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# Session de problèmes

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## SESSION DE PROBLEMES

On trouvera ci-dessous quelques uns des problèmes proposés à cette session.

**E. Corominas** : A minimal automorphic poset is a poset A without the fixed point property whose all the strict retracts of A have the fixed point property Examples include the crowns.

Problem 1 - Describe the minimal automorphic posets where only finitely many cycles are allowed. Same problem when the height of the posets is finite.

Problem 2 - Is a minimal automorphic finite poset A isomorphic to a retract of $A \times A$.

Conjecture : If K is a retract of $A \times A$ isomorphic to A and the projections are also isomorphic to A then there are exactly two retractions from $A \times A$ onto K.

**B. Courcelle** :

Problem 1 - Decide whether or not two regular langages on $\{o,1\}$ are order isomorphic with respect to the lexicographic ordering $<_{lex}$.

Problem 2 - Describe the equational rules for rational expressions defining the frontiers of regular trees.

**F. Galvin** :

If r and s are positive integers and $\mathcal{U}$ is a (non principal) ultrafilter on $\omega$, let $G_{r,s}(\mathcal{U})$ be the following game of length $\omega$ . At move n, first White chooses a set $W_n \in [\omega \setminus \bigcup_{i<n} B_i]^r$ , and then Black chooses a set $B_n \in [\omega \setminus \bigcup_{i<n} W_i]^s$ . White wins if $\bigcup_{n<\omega} W_n \in \mathcal{U}$ . If $r > 2s$, there is an ultrafilter $\mathcal{U}$ such that White has a winning strategy in $G_{r,s}(\mathcal{U})$ ; this is an unpublished result of F. Galvin, S. Hechler, and R. McKenzie. It is easy to see that White cannot have a winning strategy if $r < s$.

Problem - What happens for $s < r < 2s$ ? Is there an ultrafilter $\mathcal{U}$ such that White has a winning strategy in $G_{3,2}(\mathcal{U})$ ?

**A. Hajnal :**

Problem 1 - Let $G = (V,E), H$ be graphs. $G \rightarrowtail (H)^1_\kappa$ is the following statement :

$\forall f : V \longrightarrow \kappa \, \exists \xi < \kappa$ $H$ is isomorphic to an induced subgraph of $G \mid f^{-1}(\{\xi\})$.

P. Komjàth proved that for all $3 \leq n \leq \aleph_0$

$\forall H \ \forall \kappa \ K_n \not\subset H \Rightarrow \exists G, K_n \not\subset G \wedge G \rightarrowtail (H)^1_\kappa$ .

This was recently extended by the author and Komjath for arbitrary n. However , the cardinality of $G$ in general is larger than that of $H$ . Is it true that for all countable $H$ not containing a $K_3$ there is a countable $G$ not containing a $K_3$ such that

$$G \rightarrowtail (H)^1_2 \ ?$$

Problem 2 . Let $G_i = (V_i, E_i)$ be graphs for $i < 2$. $G_0 \times G_1 = (V_0 \times V_1, E_0 * E_1)$ where $\{(x_0,x_1)\ (y_0,y_1)\} \in E_0 * E_1 \Leftrightarrow \{x_0,y_0\} \in E_0 \ \wedge \ \{x_1,y_1\} \in E_1$.

The author proved in a forthcoming volume of Combinatorica that there exist graphs $G_i : i<2$ with $\mathrm{Chr}(G_i) = \aleph_1$ for $i<2$ such that

$$\mathrm{Chr}(G_0 \times G_1) = \aleph_0.$$

L. Soukup proved that it is consistent with ZFC + GCH that $\exists G_i \ i<2$ with $\mathrm{Chr}(G_0 \times G_1) = \aleph_0 \wedge \mathrm{Chr}(G_i) = \aleph_2$.

Is there a natural bound in ZFC for $\mathrm{Chr}(G_i)$ if we know that $\mathrm{Chr}(G_0 \times G_1) = \aleph_0$ ?

## B. VOIGT.

Let $[\omega]^{\omega}$ be the set of all strictly increasing maps $f : \omega \to \omega$ (i.e. the set of all infinite subsets of $\omega$ ). It becomes a polish space with the metric defined by $d(f,g) = \frac{1}{i+1}$ where $i = \text{Min}\ \{j/j < \omega$ and $f(i) \neq g(i)\}$ (This gives the usual Tychonoff topology).

Problem : Is it true that every set $\mathfrak{B} \subseteq [\omega]^{\omega}$ having the property of Baire in the restricted sense (i.e. for all $\mathfrak{A} \subseteq [\omega]^{\omega}$ the intersection $\mathfrak{A} \cap \mathfrak{B}$ is Baire w.r.t. $\mathfrak{A}$) is Ramsey ? The set $\mathfrak{B}$ is Ramsey means that there is $f \in [\omega]^{\omega}$ such that either $f.[\omega]^{\omega} \subseteq B$ (i.e. all infinite subsets of f belongs to B).or $f.[\omega]^{\omega} \cap B = \emptyset$ (i.e. non infinite subset of f belongs to B)

Motivation.

(1) The answer is YES for B being analytic.

(2) The answer is NO for B being Baire (in general).

(3) I cannot think of any counterexample.

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