PHILOSOPHIA SCIENTIÆ

I. M. JAMES

Reflections on the "History of topology"

Philosophia Scientiæ, tome 2, nº 3 (1997), p. 41-49
<http://www.numdam.org/item?id=PHSC_1997__2_3_41_0>

Reflections on the 'History of Topology'

I.M. James
Mathematical Institute
University of Oxford

Philosophia Scientiae, 2 (3), 1997, 41-49

Abstract. The history of topology is a relatively inexplored part of the history of mathematics. I am editor of a volume of studies of the subject which will be published in the summer of 1998. In this lecture, I describe the background to this project.

Résumé. L'histoire de la topologie est une partie relativement inexplorée de l'histoire des mathématiques. Je suis rédacteur en chef d'un volume d'études sur le sujet qui sera publié en été 1998. Dans cette conférence, je présente un exposé du contexte de ce projet.

Not very long ago I was asked by my publisher whether I would like to compile a book on the history of topology, the branch of mathematics I know best. Until then I had not thought of undertaking any studies of a historical nature, although I had always been interested in the history of mathematics in a general way. Although very conscious of my own inadequacies to undertake such a task I was sufficiently attracted by the proposal to respond positively. A year and a half later the project is going well, and should be complete by the middle of 1998. The observations I would like to share with you today have arisen in the course of this work, which will be the source of most of my illustrations, but I hope that what I am going to say will have some bearing on other similar projects in the history of mathematics and in the history of science generally.

I should like to begin by describing the way in which I have approached the task of compiling a history of topology. As far as I could discover, little or no space is devoted to topology in the standard histories of mathematics. Some important monographs and substantial articles have been published on various aspects of the subject, such as the development of particular concepts, for example homology. There are also overviews by Jean Dieudonné [Dieudonné 1994] and Guy Hirsch [Hirsch 1978]. However in many ways I found myself in the position of a pioneer, trying to compile a full-scale history of topology for the first time.

I decided at an early stage that it would be best to concentrate on classical topology, meaning algebraic, differential and geometric topology, because I was aware that a multi-volume history of point-set (or general) topology was already under way. In fact that other kind of topology is a comparatively recent development with a rather different culture so that the separation of the two projects may be no bad thing.

Naturally the first thing I had to do was to get to know the existing literature. Oxford, my own university, has a superb library for some purposes but unfortunately there are serious defects in its holdings of material which might be useful for the history of mathematics, only partially met in other libraries in my country.

Probably every library has its defects but by spending time at a number of good libraries in different countries I was able to find most of what I needed. My office is now full of copies of practically all the published material which I thought might be relevant, and which I was unable to find in the Oxford libraries.

However I was conscious that I might be missing important material either because it had not been published in any of the ordinary literature or because it had not been published at all. What treasures might there not be among the multitude of scientific journals published in the nineteenth century or among theses tucked away in university libraries? What dusty bundles of papers might be waiting for someone to go through them? Ideally I would have liked to seek out this kind of material, but that would have taken years.

The next step was to try and identify topics for the planned volume, and here it was obviously necessary to consult others. After doing so I ended up with about fifty different topics for articles. Following a further round of consultations I had identified two or three possible authors for each topic, and then began the slow process of signing up the contributors. Of course another way to have proceeded would have been to have identified potential contributors first and offer them a fairly a free hand as to what to write about. I understand that is the procedure adopted by Charles Aull and Robert Löwen, the editors of the projected history of general topology. I still have one or two orphan topics but now the list of contributors is essentially complete. There is a good international spread, balanced fairly evenly between professional historians and regular mathematicians with historical interests.

Right at the start I had given much thought to the kind of readership for which the volume should be designed. Most mathematicians are familiar with the Mathematical Intelligencer, which often contains articles of historical interest. It seemed to me that this gave an idea of what to aim at, a more general readership than that which is enjoyed by a specialist journal like *Historia Mathematica*, and I tried to convey this to my contributors.

For the historians this meant suggesting that they might forego, on this occasion, the full scholarly apparatus of footnotes etc., while to the regular mathematicians I have tried to suggest that they would need to apply the same standards of care to the historical record as they would in their normal mathematical writing. There is always a danger, in a compilation of articles on many different topics by many different people, that the volume will turn out to be too much of a miscellany. Without attempting to lay down detailed guidance I offered to suggest to contributors one or two articles in the literature which they might wish to consider taking as a model.

Of course the question arose as to the period to be covered. I could see no advantage in laying down any hard and fast rule. To end with the year 1950, say, might be appropriate for some topics but would be quite unsuitable for others. Topology is very largely a twentieth century creation. After the end of the second world war there was an explosive development of research activity, which still continues. It could be argued that the time is not yet ripe for a historical treatment of such material, but I prefer to say that a different kind of treatment may be required for the more recent period, making greater use of first-hand knowledge.

Last year, as it happens, I took part in an interesting conference at Nice on the development of mathematics in the twentieth century. A particular feature of this well-attended meeting was that it was largely devoted to first-hand accounts by those who had been directly involved. Such accounts of significant events can be invaluable. I would like illustrate this point with some quotations from the fascinating biographical memoir of Poul Heegard which Ellen and Hans Munkholm have written for my book:

> When we started our investigation of Heegard's life and career, it was easy enough to locate his mathematical publications, but we found only a few accounts of his life in general. In particular we could locate only one obituary. We then searched the Internet for persons carrying the name of Heegard. This led us to contact a number of e-mail addresses in Norway, Denmark, USA, Sweden and Switzerland. A few of the persons we reached in this way knew that they were related to 'our' Heegard. Among these was Poul E. Heegard, a Ph.D. student of computer science at Trondheim University, Norway, and a great grandson of Poul Heegard. He gave us the very welcome news that Poul Heegard had actually left roughly 130 pages of handwritten notes, and he generously supplied us with a copy. The notes were written in 1945 (in Norwegian) when Heegard was 73 years old and they were meant as a family history told to his children and grandchildren, but they do contain a lot of information which is relevant to our study.

Unfortunately a few pages are missing at two critical points in Heegard's life, first at the time of his resignation from the University of Copenhagen, and secondly right at the end, on the day of the German invasion of Norway. However quite apart from their human interest the notes give valuable insights into the relations between pioneers in the development of combinatorial topology at a crucial period. For example it is interesting to read that when he was asked to report on Analysis Situs in the *Enzyklopädie der Mathematischen Wissenschaften*, Heegard accepted, and

> started the work with great pleasure and...finished an outline and bibliography. However, it was difficult to get the time and quiet needed to work out the theoretical introduction [Heegard was teaching eight hours a day, six days a week]. Moreover, quite senselessly, I let myself be influenced by a number of malicious comments on my work in topology. Therefore I asked Franz Meyer [the *Enzyklopädie* editor] for an assistant. It was then arranged that I should write the article with the young German mathematician Max Dehn, Dr. from Göttingen.

Heegard goes on to describe how, in the summer of 1905, he went down to Kiel, where Dehn was *Privatdozent*, to work with him and comments "I now initiated him into my viewpoints and he began to work on the general introduction, which he finished beautifully during the next winter." Here, as the Munkholms point out, Heegard seems to think of Dehn as a junior author, but in the article itself we read that "Of the two authors, Heegard did the preliminary literature studies, and also took an essential part in the work. Responsibility for the final form of the article is Dehn's." Moreover, as the Munkholms go on to observe, a different version can be found in the Heegard obituary [Johansson 1948] which has Heegard and Dehn discussing foundational problems in topology in a train when:

> Dehn believed that one should postulate just enough to let the topological essence stand out clearly, something which had never been done before. Here, in the railroad compartment, combinatorial topology was created. Heegard was enthusiastic, and proposed that they would write the article jointly.

Let me give another example where a first-hand account exists of a milestone in the development of the subject. At a meeting of the Moscow Mathematical Society on 5 September 1935 Aleksandrov gave a Memorial Address for Emmy Noether which, in translation [Noether 1983] by Neil and Ann Koblitz, included the following passage:

> In the summers of 1926 and 1927 she went to the courses on topology which Hopf and I gave at Göttingen. She rapidly became oriented in a field which was completely new to her, and she continually made observations, both deep and subtle. When in the course of our lectures she first became aquainted with a systematic construction of combinatorial topology, she immediately observed that it would be worthwhile to study directly the groups of algebraic complexes and cycles of a given polyhedron and the subgroup of the cycle group consisting of cycles homologous to zero. This observation now seems self-evident. But in those years (1925-1928) this was a completely new point of view, which did not immediately encounter a sympathetic response on the part of many authoritative

> topologists. Hopf and I immediately adopted Emmy Noether's view in this matter, but for some time we were among the small number of mathematicians who shared this viewpoint. These days it would never occur to anyone to construct combinatorial topology in any way other than through the theory of abelian groups; it is thus all the more fitting that it was Emmy Noether who first had the idea of such a construction. At the same time she noticed how simple and transparent the proof of the Euler-Poincaré formula becomes if one makes systematic use of the concept of Betti group. Her remarks in this connection inspired Hopf completely to rework his original proof of the well-known fixed point formula, discovered by Lefschetz in the case of manifolds and generalized by Hopf to the case of arbitrary polyhedra. Hopf's work *Eine Verallgemeinerung der Euler-Poincaréschen Formel*, published in Göttinger Nachrichten in 1928, bears the imprint of these remarks of Emmy Noether.

Again we observe that material of this kind must not be accepted uncritically, since Aleksandrov gives a different version in his autobiography [Aleksandrov 1979 and 1980], ably translated by Ann Dowker, where we read:

> In the middle of December (1925) Emmy Noether came to spend a month in Blaricum [the village near Amsterdam where Brouwer lived]. This was a brilliant addition to the group of mathematicians around Brouwer. I remember a dinner at Brouwer's in her honour during which she explained the definition of the Betti groups of complexes, which spread around quickly and completely transformed the whole of topology.

In neither version does Aleksandrov give any credit to Leopold Vietoris, who may well have invented homology groups simultaneously or even earlier.

In the present audience I am sure I do not need to emphasize the importance of preserving the correspondence and other papers of important scientists. It cannot safely be left to family or colleagues. In my country there is a unit, at the University of Bath, which catalogues such material and ensures it is deposited somewhere where future scholars can find it, for example a university library. An archive of such material for mathematics, at least, has been started at the University of Texas at Austin, which rather specializes in that kind of thing. Last year I paid a visit to the archives of the Rockefeller Foundation, not far from New York, which contains much interesting material and is used as a deposit by other Foundations as well. I was particularly looking for papers about people such as Aleksandrov, Hopf and Hurewicz who held Fellowships at various times between the wars which enabled them to travel from Moscow, Berlin or Vienna, as the case may be, and work in the Netherlands with Brouwer. However the task of first tracking down, and then working through, such material is laborious even at a proper archival centre, with the help of professional staff.

To have included even short biographies of all the topologists mentioned by the contributors to my book would have been impracticable. To try and select the most important would have seemed presumptuous. After careful consideration I decided, reluctantly, to exclude living persons. Even then the numbers would have been too great so I made a selection of around thirty who are certainly not minor figures and whose lives are not only interesting in themselves but in some way illustrative of the circumstances of the time. I have permitted myself to regard someone as a topologist, for this purpose, if topology was one of their main mathematical interests at some stage in their lives even if it was not the only one. After much heart-searching I ended up with the following list:

Adams, Alexander, Betti, Borsuk, Brouwer, Cech, Dehn, Dowker, Ehresmann, Freudenthal, Heegard, Hopf, Hurewicz, Jordan, Lefschetz, Listing, Möbius, Morse, Newman, Nielsen, Poincaré, Reidemeister, de Rham, Riemann, Seifert, Steenrod, Tietze, Whitehead, Whitney, Wirtinger.

In some cases it was possible to arrange for a fresh biography to be written, especially if new material has come to light. In others I have written one myself, based on what has already been published but supplemented, wherever possible, by further information from those who knew the person concerned. In addition there will be separate articles about the Japanese and Russian schools. Another editor might well have chosen differently; indeed I am not sure that the above list will be my final one.

Before starting to write about the lives of individual mathematicians it is necessary to know something about the times in which they lived. We need to understand the impact of major historical events. We also need to understand the way that scientific education and research was organized, for example in the German-speaking area of Europe in the nineteenth century. Moreover each of the major centres of excellence merits a careful study.

What I am saying here is, I hope, consistent with the ideas expressed in Thomas Kuhn's influential essay [Kuhn 1970], in which he introduced the concept of 'paradigm', in relation to the history of science. As I understand it the historians now focus more on the larger communities that produce and communicate knowledge than they did earlier. Why, for example, do particular institutions achieve excellence in particular areas of science, usually for a limited period? In the case of topology, there was a golden age, approximately the first thirty years of this century. at the University of Vienna, and another one slightly later at Princeton. It would be interesting to compare these, and to try and understand the reasons why they occurred.

However the study of particular institutions is not the only approach. The social scientists have adopted the notion of 'invisible college', which offers an alternative way to describe how academic schools of research develop and operate. For example some of them made a study of the network of people devoted to research into the theory of finite groups. They found, not surprisingly, that the great majority could be arranged in a small number of inter-related 'family trees', the academic descendants of certain individuals who were the pioneers in this particular discipline. One of the major trees in topology is that headed by J.H.C. Whitehead, which numbers over a hundred individuals. Whitehead himself was most influenced by Alexander and Veblen, ultimately by Poincaré. However everyone was influenced by Poincaré, so with him the concept rather breaks down.

The professional historians of mathematics I have consulted have been most friendly and helpful in offering advice and encouragement and in responding to requests for information. Everything I have said today will no doubt be familiar to them, but by coming into the historical field at this stage in my career without having served an apprenticeship I have had to follow a steep learning curve, indeed the process is still continuing. They would agree, I know, that there is a problem in the lack of communication between the professional historians on the one hand and the regular mathematicians on the other. The problem is more acute in some countries than others. Is it anything to do with the notion that when you get too old to do scientific research you can always start to take an interest in the history of your subject? Could more be done to interest the ordinary working mathematician in the work of the historians?

The problem is to raise the level of consciousness from something close to zero. I was struck by the remark of one of my contributors, who leads a major research group at one of the top American universities, that his research students seemed uninterested in even the greatest achievements in their subject of only thirty or forty years ago, except insofar as these might directly enter into their research. He referred specifically to the successful resolution of several of the problems in topology which Poincaré regarded as fundamental, largely achieved in the twenty wonderful years after the end of the second world war.

I am arranging for a conference to be held in conjunction with the publication of my volume, so that the articles in it may be discussed and topics for future research identified. Much has been written about Brouwer, for example, but hardly anything about Alexander, the leading topologist in Princeton's golden age. Why did the Viennese school lose its former glory? What did the terms 'combinatorial topology' and 'point-set topology' mean in the early years of the century? There are plenty of such topics awaiting the attention of a suitably qualified investigator.

I hope these reflections, arising from the work I have done on this project, will provoke some comments from members of the audience. However successful, a compilation of separate articles by different people cannot take the place of a book by one or two authors in which the subject is treated as a whole. Such a book is certainly needed, although it may take many years to complete. I greatly hope that before long someone may embark on this, and find the work I have done makes a useful beginning.

References

Aleksandrov, Paul

1979 Autobiography. *Russian mathematical Surveys* 34:6, 219-250.

1980 Autobiography. *Russian mathematical Surveys* 35:3, 315-358.

Dieudonné, J.A.

1994 Une Brève Histoire de la Topologie, in *Development of Mathematics 1900-1950*, Birkhauser.

Hirsch, G.

1978 Topologie, in *Abrégé d'histoire des mathématiques 1700-1900* (ed. Dieudonné), Paris : Hermann.

Johansson, I.

1948 Minnetale over Poul Heegard, Det norske Videnskabsakademi i Oslo, *Arbok*, 38-47.

Kuhn, Thomas

1970 *The Structure of Scientific Revolutions*, Univ. Chicago Press.

Noether, Emmy

1983 *Gesammelte Abhandlungen*, Springer.