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ON A METHOD OF MOMENTS

by

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By a linear system we shall understand a system of three Banach spaces reals X, , Y and two continuous linear operators A, B

$$(X \underset{A}{\to} \quad \underset{B}{\to} Y)$$

In our considerations the space will not play any role, because we shall not consider constrains on trajectories. For this reason we write briefly C = BA.

Let y_o CX. If we are looking for a minimal norm solution of equation $Cu = y_o$, we said that we consider a minimum norm problem.

The first step for the solving of minimum norm problem is to find

(1) $a = \inf \{ \|u\| : Cu = y_o \}$

In the case when Y is one dimensional it can be easily reduced to a well known problem of calculating of the norm of the functional C.

A case when Y is finite dimensional was reduced to the one dimensional case by M.G. KREIN [1] by formula

(2) $\inf \{ \|u\| : Cu = y_o \} = \sup\limits_{\mathcal{C} \in Y^*} \inf \{ \|u\| : \mathcal{C}(Cu) = \mathcal{C}(y_o) \}$

In the theory of control formula (2) was used first time by N.N. KRASSOWSKI [3]. A.G. BUTKOWSKI [2] has proved formula (2) for $X = L^p$ and $Y = \ell^p$.

It was shown in [4] that formula (2) holds in general provided that the image Γ of the closed unit ball $K = \{u : \|u\| \leq 1\}$ is closed.

If Γ is not closed formula (2) may not hold. It follows from the following example [5]. Let $X = \ell$ and let Y be an arbitrary infinite dimensional Banach space. Let $y_o, y_1, \dots, y_n, \dots$ be a sequence of strongly linearly independent elements (i.e. such that if a series $\sum_{n=0}^{\infty} t_n y_n$ is convergent to 0, then $t_n = 0$, $n = 0, 1, \dots$) convergent to y_o. Such sequences exist in each Banach spaces of infinite dimension. In fact as follows from a Banach theorem each infinite dimensional Banach contains an infinite dimensional subspace with a basis $\{e_n\}$ $n = 0, 1, 2, \dots$ Let us put $y_o = e_o$

$y_n = e_o + \frac{1}{n} e_n$. It is easy to verify that the sequence y_n has desidered properties.

Let

$$C(\{\ell_n\}) = \frac{1}{2} t_o y_o + \sum_{n=1}^{\infty} t_n y_n$$

The operator C is one to one. It is easy to verify and that

$$\inf \{\|u\| : Cu = y_o\} = 2$$

and that on the other hand for each $\mathcal{C} \in Y^*$

$$\inf \{\|u\| : \mathcal{C}(Cu) = \mathcal{C}(y_o)\} = 1$$

as follows from the fact that $y_n \to y_o$ and $\mathcal{C}$ is continuous.

Similar example was done by I. SINGER [6].

We say that maximum principle of Pontrjagin holds if there is $\mathcal{C}_o \in Y^*$ such that

(3) $\inf \{\|u\| : Cu = y_o\} = \inf \{\|u\| : \mathcal{C}_o(Cu) = \mathcal{C}_o(y_o)\}$.

The set CX is closed if and only if Γ has interior in CX . In this case using Hahn-Banach theorem one may easily to show that the principle of maximum holds for all y_o CX . Thus the principle of maximum holds if either Y or X are finite dimensional .

If the set CX is not closed, there is such $y_o \in CX$ that the principle of maximum does not hold. It means that for all $\mathcal{C} \in Y^*$

(4) $\inf \{ \|u\| : Cu = y_o \} > \inf \{ \|u\| : \mathcal{C}(Cu) = \mathcal{C}(y_o) \}$

It is a consequence of a following theorem of WOJTASZCZYK [7].

Let Y be a Banach space. Let Γ be a closed set in Y such that 0 is an algebraically internal point in Γ and let $\overline{\text{lin}\, \Gamma} = Y$. If for each point y_o of the algebraic boundary of Γ, there is a continuous linear functional f such that $f(y_o) \geq f(x)$ for all $x \in \Gamma$, then has interior in the norm sense.

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