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On the construction of factorial designs using abelian group theory

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1. - Introduction

In many experimental situations the investigator would like to examine the effects of many variables (FACTORS) simultaneously and estimate the way they interact with one another. For example the factors may be drugs, fertilizers, pressure and temperature, chemical reactants etc.

Assume tath for each of them several different modalities or degrees of intensity (LEVELS) are possible, like for instance different concentrations of a drug, or degrees of temperature etc. In particular a factor will always have at least two levels : present-absent. FACTORIAL EXPERIMENTS are those in which factors are examined together, as opposed to one at a time. This allows greater precision and enables us to test their independence and estimate their interactions (see for instance [10] and [11]) If $A_1, A_2, \dots, A_m$ denote the factors and s_i (i $= 1, 2, \dots, m$) is the number of levels of factor A_i, then we speak of an $s_1 \times s_2 \times \dots \times s_m$-experiment.

The problem of designing, i.e. planning, a factorial experiment is that of assigning TREATMENTS, i.e. combinations of factors each at a given level, to experimental units in a way that will make it possible, once the experiment is carried out, to draw conclusions from the data with maximum precision. The following difficulties may arise :

a) The total number of treatments $(= s_1 s_2 \dots s_m)$ is too high, taking into account the need to replicate the experiment ;

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b) the experimental units (PLOTS) are not homogeneous, due to the presence of so-called SUB-EXPERIMENTAL FACTORS i. e. environmental variables, like fertility of the soil, humidity of the air, which may interfere with the outcome of the experiment but are beyond the investigator's control. Usually it is assumed that experimental factors and sub-experimental ones do not interact.

To overcome problems *a*) and *b*) we must sacrifice some of the information on the less interesting effects and/or interactions through the well known techniques of ALIASING and CONFOUNDING.

An extensive literature deals with FACTORIAL DESIGNS for 2^m-experiments, 3^m-experiments and the so-called mixed $2^m \times 3^n$-experiments. If $s_1 = s_2 = \dots = s_m =$ a prime power, a well-known method of constructing aliased and confounded designs exists, based on finite affine and projective geometries. (See [1] and [2]) .

Recently, more general constructions have been given, based on abelian groups, which apply to *all* types of factorial experiments see [4],[5].

The purpose of this paper is to modify the ideas and methods of [3], [4], [5] and [6] to account for the use of pseudofactors, showing that greater flexibility in the construction and — in some cases — new results can be obtained in this way. The language and symbols employed will be those of [7] and [8] which make wide use of duality and bilinear forms in modules. For this reason some theorems of finite abelian group theory are briefly recalled in 2. Familiarity with the terminology of experimental designs will be assumed.

The philosophy underlying the paper is to emphasize the rôle of abstract algebra in translating experimental problems into rigorous scientific language : the theory of groups provides not only greater mathematical elegance and economy of thought, but also construction methods that are very general indeed.

2. – Notation

In the sequel we shall use the following notation and well-known results.

Let $G_i\,(+)$ $(i = 1, \dots, m)$ be cyclic groups of finite order s_i. Consider the group

$$G = G_1 \oplus G_2 \oplus \dots \oplus G_m .$$

Then

$$|G| = s_1 \, s_2 \dots s_m .$$

Let γ_t be a common multiple of $s_1, \dots, s_m$. Then each G_i and G can be regarded as Z_{γ_t}-modules. Every subgroup of G is a Z_{γ_t}-submodule of G. To make our computations simpler we can imagine that G_i is embedded in Z_{γ_t}, by taking

$$G_i = \{0\,, \gamma_t/s_i\,, 2\gamma_t/s_i\,, \dots\,, (s_i - 1)\,\gamma_t/s_i\}$$

where all the integers will be taken mod. γ_t.

An element of G will be denoted by an m-tuple (or row vector)

$$\underline{x} = (x_1\,, \dots\,, x_m) \qquad x_i \in G_i\,.$$

Assume $[\cdot,\cdot]$ is a symmetric Z_{γ_t}-bilinear form defined over G :

$[.\,,.]$ is said to be non-degenerate if $[\underline{x}\,, \underline{z}] = 0\ \forall \underline{z} \Rightarrow \underline{x} = \underline{0}$.

If $\Delta_t = \begin{pmatrix} s_1/\gamma_t & & & \\ & s_2/\gamma_t\,. & 0 & \\ & & \ddots & \\ 0 & & & s_m/\gamma_t \end{pmatrix}$ it is easy to convince ourselves that $[\underline{x}\,, \underline{z}] = \underline{x}\,\Delta_t\,\underline{z}^T$ defines a non-degenerate symmetric bilinear form in G. For every subgroup $H \leq G$, we define

$$H^\perp = \{\underline{x} \in G \mid [\underline{x}\,, \underline{h}] = 0\ \forall h \in H\}\,.$$

$H^\perp$ is a subgroup of G. Analogous results to the vector space case hold for non-degenerate bilinear forms over finitely generated abelian groups, i.e.

$$H \leq K \Rightarrow H^\perp \geq K^\perp$$

$$H^\perp \cap K^\perp = (H + K)^\perp$$

$$(H \cap K)^\perp = H^\perp + K^\perp \qquad \forall\, H\,, K \leq G$$

$$H^{\perp\perp} = H$$

$$|H| \cdot |H^\perp| = |G|\,, \text{ etc.}$$

Given two groups G and V, both Z_{γ_t}-modules, both with a non-degenerate bilinear form $[\cdot\ ,\ \cdot]_t$ and $[\text{-}\ ,\ \text{-}]_p$ respectively, and given a group morphism, i.e. a Z_{γ_t}-morphism

$$\varnothing : V \to G \text{, then there exists a morphism}$$

$$\varnothing_* : G \to V \ s.\,t.\ \forall \underline{x} \in G,\ \ \forall \underline{u} \in V$$

$$[\varnothing\ (\underline{u})\ ,\ \underline{x}]_t = [\underline{u}\ ,\ \varnothing^*\ (\underline{x})]_p\ .$$

We use $\perp_t, \perp_p$ respectively to indicate orthogonality in G and in V. The following holds : $\forall H \leq G$

$$[\varnothing_*\ (H)]\perp_p = \varnothing^{-1}\ (H\perp_t)$$

and $\forall W \leq V$

$$\varnothing\ (W\perp_p) = (\varnothing_*^{-1}\ (W))\perp_t\ .$$

Also, $\varnothing_*$ is 1-1 $\Leftrightarrow \varnothing$ is onto

$\varnothing_*$ is onto $\Leftrightarrow \varnothing$ is one-one.

3. – Construction and identification of effects and interactions in a factorial design with pseudofactors

It is well known that if the number of levels s of a factor A is not a prime — say $s = r_1 r_2$ — then the effect of A can be regarded as if it were due to the effects of two « pseudofactors » A_1 and A_2 at levels r_1 and r_2 respectively and to their interaction $A_1 \times A_2$. Also, given any other factor B, the degrees of freedom for the « pseudo-interactions » $A_1 \times B$, $A_2 \times B$ and $A_1 \times A_2 \times B$ will represent the interaction $A \times B$. Let $A_1, A_2, \dots, A_m$ be factors of an experiment at levels $s_1, s_2, \dots, s_m$ respectively.

If

$$s_1 = p_1^{\alpha_{11}} p_2^{\alpha_{12}} \dots p_n^{\alpha_{1n}}$$

$$s_2 = p_1^{\alpha_{21}} p_2^{\alpha_{22}} \dots p_n^{\alpha_{2n}} \qquad a_{ij} \geq 0$$

$$\cdots\cdots\cdots$$

$$s_m = p_1^{\alpha_{m1}} \dots\dots p_n^{\alpha_{mn}} \qquad p_j \text{ primes}$$

we can think of factor A_i at s_i levels as corresponding to pseudo-factors

$$A_{11}^{(i)} \ldots A_{1\alpha_{i},1}^{(i)} \text{ at levels } p_1$$

$$\cdots\cdots \qquad \cdots\cdots$$

$$A_{n,1}^{(i)} \ldots A_{n\alpha_{i},n}^{(i)} \text{ at levels } p_n$$

and all their interactions. We want to show that similar results to those of [8] hold for this type of design.

Consider the elementary abelian group G_i of order s_i.

Elements of G_i will be denoted by vectors $\underline{x}_i$ and will identify the s_i levels of factor A_i by the arbitrary choice of a 1-1 correspondence between levels and elements of G_i. In G_i we can introduce a non-degenerate symmetric bilinear form $[\underline{x}_i , \underline{y}_i]_i = \underline{x}_i \Delta_i \underline{y}_i^T$ with Δ_i as in 2. and $\gamma_t = p_1 p_2 \ldots p_n$.

The group $G = \oplus_{i=1}^{n} G_i$ is again elementary abelian and a $\mathbb{Z}_{\gamma_t}$-module.

Treatment combinations can be identified with elements $\underline{x} = (\underline{x}_1 , \ldots , \underline{x}_m)$ of G. For instance if A has 4 levels and B has 3 levels, the vector 002 will denote the treatment usually indicated by b, 032, 302, 332 will denote ab, $a^2 b$, $a^3 b$ in some order etc.

Furthermore, each cyclic group $\langle \underline{x} \rangle$ can be taken to represent some degres of freedom for a main effect or interaction as follows : let $[\cdot , \cdot]_t$ be the non degenerate symmetric bilinear form defined by the matrix

$$\Delta_t = \begin{pmatrix} \Delta_1 & & & 0 \\ & \Delta_2 & & \\ & & \ddots & \\ 0 & & & \Delta_m \end{pmatrix}.$$

and let $\langle \underline{x} \rangle^{\perp_t} = \{\underline{z} ; [\underline{z} , \underline{x}]_t = 0\} = H$. Clearly H does not depend on the choice of the generator $\underline{x}$.

Denote by $T(\underline{x})$ the set of contrasts among « strata » of treatments corresponding to cosets of H in G, i.e. $T(\underline{x})$ = set of all real valued functions such that the sum of their values is 0 (contrasts) and are constant on cosets of H in G. Thinking of $T(\underline{x})$ as a vector space, we talk of independent contrasts (= degrees of freedom) and orthogonal ones.

Since the number of cosets of $\langle \underline{x} \rangle^{\perp_t}$ in G is $|G/\langle x \rangle^{\perp_t}| = |\langle \underline{x} \rangle|$, $T(\underline{x})$ determines s-1 degrees of freedom where s = order of $\underline{x}$.

Define $\tilde{T}$ $(\underline{x})$ = set of contrasts among cosets of T $(\underline{x})$ in G which are orthogonal to the ones in T $(r. x)$ for each r dividing s.

It is enough to say that we take those that are orthogonal to the ones in T $(r_1 \underline{x})$ ... T $(r_k \underline{x})$ for r_1, ... , r_k proper prime divisors of s.

In other words we consider all subgroups $\langle r \cdot \underline{x} \rangle$ of $\langle \underline{x} \rangle$. Clearly

$$T\ (r\ .\ \underline{x}) \subseteq T\ (\underline{x})$$

since $\langle r \cdot \underline{x} \rangle \leq \langle \underline{x} \rangle \Rightarrow \langle \underline{x} \rangle^{\perp}$ is a subgroup of $\langle r \cdot \underline{x} \rangle^{\perp}$; hence each coset of $\langle r\, \underline{x} \rangle^{\perp}$ in G is the union of cosets of $\langle \underline{x} \rangle^{\perp}$ in G.

Claim: *a*) the contrasts in $\tilde{T}$ $(\underline{x})$ belong to the interaction of pseudo-factors $A_{i_1}, A_{i_2}, \ldots, A_{i_b}$ corresponding to non-zero entries in $\underline{x} = (\underline{x}_1, \ldots, \underline{x}_m)$,

b) Two sets $\tilde{T}(\underline{x})$ and $\tilde{T}(\underline{x}')$ with $\langle \underline{x} \rangle \neq \langle \underline{x}' \rangle$ are orthogonal,

c) the sets $\tilde{T}(\underline{x})$ exhaust all the treatment degrees of freedom.

The proof of these statements does not appear to depend on the use of pseudo factors, hence a parallel proof to the one outlined in [8] can be given.

4. – An example

A simple example will help illustrate the theory and notation so far: a 3×6 experiment with factors A and B, and pseudofactors A, B_1, B_2 at levels 3, 3, 2 respectively. We can write the treatments as

000	020	040	:	003	023	043		
200	220	240	:	203	223	243		Table I
400	420	440	:	403	423	443		

Take $\underline{x} = 200 : \langle \underline{x} \rangle^{\perp_t} = \{\underline{z} : z_1 = 0\}$ = first row of Table 1. Treatment strata which give 2 d. f. for the main effect of A are the rows in the above table. Similarly taking $\underline{y} = 003$ we find one d. f. for B given by the contrasts between the first half and second half of the table.

Now let $\underline{w} = 203$. Strata for T $(\underline{w})$ are cosets of $\langle \underline{w} \rangle^{\perp_t} = \{\underline{z} : z_1 + z_3 \equiv 0 \text{ mod. } 6\} = \{000, 020, 040\}$, hence half rows in the above table. But $\langle \underline{w} \rangle \geq \langle \underline{x} \rangle$ and $\langle \underline{w} \rangle \geq \langle \underline{y} \rangle$ so $\tilde{T}$ $(\underline{w})$ is given by the contrasts of $T(\underline{w})$ orthogonal to those in $T(\underline{x})$ and $T(\underline{y})$. Thus

$\tilde{T}(203) = \{c_1(000, 020, 040) - c_1(003, 023, 043) + c_2(200, 220, 240) - c_2(203, 223, 243) + c_3(400, 420, 440) - c_3(403, 423, 443), c_1, c_2, c_3 \in \mathbb{R}, c_1 + c_2 + c_3 = 0\}$ gives 2 d. f. for $A \times B$.

5. – Plot factors and the design key

Similarly to our construction of G we can construct a group V whose generators are the "plot" pseudofactors. More specifically, assume the N experimental units are arranged in blocks, replications, rows, columns, split plots and other groupings. We can assume that there are n "plot factors" $P_1, \ldots, P_n$ each at q_i levels and each plot is specified by assigning one of the levels for each P_i. Thus we have $V = V_1 \oplus \ldots \oplus V_n$, V_i = elementary abelian group of order q_i. Let γ_p be defined in analogy with γ_t and similariy for the matrix Δ_p. Plot effects and interactions are identified as before, by sets $\tilde{P}(\underline{y})$ $\underline{y} \in V$.

By *design key* we shall mean a map $\varnothing$ which assigns to each plot, i.e. each $\underline{y} \in V$, a treatment combination $\underline{x} \in G$. Then $\varnothing : V \to G$.

Requirement. We want $\varnothing$ to be such that it induces a 1-1 correspondence between $T(\underline{x})$ and $P(\underline{y})$. We say that $P(\underline{y})$ is the *plot alias* of $T(x)$.

Critical Assumption: We shall consider the case in which $\varnothing$ is a group morphism.

Then $\exists\, \varnothing_* : G \to V$ s. t.

$$[\varnothing(\underline{u}), x]_t = [\underline{u}, \varnothing_*(\underline{x})]_p$$

Under our condition we can show that (with a slight abuse of notation)

$$\varnothing_*\big(T(\underline{x})\big) = P(\underline{y}) \text{ where } \underline{x} = \varnothing(\underline{y})$$

so that $\varnothing_*\big(\tilde{T}(\underline{x})\big) = \tilde{P}(y)$ and the degrees of freedom determined by $\underline{x}$ are estimated by contrasts in $\tilde{P}(\underline{y})$.

6. – Aliasing, confounding and replications

Under our hypotheses the degrees of freedom for an effect or interaction identified by $\langle \underline{x} \rangle$ are *confounded with blocks* if the plot alias of $\underline{x}$ identifies a block effect $\underline{y}$. This can be extended to confounding with rows, columns, whole plots etc. In analogy with the results of [7] we can show that in order to design the experiment, we form a subgroup W of G whose elements represent the interactions we are not interested in and take the orthogonal subgroup $W\perp_t$ and its cosets as sets of treatments in each block respectively. $W\perp_t$ will be called the PRINCIPAL BLOCK.

If $\varnothing$ is *not* onto, i.e. not all treatments are actually used in the experiment, then $\varnothing_*$ is not one-one, i.e. $\exists\, \underline{x} \neq 0$ s. t. $\varnothing_*(\underline{x}) = \underline{0}$. Then the d. f. determined by $\underline{x}$ are *aliased with the mean* since the treatment strata for estimating such degrees of freedom are given by cosets of $\langle 0 \rangle \perp_p$ in V, i.e. the cosets of V in V, i.e. all the elements of V.

All elements of ker $\varnothing_*$ are aliased with the mean. They represent the *defining contrasts*. More in general, if two elements $\underline{x}$, $\underline{x}'$ belong to the same coset of ker $\varnothing_*$ in G, they are *aliased with each other*. Again we can show that given a fractional replication of a design constructed with the above methods, ker $\varnothing_* = S\perp_t$ where $S =$ the set of treatments actually occurring, so that aliased effects and interactions are those represented by subgroups $\langle \underline{x} \rangle$ with $\underline{x} \in S\perp_t$. Conversely, starting from the interactions assumed absent we form a group H and take $H\perp_t$ as the set of treatments of the design.

Aliasing and confounding can be present in the same design.

Replications: If $\varnothing$ is not one-one, different plots receive the same treatment, i.e. the design is replicated. Then $\varnothing_*$ is not onto and there are plot effects that are not aliases of any treatment effect. Two plots of V receive the same treatment iff they are in the same coset of ker $\varnothing$ in V, so that each treatment combination which is used in the design is replicated the same number of times. If, however, we want to confound different d.f. in different replications, we must change the design key.

Thus problems related to aliasing, confounding and replicating are translated into finding a subgroup orthogonal to a given one.

It is important to observe that there is a ring isomorphism

$$\mathbb{Z}_{\gamma_t} \to GF(p_1) \oplus GF(p_2) \oplus \ldots \oplus GF(p_n)$$

so that every equation of the type $\underline{x}\, \Delta_t \underline{y}^T = 0$ now reads as n equations, one for each field $GF(p_i)$.

7. – Application of the theory to the construction of new designs

Consider a whole replicate of a $2 \times 3 \times 4 \times 6$—experiment. A design for such an experiment has been given by R. Bailey in [7] with the cyclic group construction of [5] and the use of Sylow subgroups. She finds a design with blocks neither too large nor too small (the block size is 12) such that main effects are totally unconfounded and such that it confounds the following
$A \times C\,(1)$, $A \times D\ (1)$, $C \times D\ (1)$, $B \times D\ (2)$, $A \times B \times D\,(2)$, $B \times C \times D\,(2)$, $A \times B \times C \times D\,(2)$.

Now let A, C_1, C_2, D_1 denote pseudo factors at 2 levels and B, D_2 pseudo factors at 3 levels.

Using blocks of size 12, $144/12 - 1 = 11$ degrees of freedom must be confounded with blocks. The experiment may be conceptually split into two «sub-experiments»; a 2^4—experiment E_1, with factors A, C_1, C_2, D_1 in blocks of 4 plots, and a 3^2—experiment E_2 with factors B and D_2 in blocks of 3 plots.

To confound interactions as high as possible in E_1, we take $A \times$ $\times\, C_2 \times D_1$ and $C_1 \times C_2 \times D_1$: in this way, $A \times C_1$ will also be confounded. The equations defining the principal block of E_1 will be

$$\begin{cases} x + z'' + t' = 0 \\ z' + z'' + t' = 0 \end{cases} \qquad \text{over } GF(2)$$

where x, z', z'', t' represent the levels of A, C_1, C_2, D_1 respectively. The solutions are

$$\{(0000),\quad (0011),\quad (1101),\quad (1110)\}$$

To confound a high order interaction in E_2 we choose $B \times D_2$ giving the equation

$$y + t'' = 0 \quad \text{over } GF(3)$$

for the principal block, with solutions

$$\{(00),\quad (12),\quad (21)\}$$

The principal block in the overall experiment will receive treatments

$$
\begin{array}{llll}
(0,0,00,00), & (0,0,03,30), & (3,0,30,30), & (3,0,33,00) \\
(0,2,00,04), & (0,2,03,34), & (3,2,30,34), & (3,2,33,04) \\
(0,4,00,02), & (0,4,03,32), & (3,4,30,32), & (3,4,33,02)
\end{array}
$$

The following pseudo degrees of freedom are confounded with blocks:

$$
\left.\begin{array}{ll}
A \times C_2 \times D_1 & (1) \\
C_1 \times C_2 \times D_1 & (1) \\
A \times C_1 & (1)
\end{array}\right. \quad \text{in } E_1
$$

and

$$
B \times D_2 \qquad (2) \qquad \text{in } E_2
$$

and therefore also

$$
\begin{array}{ll}
A \times B \times C_1 \times D_2 & (2) \\
A \times B \times C_2 \times D_1 \times D_2 & (2) \\
B \times C_1 \times C_2 \times D_1 \times D_2 & (2)
\end{array}
$$

i.e. the following degrees of freedom for the overall experiment:

$$
\begin{array}{ll}
A \times C & (1) \\
C \times D & (1) \\
B \times D & (2) \\
A \times C \times D & (1) \\
B \times C \times D & (2) \\
A \times B \times C \times D & (4)
\end{array}
$$

A comparison of this with the design obtained by R. Bailey shows that more degrees of freedom for higher order interactions are confounded here, which in general is advantageous. R. Bailey's design can be obtained as a particular case of the pseudo factor method taking equations $x + z' = 0$ and $x + t' = 0$ over $GF(2)$.

8. – Final remarks

The method of [4] and [5] differs from the one employed here because the groups G_i and V_i are cyclic instead of elementary abelian.

Elementary abelian groups have more subgroups than any other group of the same order: thus the introduction of pseudofactors allows more flexibility in the construction of designs. Besides, they provide a conceptual link between the «new» and the «old» theory: by virtue of the ring isomorphism between $\mathbb{Z}_{p_1 p_2 \ldots p_n}$ and $\bigoplus_{i=1}^{m} GF(p_i)$ the group construction of factorial designs with pseudo factors leads us back to Bose′ s finite geometry method. It also suggests all the possible «intermediate» constructions that can be achieved, endowing G with other possible abelian group structures.

What is not clear is in what way the construction depends on the choice of the bilinear form in G, i.e. whether it is possible to change the form and obtain meaningful and useful results.

Finally, note that we have not considered the special case of factors with quantitative equally spaced levels: it has been shown in [7] that for such experiments, isomorphic designs may give different information on the linear, quadratic etc. components of interactions, depending on how we label the factor levels. Also, designs have been obtained without the critical assumption that the design key be a homomorphism: see for instance [11] p. 205 and [6]. In some cases a degree of freedom for a main effect or interaction is only partially confounded or aliased in a given replication.

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