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# A Note on Gettier's Problem

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As is well known Gettier's problem is the question whether justified true belief is knowledge. In his short and clear article [Gettier 1963] he tries to show with two rather artificial counterexamples that the answer is negative. This note is to show first that there are a lot of non-artificial genuine examples in the history of science which show clearly that (well) justified true belief is not a sufficient condition for knowledge. And secondly that one type of argumentation Gettier uses leads when applied in other areas to paradoxical situations and the other is not a counterexample. Thus though Gettier is right in giving a negative answer to the problem his (artificial) examples can hardly be accepted as real counterexamples.

## 1. Gettier's Claim

In the mentioned essay Gettier claims that the following three conditions

(i) p is true

(ii) S believes that p

(iii) S is justified in believing that p

are (together) not a sufficient condition for S knows that p. According to him instead of (ii) one may also substitute 'S accepts p' or 'S is sure that p is true' and instead of (iii) one may substitute 'S has adequate evidence for p' and 'S has the right to be sure that p'.

In addition Gettier states the following two points :

(iv) It is possible for a person to be justified in believing a proposition that is in fact false.

(v) For any proposition p, if S is justified in believing p and p entails q, and S deduces q from p and accepts q as a result of this deduction, then S is justified in believing q.

With the help of (iv) and (v) Gettier constructs two rather artificial counterexamples to show that (i), (ii) and (iii) together are not a sufficient condition of knowledge.

## 2. Examples

The examples below are examples for scientifically justified strong (true) belief. This kind of belief is characterized by the condition that if someone believes something, then he does not yet know it and if he knows it, he does not (or need not any more) believe it. I shall call this kind of belief G-belief. There is a second kind of belief which is weaker and is characterized by the condition, that if someone knows something, he also believes it, but if he does not believe it, he also does not know it. This second kind may be translated also by « holding it to be true ». This kind of belief I shall call B-belief. Since someone who knows something holds it to be true, knowledge implies B-belief. But if someone believes something strongly and with justification then he also holds it to be true. Therefore also G-belief implies B-belief.[1]

2.1 Before the proof of the independence of the Continuum Hypothesis (from the axioms of set theory) was given, v. Neumann believed (but didn't know), that the Continuum Hypothesis is independent. After Gödel proved the first part [Gödel 1940], i.e. that the General Continuum Hypothesis (GCH) can be consistently added to the axioms of Neumann-Bernays-Gödel-Set Theory (even if very strong axioms of infinity are used), v. Neumann wrote :

> « Two surmised theorems of set theory, or rather two principles, the so-called 'Principle of Choice' and the so-called 'Continuum Hypothesis' resisted for about 50 years all attempts of demonstration. Gödel proved that neither of the two can be disproved with mathematical means. For one of them we know that it can not be proved either, for the other the same seems likely, although it does not seem likely, that a lesser man than Gödel will be able to prove this »[2]

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1 Elsewhere I have given a consistent system of these notions of belief together with a strong notion of knowledge and a very weak notion of assumption. Cf. [Weingartner 1981]. The present note however is independent of this system and the definitions and theorems there.

2 [Neumann 1951], in [Bulloff / Holyoke / Hahn 1969]. « The Tribute to Dr. Gödel » from which the passage is cited was given by v. Neumann in March 1951 on the occasion of the presentation of the Albert Einstein Award to Gödel. It appeared in print in the volume *Foundations of Mathematics* (ed.

The belief of von Neumann that GCH was independent of the axioms of Set Theory is a good example of strong belief which was scientifically justified. Even if there was already some justification before Gödel's proof of the first part (consistency with the axioms of Set Theory) there was a more severe justification[3] after Gödel's proof of 1939 (which appeared 1940).

But after the proof of the second part — that also the negation of GCH can be consistently added to the axioms of Set Theory (it holds for both systems, that of Zermelo-Fraenkel and that of Neumann-Bernays-Gödel) — was given by Paul Cohen in 1963 [Cohen 1963/64] von Neumann didn't any more believe it, but knew that GCH was independent (from the axioms of Set Theory). The proof of Cohen showed that it was true that GCH is independent. After the proof was given justified belief was replaced by knowledge. In general all correct mathematical conjectures are examples for Gettier's point : that justified — even very well scientifically justified — belief which later is proved to be true is not a sufficient condition for knowledge. Without proof no mathematician would claim to have knowledge. The most recent and celebrated example is Fermat's conjecture that there are no solutions for $x^n + y^n = z^n$ where $n > 2$. Even Fermat might have had justified belief in it, but certainly many mathematicians who had proofs for some integers > 2 like Euler had justified belief not to speak of the many mathematicians of the 20th century who contributed important preconditional results for the final proof which was carried through in a second attempt by Wiles in autumn 1994.

2.2 Poincaré wrote two important essays in 1905 ; both had the title : « Sur la dynamique de l'électron »[4]. One of his important beliefs which he defended there but for which he had no real proof in a theory (but considerable evidence from a greater theoretical context) was this :

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Bulloff et al.), a collection of papers given at a symposium commemorating the sixtieth birthday of Kurt Gödel.

[3] It should be mentioned that « justification » is always to be understood in a relative and partial sense here, not in an absolute sense of something being finally justified.

[4] The first appeared in 1905 in *C. R. Ac. Sci.* Paris 140, page 1504ff. The second in 1906 in *Rend. Circ. Math. Palermo* 21, page 129ff.

> « It seems that this impossibility of demonstrating absolute motion is a general law of nature. »[5]

In these papers Poincaré modified and completed the Lorentz-Transformations. This shows as is well known that Poincaré simultaneously with Einstein understood some essential features of Special Relativity without breaking through to the theory. Thus this is a good example of a true justified belief which could not be called knowledge.

2.3 The General Theory of Relativity (completed by Einstein in 1915), made three important predictions : (a) The perihelion of Mercury, (b) the deviation of light rays which pass close to big masses and (c) the red-shift of the light reaching us from distant stars.[6] The first (a) was known as an effect (not explainable with Newton's Theory) before Einstein's Theory was created. And so the prediction (and explanation) because of stronger gravitation since Mercury is much closer to the sun by General Relativity was a success immediately. In this case Einstein knew the positive result of the test of his theory. In the case of (b) and (c) he strongly believed, that they are correct and there will be a positive test possible too. In 1919 came the first confirmation of the prediction (b), the deviation of light rays : a British expedition of astronomers observed a total eclipse of the sun in Africa and confirmed the effect, that light rays from a star which run very close to the sun are deviated towards the sun (in general towards great masses). Later better and more exact confirmations of (b) were obtained.[7]

In 1922 the Russian meteorologist Alexander Friedmann believed (but did not know and predicted (on the basis of Einstein's picture of dynamic space)), that the entire universe is in dynamic change. In 1929 the American astronomer Edwin Hubble confirmed this prediction. He found out, that the light reaching us from distant stars, is shifted towards red of the spectrum (red-shift) and that this red-shift is proportional to the distance of the emitting star(s)

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5 Cited in [Pais 1982], page 129.

6 With the development of experimental techniques (in the second half of the 20th century) a number of further experimental tests posed by General Relativity could be performed with significant results. Cf. [Shapiro 1980].

7 For details see [Pais 1982], 272 ff. and [Wilkinson 1980].

(galaxy). This red-shift has a simple explanation as the Doppler-Effect caused by the recession of distant stars (galaxies) away from us. This was the confirmation of (c) which was later again confirmed many times. Thus after these positive results of testing the predictions Einstein knew, that predictions (b) and (c) are correct and are positively confirmed by tests. And this means also, that he did not and needed not believe (G-believe) that anymore, since there is sufficient justification to say that he knows now.[8]

The history of natural sciences is full of examples which prove the negative answer to Gettier's problem. Another interesting case is gravitation. That any two masses are attracted by each other was claimed and strongly believed by Newton in his theory. It was scientifically justified belief in the best sense. But only 111 years later Cavendish could prove it with a famous experiment.

In fact it is rather strange that Gettier himself and many others who commented on his problem since 1963 did not discover these areas (of mathematics and natural science) where genuine examples lie at hand.

2.4 There are also famous examples as instances of Gettier's claim (iv) in the history of science. One well known case is that of Frege's belief in his axiom V of his *Grundgesetze der Arithmetik.* That this was a strong and scientifically justified belief is clear from what he says in his *Introduction* (of volume I) and in his *Appendix* (to volume II) after he got the letter from Bertrand Russell showing him a contradictory consequence of his axiom :

> « A dispute can arise, so far as I can see, only with regard to my Basic Law concerning courses-of-values (V), which logicians perhaps have not yet expressly enunciated, and yet is what people have in mind, for example, where they speak of the extensions of concepts. I hold that it is a law of pure logic. In any event the place is pointed out where the decision must be made »[9]
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> « Hardly anything more unwelcome can befall a scientific writer than that one of the foundations of his edifice be shaken after the work is finished. I have been placed in this position by a letter of Mr. Bertrand

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8 For recent tests see [Shapiro 1980]. For historical facts see [Pais 1982], 180 and 196.

9 [Frege 1967]. Introduction, pages 3–4.

> Russell just as the printing of this [second] volume was nearing completion. It is a matter of my Basic Law (V). I have never concealed from myself its lack of the self-evidence which the others possess, and which must properly be demanded of a law of logic, and in fact I pointed out this weakness in the Introduction to the first volume (pages 3-4, above). I should gladly have relinquished this foundation if I had known of any substitute for it. And even now I do not see how arithmetic can be scientifically founded, how numbers can be conceived as logical objects and brought under study, unless we are allowed — at least conditionally — the transition from a concept to its extension. »[10]

## 3 Gettier's « Counterexamples »

### 3.1 Valid but irrelevant arguments

The second counterexample (case II) used by Gettier is an argument which contains a typical irrelevant move or an irrelevant principle. The argument used by Gettier is : Jones owns a Ford ; therefore : Either Jones owns a Ford, or Brown is in Boston (and similar with the other alternatives : or Brown is in Barcelona ; or Brown is in Brest-Litovsk). The general form of this argument is : p, therefore : p v q (or therefore : p v r ; p v s). This is the logically valid principle of addition. I call this valid form an irrelevant one because one can replace a part of the conclusion (q) by any proposition whatsoever *salva validitate*. More accurately : a valid argument $A \vdash \alpha$ is irrelevant if there is some predicate-letter or propositional variable in $\alpha$ which can be replaced at some of its occurrences by an arbitrary predicate (of same arity) or an arbitrary propositional variable *salva validitate*. One can show that many paradoxes in different areas like the theory of confirmation, explanation, law statements, disposition predicates, verisimilitude, epistemic and deontic logic ... etc. are caused by irrelevant arguments in the above sense. Specifically the irrelevant principle of addition is the culprit for Hesse's paradox of confirmation, for Goodman's paradox for the Ross-paradox and the paradox of Free-Choice in Deontic Logic.[11]

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[10] [Frege 1967], page 127 (Appendix of Vol. II).

[11] This was shown in Weingartner-Schurz (1986) page 14 f., 29 and 35 respectively. The relevance criteria applied in this essay have been replaced

Since the argument in Gettier's second counterexample (case II) uses essentially the principle of addition and since it can be shown that the principle of addition applied to other areas leads to well known paradoxes the (artificial) counterexample of case II cannot be accepted as a serious counterexample. In fact neither scientific nor everyday language discourse uses the principle of addition in the sense that q is completely arbitrary and independent of the premise. In general I want to emphasize that scientists if they speak of consequences of scientific theories, do have in mind something much more restricted than that what (classical) logic permits to be an element of the consequence class. Philosophers of science, on the other hand, when they describe what scientists do — explaining and confirming hypothesis, establishing laws etc. — allow all the consequences which logic permits. My thesis is that this is the main reason for most of the well known paradoxes in the theory of explanation, confirmation, law, statements, disposition predicates etc. discussed by philosophers of science and logicians.[12]

3.2 Case I is not a counterexample

The first « counterexample » (case I) used by Gettier is an argument which uses a kind of virtual belief (i.e. a belief which is only a part of Smith's belief and not really his belief). Therefore this argument is not a counterexample. This can be shown in the following way : Smith has strong evidence (with justification) for proposition (d) : Jones is the man who will get the job [$G_j$] and Jones has ten coins in his pocket [$C_j$]. Gettier assumes that $G_j$ is false and (agreeable, though he does not mention it) that only a single person can set the job. Smith infers from (d) (e) : The man who will get the job has ten coins in his pocket [$C(\iota x\, Gx)$]. Gettier claims that Smith believes that (e) and is justified in believing that (e). This is a half-truth. Why? In the inference form (d) to (e) there is a loss of information, i.e. what is lost is $G_j$. Or in the inference form $G_j \wedge C_j$ to $C(\iota x\, Gx)$ the information $\iota x\, Gx = j$ is lost. But clearly when Smith believes in (e) he believes in (e) $\wedge\, G_j$ or (e) $\wedge\, \iota x\, Gx = j$ (because he

by a more powerful criterion later. Cf. [Schurz-Weingartner 1987], page 54 and [Weingartner 1993] and [Weingartner 1994].

[12] I have substantiated this theses in detail elsewhere. Cf. note 11 and [Weingartner 1988].

believes in (e) in virtue of believing in (d)). Thus in fact Smith has a false belief ; i.e. Smith does not have justified true belief, since he in fact believes in $(e) \wedge G_j$ (or : $(e) \wedge \iota x\, Gx = j$) which is not true. Therefore since Smith's belief is not true case I is not a counterexample.

3.3 Gettier's inference rule for beliefs is problematic. Although I think that rule (v) (chapter 1, this essay) is applicable and correct in most cases there are examples which may suggest some further restriction. Frege strongly believed in his fifth axiom of his *Grundgesetze der Arithmetik* (see 2.4). After Russell showed him that a contradiction follows from it rule (v) forces us to say either that Frege gave up his belief in the axiom or believed in a contradiction. Though we will deny the latter it is hard to say that Frege gave up his belief in his axiom of comprehension because he thought it is the only way of building up mathematics and tried to find a « way out ».[13]

In general it seems to me that rule (v) leads to difficulties in situations in science where we still believe in a theory or hypothesis — because it has high explanatory power — although it has some false or even paradoxical consequences which we are not able to explain so far.

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13 Cf. the cited passage in chapter 2.3 above from the appendix. Concerning Frege's way out in his appendix Quine showed that it wasn't really a way out. Cf. [Quine 1955].

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