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ON LARGE CHROMATIC GRAPHS NOT CONTAINING PRESCRIBED SUBGRAPHS

by A. HAJNAL

Abstract

This is a joint work of the author, I. JUHASZ and S. SHELAH.

The following statements are consistent with ZFC+GCH relative to the existence of a supercompact cardinal.

(A) $\exists S \subset \aleph_{\omega+1}\ \exists (A_\alpha : \alpha \in S)$ such that

(1) S is stationary in $\aleph_{\omega+1} \wedge \forall \alpha \in S \quad \mathrm{cf}(\alpha) = \omega_1$

(2) $\forall \alpha \in S \quad \mathrm{tp}\ A_\alpha = \omega_1 \wedge \bigcup A_\alpha = \alpha$

(3) $\forall \alpha \neq \beta \in S \ |A_\alpha \cap A_\beta| < \aleph_0$

(B) There exists a family $\mathcal{F} \subset [\aleph_{\omega+1}]^{\omega_1}$ such that for $F \neq F' \in \mathcal{F}$ $|F \cap F'| < \aleph_0$ and $\mathcal{F}$ does not possess property B.

(C) There existe a T_1 space X of cardinality $\aleph_{\omega+1}$ such that the family of all closed Cantor subsets of X does not possess property B.

(D) There exists a graph $\mathcal{G} = (\aleph_{\omega+1}, E)$ of chromatic number $\aleph_2$ such that $K_{\aleph_0,\aleph_0} \not\subseteq \mathcal{G}$.

It is known that $\square_{\aleph_\omega}$ implies the negations of these statements. (A), (C) and (D) answer questions stated in the Erdös-Hajnal problem list. (B), (C), and (D) follow from (A) using combinatorial arguments.

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