

# A Generalization of Hume's Thesis<sup>1</sup>

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More specifically, I will consider the Hume thesis concerning the is/ought relation. This reservation is essential, because we have also another thesis stated by Hume, namely the principle (roughly speaking) that sets with the same cardinality have the same number of elements. Hume's thesis about the is/ought relation is expressed in the following way :

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<sup>1</sup>An extended version of this paper is forthcoming in [Woleński 2006] under the same title.

“I cannot forbear adding to those reasonings an observation, which may, perhaps be found of some importance”. In every system of morality, which I have hitherto met with, I have always remark’d that the author proceeds for some time in the ordinary way of reasoning, and establishes the being of a God, or makes observations concerning human affairs; when of a sudden I am surpris’d to find, that instead of usual copulations of propositions, *is*, and *not*, I meet with no proposition that is not connected with an *ought*, or *ought not*. This change is imperceptible; but is, however, of the last consequence. For as this *ought*, or *ought not*, expresses some new relation or affirmation, ’ts necessary that it shou’d be observed and explain’d; and at the same time that a reason should be given, for what seems altogether inconceivable, how this new relation can be a deduction from others, which are entirely different from it.” [Hume 1951, 469]

Perhaps it is important to note that the quoted fragment says nothing about the relation between norms as specific linguistic utterances and declarative sentences. In particular, Hume did not characterize ought-sentences as norms or imperatives. He only contrasted two kinds of copulations, relations or affirmations, one expressed by “is” and “is not” and another, which is expressed by “ought” and “ought not”. What Hume does in his statement, consists in pointing out that ought-sentences are not deducible (“seems altogether inconceivable how this new relation can be deduced from others, which are entirely different from it”).

A similar question was considered by Poincaré in his essay about morality and science.<sup>2</sup> According to Poincaré, a correct logical inference with an imperative as its conclusion requires an imperative premise.<sup>3</sup> However, this metalogical rule is defined for a language in which occurs imperatives and declarative sentences, not only the latter. Thus, there is an essential difference between Hume and Poincaré in their approaches to the is/ought problem. I will take Hume’s pattern. My aim is to generalize the Hume’s thesis for a broad class of modal sentences, that is the variety which includes at least deontic, epistemic and interrogative modalities.

I start with deontic modalities for which the Hume thesis was originally formulated. As usually, by deontic sentences I understand instantiations of the following schematic formulas (*A* is an arbitrary non-deontic sentential expression) : *OA* — it is obligatory that *A*; *FA* — is prohi-

<sup>2</sup>See [Poincaré 1910], chapter 8.

<sup>3</sup>This principle was implicitly adopted by Aristotle in his treatment of normative syllogisms as reconstructed [Kalinowski 1953, 163–173].

bited (forbidden that  $A$ );  $PA$  — it is permitted that  $A$ . I assume the standard deontic logic, which validates the following equivalences (as definitions) :  $FA \Leftrightarrow O\neg A$ ;  $OA \Leftrightarrow O\neg A$ ;  $PA \Leftrightarrow \neg O\neg A$ ;  $OA \Leftrightarrow \neg P\neg A$ ;  $PA \Leftrightarrow \neg FA$ ;  $FA \Leftrightarrow \neg PA$ . These dependencies generate the wellknown logical square (or square of oppositions) (D) for deontic sentences (it is isomorphic with the logical square for alethic modal sentences) (see Fig. 1 5 on page 111).

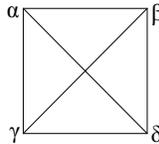


FIG. 1 – (D)

The interpretation :  $\alpha - OA$ ,  $\beta - FA$ ,  $\gamma - PA$ ,  $\delta - P\neg A$ . We have the following facts (the symbol  $\vdash$  indicates that the formula occurring after it is a logical theorem) :

- (1)  $\vdash \neg(\alpha \wedge \beta)$  (obligation and prohibition are contraries) ;
- (2)  $\vdash (\alpha \Rightarrow \gamma)$  (obligation entails permission) ;
- (3)  $\vdash (\beta \Rightarrow \delta)$  (prohibition entails permission not) ;
- (4)  $\vdash (\alpha \Leftrightarrow \neg\delta)$  (obligation and permission not are contradictories) ;
- (5)  $\vdash (\beta \Leftrightarrow \neg\gamma)$  (prohibition and permission are contradictories) ;
- (6)  $\vdash (\gamma \vee \delta)$  (permission and permission not are complementaries).

The Hume thesis cannot be formulated within (D), because it concerns the relation between  $A$  and  $OA$ . More precisely, the Hume's thesis asserts (\*)  $\neg \vdash (A \Rightarrow OA)$ . We should consider another diagram, namely (D1) (see Fig. 2).

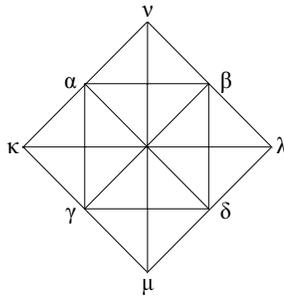


FIG. 2 – (D1)

The point  $\kappa$  is for  $A$ , the point  $\lambda$  for  $\neg A$ , the point  $\nu$  for  $\alpha \vee \beta$  (that is,  $OA \vee FA$ ; normative determination, symbolically  $DA$ ) and  $\mu$  for  $\gamma \wedge \delta$  (that is,  $PA \wedge P\neg A$ ; normative indifference; symbolically  $IA$ ). We have new theorems, namely  $\alpha \Rightarrow \nu$  (obligation entails determination);  $\beta \Rightarrow \nu$  (prohibition entail determination);  $\mu \Rightarrow \gamma$  (indifference entails permission);  $\mu \Rightarrow \delta$  (indifference entails permission not). However, (D1) does not suggest anything about validity of (\*). Moreover, we have also questions concerning the status of (a)  $\alpha \Rightarrow \kappa$ , that is,  $OA \Rightarrow A$ ; (b)  $\kappa \Rightarrow \gamma$ , that is,  $A \Rightarrow PA$ ; (c)  $\gamma \Rightarrow \kappa$ , that is,  $PA \Rightarrow A$ , and similarly, for  $\beta$ ,  $\lambda$  and  $\delta$ ; (d)  $\kappa \Rightarrow \nu$ ; (e)  $\nu \Rightarrow \kappa$ , (e)  $\kappa \Rightarrow \mu$ ; (f)  $\mu \Rightarrow \kappa$ , and similarly, for  $\lambda$  and  $\mu$ . These formulas are not proper for deontic logic, although alethic counterparts of (a) and (b) are valid.

In order to investigate the problem, one must appeal to semantics.

The best tools are provided by possible world semantics. Not entering too deeply into formal details, let us assume that we have the ordered triple (Kripke frame)  $S = \langle K, W^*, R \rangle$ , where  $K$  is a non-empty set of items called possible worlds,  $W^*$  is a distinguished element of  $K$ , usually interpreted as the real world, and  $R$  is a binary relation defined on  $K$  (the accessibility or alternativeness relation).  $S$  is a deontic frame if and only if  $R$  is not reflexive, that is, it is not generally true that  $WRW$ . In particular, we assume that not  $W^*RW^*$ . This assumption immediately excludes (a) as not a tautology of deontic logic. Now we define:  $OA$  is true in  $W^*$  if and only if  $A$  is true in every world  $W$  such that  $WRW^*$ . Intuitively, the sentence "it is obligatory that  $A$ " is true in the real world  $W^*$  if and only if  $A$  is true in every world  $W$  being a deontic alternative to  $W^*$ , that is, in the world which all obligations valid in the real world are satisfied. Accordingly, the sentence  $PA$  is true in  $W^*$  if and only if there is a world  $W$  such that  $WRW^*$  and  $A$  is true in  $W$ .<sup>4</sup> These intuitive constraints exclude (\*), (b) – (f) as deontic tautologies.<sup>5</sup> In the light of this analysis the Hume thesis can be generalized to the statement (the letter  $N$  denotes one of the symbols  $O, F, P, P\lrcorner$ ; moreover, if  $N$  is  $F$  or  $P\lrcorner$ , then  $A$  is to be replaced by  $\lrcorner A$  outside of the given deontic operator) :

- (GHT)(a)  $\lrcorner \vdash A \Rightarrow NA$ ;
- (b)  $\lrcorner \vdash NA \Rightarrow A$ .

I will call (GHTa) as the simple Hume thesis, and (GHTb) as the converse Hume thesis. The particular cases of both are summarized by the following list :

- (7)  $\lrcorner \vdash (A \Rightarrow OA)$  (the simple Hume thesis for obligation) ;
- (8)  $\lrcorner \vdash (\lrcorner A \Rightarrow FA)$  (the simple Hume thesis for prohibition) ;
- (9)  $\lrcorner \vdash (OA \Rightarrow A)$  (the converse Hume thesis for obligation) ;
- (10)  $\lrcorner \vdash (FA \Rightarrow \lrcorner A)$  (the converse Hume thesis for prohibition) ;

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<sup>4</sup>The truth-conditions for  $FA, P\lrcorner A, DA$  and  $IA$  follow immediately from the definitions for  $OA$  and  $PA$ .

<sup>5</sup>Some instances of (b) and (c) are valid. If  $A$  is a tautology,  $A \Rightarrow PA, PA \Rightarrow A$  and  $OA \Rightarrow A$  are universally true. Moreover, if something is obligatory and  $A$  is a tautology, then  $A \Rightarrow OA$  is also valid. However, (\*), (a) – (c) are not universally valid.

- prohibition);
- (11)  $\neg \vdash (A \Rightarrow PA)$  (the simple Hume thesis for permission);
- (12)  $\neg \vdash (\neg A \Rightarrow P\neg A)$  (the simple Hume thesis for permission not);
- (13)  $\neg \vdash (PA \Rightarrow A)$  (the converse Hume thesis for permission);
- (14)  $\neg \vdash (P\neg A \Rightarrow \neg A)$  (the converse Hume thesis for permission not);
- (15)  $\neg \vdash (A \Rightarrow DA)$  (the first simple Hume thesis for determination);
- (16)  $\neg \vdash (\neg A \Rightarrow DA)$  (the second simple Hume thesis for determination);
- (17)  $\neg \vdash (DA \Rightarrow A)$  (the first converse Hume thesis for determination);
- (18)  $\neg \vdash (DA \Rightarrow \neg A)$  (the second converse Hume thesis for determination);
- (19)  $\neg \vdash (A \Rightarrow IA)$  (the first simple Hume thesis for indifference);
- (20)  $\neg \vdash (\neg A \Rightarrow IA)$  (the second simple Hume thesis for indifference);
- (21)  $\neg \vdash (IA \Rightarrow A)$  (the first converse Hume thesis for indifference);
- (22)  $\neg \vdash (IA \Rightarrow \neg A)$  (the second converse Hume thesis for indifference).

It is possible to go further. If we assume normal modal logic, (a) holds for necessity, but (b) for possibility. The converse of (a) is valid only for very special modal logics, but (c) invalid everywhere. However, alethic modal logic is an exception as far as the matter concerns (a) and (b). Most modal contexts behaves like deontic sentences and satisfies (GTM). Thus, the sentence  $A$  does not entail “I believe (ask, suppose, assert, etc.) that  $A$ ”. The converse dependence holds neither. There are some dubious case. For example, according to the classical definition of knowledge, the sentence “I know that  $A$ ” entails  $A$ . On Frege’s account of assertion, its logical force demands that “I assert  $A$ ” entails  $A$ . I am inclined to think that these account are not correct. The logical entailment from “I know that  $A$ ” to  $A$  cannot be justified by definition only. It requires a

semantic basis. Of course, it is possible to give it, but the principle “if I know that  $A$ , then  $\neg A$  is possible” seems to be more plausible. Frege’s account of assertion is open to a similar criticism. If these remarks are right, (GHT) is a very general principle, which is fairly important for many philosophical issues. Let me mention only that this thesis very strongly challenges naturalism in epistemology.

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