Corollary.</u> $U(\mu)$ is prolonged analytically as $\mathcal{L}(C^\infty(\Gamma),C^\infty(\bar{\Omega}))$ valued function into

$$\bigcup_{|k|:\text{large}} (G_k - \{\mu;\ \mathcal{P}(\mu)-\gamma(k,\mu)=0\}).$$

<u>Theorem 2.2.</u> Suppose that $\mu\in G_k$ and $\mathcal{P}(\mu)-\gamma(k,\mu)=0$ for $|k|$ large. Then we have

$$\dim\{u;\ \mu\text{-outgoing solution of (2.1) for } g=0\} = 1.$$

Note that the zeros of $\mathcal{P}(\mu)=0$ are $\{iz_j,\ j=0,\pm 1,\pm 2,\cdots\cdots\}$ and

$$\left|\frac{d}{d\mu}(\mathcal{P}(\mu) - \gamma(k,\mu))_{\mu=iz_j}\right| \geq d - C|k|^{-1}.$$

By setting $k=-\frac{\pi}{d}j$ we have that $\mathcal{P}(\mu)-\gamma(k,\mu)=0$ has only one zero in iD_j and it is simple. Denote it by $i\zeta_j$ and we see that ζ_j has an asymptotic expansion (1.2).

Theorem 1 is immediately derived from Theorems 2.1 and 2.2 if we recall the relationships between $\mathcal{S}(z)$ and $U(\mu)$ shown in Lax and Phillips[7], especially Theorem 5.1 of Chapter V, which says that $\mathcal{S}(z)$ has a pole at exactly those points z such that $\mu=iz$ is a pole of $U(\mu)$.

§3. Sketch of the proofs of Theorems 2.1 and 2.2.

3.1. Asymptotic solutions for oscillatory boundary data.

Let $a_j \in \Gamma_j$ be the points verifying

$$|a_1 - a_2| = \mathrm{dis}(\mathcal{O}_1, \mathcal{O}_2).$$

Denote by $S_j(\delta)$ for $\delta > 0$ a connected component containing a_j of

$$S_j \cap \{x;\ \mathrm{dis}(x,L) = \delta\}$$

where L is a straight line passing a_1 and a_2, and denote by $\omega(\delta)$ a domain surrounded by $\{x;\mathrm{dis}(x,L)=\delta\}$ and $S_j(\delta)$, j=1,2. Let $\upsilon_k(x)$ be a smooth function satisfying

$$\upsilon_k(x) = \begin{cases} 1 & \text{for } x \in S_1(k^{-\varepsilon}) \\ 0 & \text{for } x \notin S_1((1+\delta)k^{-\varepsilon}) \end{cases}$$

for some $\delta > 0$, $\varepsilon > 0$ small constants. Let $h(t) \in C^\infty(0,d/2)$ satisfying $h(t) \geq 0$ and $\int h(t)dt = 1$. Set

$$m(x,t;k) = e^{ik(\psi(x)-t)}\, w(x)h(t-j(x)) \tag{3.1}$$

where $\psi \in C^\infty(S_1(\delta_0))$ is a real valued function satisfying some conditions and j(x) a fixed smooth function determined by $\mathcal{O}$. We construct a sequence of functions of the form

$$u_q(x,t;k) = e^{ik(\varphi_q(x)-t)} \sum_{j=0}^{N} v_{j,q}(x,t;k)(ik)^{-j}. \tag{3.2}$$

(I) φ_q, q=0,1,$\cdots$ are determined successively by

$$\begin{cases} |\nabla \varphi_0| = 1 & \text{in } \omega(\delta) \\ \varphi_0 = \psi \quad \text{and} \quad \partial\varphi_0/\partial n > 0 & \text{on } S_1(\delta), \end{cases}$$

$$\begin{cases} |\nabla \varphi_1| = 1 & \text{on} \quad \omega(\delta) \\ \varphi_1 = \varphi_0 \ \text{ and } \ \partial\varphi_1/\partial n > 0 & \text{on} \quad S_2(\delta), \end{cases}$$

$$\begin{cases} |\nabla \varphi_2| = 1 & \text{on} \quad \omega(\delta) \\ \varphi_2 = \varphi_1 \ \text{ and } \ \partial\varphi_2/\partial n > 0 & \text{on} \quad S_1(\delta), \end{cases}$$

$$\vdots$$

(II) On amplitude functions.

Set

$$T_q = 2\frac{\partial}{\partial t} + 2\nabla\varphi_q \cdot \nabla + \Delta\varphi_q .$$

$v_{0,q}$, $q=0,1,2,\cdots$ are defined successively by

$$\begin{cases} T_0 v_{0,0} = 0 & \text{in} \quad \omega(\delta)\times\mathbb{R} \\ v_{0,0} = f(x,t) & \text{on} \quad \Gamma_1\times\mathbb{R} \end{cases}$$

where $f(x,t)=w(x)h(t-j(x))$, and for $p\geq 1$

$$\begin{cases} T_{2p-1} v_{0,2p-1} = 0 & \text{in} \quad \omega(\delta)\times\mathbb{R} \\ v_{0,2p-1} = v_{0,2p-2} & \text{on} \quad \Gamma_2\times\mathbb{R}, \end{cases}$$

$$\begin{cases} T_{2p} v_{0,2p} = 0 & \text{in} \quad \omega(\delta)\times\mathbb{R} \\ v_{0,2p} = \upsilon_k(x) v_{0,2p-1} & \text{on} \quad \Gamma_1\times\mathbb{R}. \end{cases}$$

Next for $j\geq 1$, $v_{j,q}$, $q=0,1,2,\cdots$ are defined successively for all $p\geq 0$ by

$$\begin{cases} T_{2p} v_{j,2p} = \Box v_{j-1,2p} & \text{in} \quad \omega(\delta)\times\mathbb{R} \\ v_{j,2p} = 0 & \text{on} \quad \Gamma_1\times\mathbb{R}, \end{cases}$$

$$\begin{cases} T_{2p+1} v_{j,2p+1} = \Box v_{j-1,2p+1} & \text{in} \quad \omega(\delta)\times\mathbb{R} \\ v_{j,2p+1} = v_{j,2p} & \text{on} \quad \Gamma_2\times\mathbb{R}. \end{cases}$$

On the asymptotic behavior of φ_q, $v_{j,q}$ for $q \longrightarrow \infty$, we have the following Lemmas.

<u>Lemma 3.1.</u> It holds that

$$|\varphi_{2p} - (\varphi_\infty + 2dp + d_0)|_m \leq C_m \alpha^{2p}$$

$$|\varphi_{2p+1} - (\tilde{\varphi}_\infty + (2p+1)d + d_0)|_m \leq C_m \alpha^{2p}$$

where φ_∞, $\tilde{\varphi}_\infty$ are functions independent of ψ, and they verify

$$|\nabla \varphi_\infty| = 1 \quad \text{in} \quad \omega(\delta) \quad \text{and} \quad \varphi(a_1)=0,$$

$$|\nabla \tilde{\varphi}_\infty| = 1 \quad \text{in} \quad \omega(\delta) \quad \text{and} \quad \tilde{\varphi}(a_2)=0,$$

and d_0 is a constant depending on ψ, α is a positive constant < 1.

<u>Lemma 3.2.</u> It holds that

$$|v_{j,2p}(x,t;k) - bw(A)(\lambda\tilde{\lambda})^p v_{j,\infty}(x,t-2pd-j(A)-d_\infty;k)|_m$$
$$\leq C_{j,m}(\alpha\lambda\tilde{\lambda})^p M_{m+2j},$$

$$|v_{j,2p+1}(x,t;k) - bw(A)(\lambda\tilde{\lambda})^p \tilde{v}_{j,\infty}(x,t-2pd-j(A)-d_\infty;k)|_m$$
$$\leq C_{j,m}(\alpha\lambda\tilde{\lambda})^p M_{m+2j},$$

where λ, $\tilde{\lambda}$ are constants determined by $\mathcal{O}$ such that $0 < \lambda, \tilde{\lambda} < 1$,

$$M_\ell = k^{\varepsilon\ell} \sum_{|\beta|<\ell} \sup_{\Gamma_1\times\mathbb{R}} |D^\beta_{x,t} f|,$$

$v_{j,\infty}$ and $\tilde{v}_{j,\infty}$ are functions of the form

$$v_{j,\infty}(x,t;k) = \sum_{\ell=0}^{2j} a_{j,\ell}(x,k)h^{(\ell)}(t-j(x)),$$

$$\tilde{v}_{j,\infty}(x,t;k) = \sum_{\ell=0}^{2j} \tilde{a}_{j,\ell}(x,k)h^{(\ell)}(t-\tilde{j}(x)),$$

and b is a constant depending on ψ, A is a point in $S_1(\delta)$ depending on ψ.

Remark that we have

$$\square u_q = e^{ik(\varphi_q - t)} (ik)^{-N} \square v_{N,q}.$$

Next we construct by a usual method asymptotic solutions for

$$(3.3) \qquad \begin{cases} \square u = 0 & \text{in} \quad \omega \times \mathbb{R} \\ u = (1-\upsilon_k(x))u_{2p}(x,t;k) & \text{on} \quad \Gamma_1 \times \mathbb{R} \end{cases}$$

Denote the asymptotic solution by u'_{2p}. Extend $\square(u_{2p}+u'_{2p})$ and $\square u_{2p+1}$ by a fixed manner into $\mathcal{O}$ so that these are smooth in $\mathbb{R}^3 \times \mathbb{R}$, and de note by u''_q the solution of

$$\begin{cases} \square u = -\square(u_q + u'_q) & \text{in } \mathbb{R}^3 \times \mathbb{R} \\ u = 0 & \text{for } t < 0 \end{cases}$$

where we set $u'_{2p+1}=0$. By taking account of the continuity from u_q to u'_q and u''_q we have a Lemma of the type Lemma 3.2 on the convergence of u'_q and u''_q. Set

$$r_q = u_q + u'_q + u''_q$$

and we have

<u>Lemma 3.3.</u> It holds that

$$|r_{2p}(x,t;k) - bw(A)e^{-ik(j(A)+d_\infty)} e^{ikd_0} (\lambda\tilde{\lambda})^p \cdot r_\infty(x,t-2pd-j(A)-d_\infty;k)|_m \leq C_m(\alpha\lambda\tilde{\lambda})^p k^{m+1}$$

$$|r_{2p+1}(x,t;k) - bw(A)e^{-ik(j(A)+d_\infty)} e^{ikd_0} (\lambda\tilde{\lambda})^p \cdot \tilde{r}_\infty(x,t-2pd-j(A)-d_\infty;k)|_m \leq C_m(\alpha\lambda\tilde{\lambda})^p k^{m+1},$$

where r_∞, $\tilde{r}_\infty$ are functions independent of ψ and w.

Set

$$r(x,t;k) = \sum_{q=0}^{\infty} (-1)^q r_q(x,t;k).$$

Evidently it holds that

$$\Box r = 0 \qquad \text{in} \quad \Omega\times\mathbb{R}.$$

We consider the Laplace transformation of r in t, that is,

(3.4) $$\hat{r}(x,\mu;k) = \int e^{-\mu t} r(x,t;k)dt.$$

We have from Lemma 3.3 the following

Proposition 3.4. Let $\mathrm{Re}\mu > 0$. Then (3.4) converges and we have a representation of $\hat{r}(x,\mu;k)$

(3.5) $\hat{r}(x,\mu;k)$

$$= bw(A)e^{-(j(A)+d_\infty)(\mu+ik)} e^{ikd_0} \mathcal{P}(\mu)^{-1} r_\infty(x,\mu;k) + s(x,\mu;k),$$

where $\hat{r}_\infty(x,\mu;k)$ is an entire function in μ independent of ψ and w, and $\hat{s}(x,\mu;k)$ is holomorphic in $\mathrm{Re}\mu > -c_0-c_1$. Moreover we have on Γ_1

$$\hat{r}(x,\mu;k) - e^{-(\mu+ik)j(x)} e^{ik\psi(x)} w(x)\hat{h}(\mu+ik)$$

$$= e^{ikd_0} bw(A)e^{-(j(A)+d_\infty)(\mu+ik)}\{\upsilon_k(x) \sum_{1\le j\le N} \sum_{0\le h\le N} a_{j,h}(x)(ik)^{-j}$$

$$\times(\mu+ik)^h \hat{h}(\mu+ik) + a_1(x,k,\mu)\} \mathcal{P}(\mu)^{-1} + e_1(x,\mu;k),$$

and on Γ_2

$$\hat{r}(x,\mu;k)=e^{ikd_0} \frac{1}{\mathcal{P}(\mu)} bw(A)e^{-(j(A)+d_\infty)(\mu+ik)} a_2(x,k,\mu) + e_2(x,k,\mu),$$

where $a_{j,h}(x)$ are smooth functions on $S_1(\delta)$, a_1 and a_2 are entire functions independent of ψ and w having an estimate

$$\sup_{x\in\Gamma_j} |a_j(x,k,\mu)| \le C_{N,R}|k|^{-N}$$

and e_1 and e_2 are holomorphic in $\mathrm{Re}\mu > -c_0 - c_1$ and satisfy

$$|e_1(x,k,\mu)| \leq \begin{cases} |k|^{-1} & \text{on } S_1((1+\delta)|k|^{-1}) \\ |k|^{-N} & \text{on } \Gamma_1 \setminus S_1((1+\delta)|k|^{-1}), \end{cases}$$

$$|e_2(x,k,\mu)| \leq |k|^{-N} \qquad \text{on } \Gamma_2.$$

3.2. Reduction to an integral equation on Γ_1.

Suppose that Γ_1 is represented as $x(\sigma)=(\sigma_1,\sigma_2,x_3(\sigma_1,\sigma_2))$ near a_1. Let $g(x) \in C_0^\infty(S_1(\delta_0))$. Then

$$g(x(\sigma)) = (2\pi)^{-2} \int e^{ik\sigma\cdot\xi}\, \hat{g}(k\xi)k^2 d\xi$$

$$= (2\pi)^{-2}\, w(x(\sigma)) \int e^{ik\sigma\cdot\xi} \hat{g}(k\xi)k^2 d\xi$$

where $w(x) \in C_0^\infty(S_1(2\delta_0))$ such that $w(x)=1$ on $S_1(\delta_0)$, and

$$\hat{g}(\xi) = \int e^{-i\sigma\cdot\xi} g(x(\sigma))d\sigma.$$

If we define $\tilde{U}_1(k,\mu)$ an operator from $L^2(S_1(\delta_0))$ into $C^\infty(\bar{\Omega})$ by

$$(\tilde{U}_1(k,\mu)g)(x) = (2\pi)^{-2} \int u(x,\xi;k,\mu)\hat{g}(k\xi)k^2 d\xi$$

where $u(x,\xi;k,\mu)$ denotes $\hat{r}(x,\mu,k)/\hat{h}(\mu+ik)$ constructed for $\psi(x(\sigma)) =\sigma\cdot\xi$. Then we have from Proposition 3.4

<u>Proposition 3.5.</u> $\tilde{U}_1(k,\mu)$ is of the form

$$(3.6) \qquad \tilde{U}_1(k,\mu)g = \frac{\hat{r}_\infty(x,\mu;k)}{\rho(\mu)} F_0(k,\mu)g + S(k,\mu)g$$

where

$$(3.7) \qquad F_0(k,\mu)g = (2\pi)^{-2} \int b(\xi)w(A(\xi))e^{-(j(A(\xi))+d_\infty(\xi))(\mu+ik)} \cdot e^{ikd_0(\xi)}\, \hat{g}(k\xi)k^2 d\xi,$$

$S(k,\mu)$ is $\mathcal{L}(L^2(S_1(\delta_0)), C^\infty(\bar{\Omega}))$-valued holomorphic function in $\mathrm{Re}\mu > -c_0-c_1$. Moreover it holds that

$$(3.8) \qquad (\mu^2 - \Delta)\tilde{U}_1 g = 0 \qquad \text{in} \quad \Omega,$$

$$(3.9) \qquad \tilde{U}_1 g = g - \frac{\alpha(x,k,\mu)}{\mathcal{P}(\mu)} F_0(k,\mu)g - E(k,\mu)g \qquad \text{on} \quad \Gamma_1$$

$$(3.10) \qquad \tilde{U}_1 g = \frac{\tilde{\alpha}(x,k,\mu)}{\mathcal{P}(\mu)} F_0(k,\mu)g + \tilde{E}(k,\mu)g \qquad \text{on} \quad \Gamma_2,$$

$$(3.11) \qquad |\alpha(x,k,\mu)F_0(k,\mu)g| \leq C|k|^{-\varepsilon} \|g\|_{L^2(\Gamma_1)}$$

$$(3.12) \qquad \|E(k,\mu)g\|_{L^2(\Gamma_1)} \leq C|k|^{-\varepsilon} \|g\|_{L^2(\Gamma_1)},$$

$$(3.13) \qquad |\tilde{\alpha}(x,k,\mu)F_0(k,\mu)g| \leq C|k|^{-N} \|g\|_{L^2(\Gamma_1)},$$

$$(3.14) \qquad \|E(k,\mu)g\|_{L^2(\Gamma_2)} \leq C|k|^{-N} \|g\|_{L^2(\Gamma_1)}$$

Note that the solution $U_2 h$ of

$$\begin{cases} (\mu^2 - \Delta)u = 0 & \text{in} \quad \mathbb{R}^3 - \overline{\mathcal{O}}_2 \\ u = h & \text{on} \quad \Gamma_2 \end{cases}$$

is continued into $\{\mu; \mathrm{Re}\mu \geq -a\log(|\mu|+1)\}$ for some $a > 0$. Then

$$U_1(k,\mu)g = \tilde{U}_1(k,\mu)g - U_2(\mu)(\tilde{U}_1(k,\mu)g|_{\Gamma_2})$$

is also of the form (3.6) and satisfies (3.8), (3.9), (3.11) and (3.12), and

$$(3.10)' \qquad U_1(k,\mu)g = 0 \qquad \text{on} \quad \Gamma_2.$$

Remark. We can extend the definition of $U_1(k,\mu)$ for any $f \in L^2(\Gamma_1)$ by using the argument in §8 of [2]. Hereafter we denote by U_1 the extended one.

3.3. Representation of $U(\mu)$.

<u>Lemma 3.6.</u> Let H and E be linear operators with $\|H\|, \|E\| < 1/2$. Then we have

$$(I - H - E)^{-1} = I + \mathcal{C}_1 + \mathcal{C}_2,$$

where

$$\mathcal{C}_1 = \mathcal{H} + \mathcal{H}\mathcal{E} + \mathcal{H}\mathcal{E}\mathcal{H} + \mathcal{H}\mathcal{E}\mathcal{H}\mathcal{E} + \cdots ,$$

$$\mathcal{C}_2 = \mathcal{E} + \mathcal{E}\mathcal{H} + \mathcal{E}\mathcal{H}\mathcal{E} + \mathcal{E}\mathcal{H}\mathcal{E}\mathcal{H} + \cdots ,$$

$$\mathcal{E} = E + E^2 + E^3 + \cdots ,$$

$$\mathcal{H} = H + H^2 + H^3 + \cdots .$$

Pose

$$H(k,\mu)g = \frac{\alpha(x,k,\mu)}{\mathcal{P}(\mu)} F_0(k,\mu)g.$$

An application of the above lemma gives

$$(3.15) \qquad (I - H - E)^{-1} = (I + \mathcal{E}) + \frac{(I+\mathcal{E})\alpha}{\mathcal{P}(\mu)-\gamma} F_0(I + \mathcal{E})$$

where

$$\gamma(k,\mu) = F_0(k,\mu)((I+\mathcal{E}(k,\mu))\alpha(\bullet,k,\mu)).$$

Evidently we have in $\mathrm{Re}\mu > 0$

$$(3.16) \qquad U(\mu) = U_1(k,\mu)(I-H(k,\mu)-E(k,\mu))^{-1}.$$

Then a substitution of (3.15) into (3.16) gives

$$U(\mu) = \frac{r_\infty(x,k,\mu)}{\mathcal{P}(\mu)-\gamma(k,\mu)} F_0(k,\mu)(I+\mathcal{E}(k,\mu)) + S(k,\mu)(I+\mathcal{E}(k,\mu)).$$

By posing

$$F(k,\mu) = F_0(k,\mu)(I+\mathcal{E}(k,\mu)),$$

$$V(k,\mu) = S(k,\mu)(I + \mathcal{E}(k,\mu)),$$

we have a representation (2.2).

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