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Existence of T-periodic solutions for a class of lagrangian systems

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Existence of T-Periodic Solutions for a Class of Lagrangian Systems.

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1. Introduction.

In this paper it will be discussed the existence of T-periodic solutions $q = q(t)$ of the Lagrangian system of ordinary differential equations:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \xi}(q, \dot{q}, t) - \frac{\partial \mathcal{L}}{\partial q}(q, \dot{q}, t) = 0 \qquad q \in C^2(\mathbb{R}, \mathbb{R}^N) \tag{1.1}$$

where $\mathcal{L}$ denotes the Lagrangian function

$$\mathcal{L}(q, \xi, t) = \frac{1}{2}\sum_{i,j=1}^{N} a_{ij}(q)\xi_i\xi_j - V(q, t)\,, \qquad q, \xi \in \mathbb{R}^N\,, \quad t \in \mathbb{R} \tag{1.2}$$

the a_{ij}'s $(i, j = 1, \ldots, N)$ being C^1 real functions on $\mathbb{R}^N$ and $V(q, t)$ a real function on $\mathbb{R}^{N+1}$, T-periodic in the t variable.

That problem has been widely studied mostly when the coefficients a_{ij} are constant; in this case (1.1) reduces to,

$$\ddot{x} + \nabla U(x, t) = 0\,. \tag{1.3}$$

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Problem (1.3) has been extensively studied when U is unbounded (see e.g. [2], [4], [13]), and in the case when U is bounded it has been studied e.g. in [5], [7], [8], [9], [10], [11].

In the general case when the a_{ij}'s are nonconstant, a discussion on problem (1.1) can be found in [6], [3], [14], although the last two ones are in a Hamiltonian setting.

The present note is devoted to the study of problem (1.1) when V is subquadratic at infinity, i.e.

$$\frac{V(q)}{|q|^2} \to 0 \quad \text{as } |q| \to \infty .$$

In this analysis, variational methods will be used, that is the T-periodic solutions of (1.1) are looked for as critical points of the action functional:

$$f(q) = \int_0^T \mathcal{L}(q, \dot{q}, t)\, dt \tag{1.4}$$

where q belongs to the Sobolev space of the T-periodic functions.

This paper is organized as follows:

a) Section 2 contains definitions and preliminary notations.

b) Section 3 is devoted to the study of problem (1.1) in the autonomous case, i.e.

$$V = V(q) .$$

In such a case f is invariant under the action of the group S^1 (namely the time translations).

With reference to problem (1.1), the existence of multiple solutions of sufficiently large prescribed T-period will be established, under the assumption that $V(q) \to +\infty$ as $|q| \to +\infty$, and the symmetry property of f will be used.

Analogous results have been obtained in [14] under more restrictive assumptions.

Moreover when the a_{ij}'s are constant, the results obtained in section 3 are a variant of those obtained by Benci in [2].

c) Section 4 deals with problem (1.1) when V is a T-periodic time-dependent bounded potential. The main theorem establishes the existence of at least one nontrivial T-periodic solution.

It must be pointed out that, when the potential is bounded, (1.4) does not satisfy the Palais-Smale condition globally.

An analogous sistuation occurs in [6] where, however, a_{ij} and V are assumed even.

d) In Section 5 problem (1.1) with an additional T-periodic « forcing » term is examined. The existence of at least one solution of the same period is then established.

2. Notations and preliminaries.

Some notations which will be used in the following sections, are now stated:

1) $|\cdot|$ denotes the Euclidean norm of $\mathbb{R}^N$ and $(\cdot|\cdot)$ its usual inner product;

2) if $1 \leqslant p < \infty$, the space

$$L^p = L^p(S^1, \mathbb{R}^N) = \left\{q\colon \mathbb{R} \to \mathbb{R}^N\colon q \ 2\pi\text{-periodic}, \int_0^{2\pi} |q(t)|^p < \infty\right\}$$

is meant to be endowed with the usual L^p norm, here denoted by $|\cdot|_p$, while $L^\infty = L^\infty(S^1, \mathbb{R}^N)$ indicates the space of the essentially bounded 2π-periodic $\mathbb{R}^N$ valued functions, endowed with the usual norm $|\cdot|_\infty$.

3) $H^1 = H^1(S^1, \mathbb{R}^N)$ represents the Sobolev space obtained by the closure of the C^∞ 2π-periodic $\mathbb{R}^N$ valued functions $q = q(t)$, endowed with the norm

$$\|q\| = \left[\int_0^{2\pi} (|\dot{q}|^2 + |q|^2)\, dt\right]^{\frac{1}{2}}$$

4) $\langle \cdot, \cdot \rangle$ indicates the duality between H^1 and H^{-1};

5) B_R indicates the closed H^1-ball of radius R centered at the origin, while ∂B_R denotes its boundary;

6) if f is a C^1 functional on H^1, $f'(q)$ denotes the Frechét derivative at $q \in H^1$.

Some shortened matrix notations are now further established for the functions a_{ij} $(i, j = 1, \ldots, N)$

$$a(q) \quad = \{a_{ij}(q)\} \qquad i, j = 1, \ldots, N \tag{2.1}$$

$$a'(q)v = \{(\nabla a_{ij}(q)|v)\} \quad i, j = 1, \ldots, N . \tag{2.2}$$

In all the theorems which will be set in the following sections, it is assumed that the a_{ij}'s satisfy:

(2.3) there exists $\mu > 0$ such that $(a(q)\xi|\xi) \geqslant \mu|\xi|^2$
for each $q \in \mathbf{R}^N$, $\xi \in \mathbf{R}^N$.

Moreover, given a Hilbert space H and a functional $f \in C^1(H, \mathbf{R})$, f is said to satisfy the Palais-Smale condition, here recalled in its weaker version, iff:

(P.S.) Any sequence $\{q_n\}$ in H such that $\{f(q_n)\}$ is bounded and $\|f'(q_n)\| \, \|q_n\| \to 0$, possesses a convergent subsequence in H.

3. Multiple free oscillations.

In this section it is examined the existence of multiple T-periodic solutions of problem (1.1), in the autonomous case.

Here the C^1 functional on H^1 defined in (1.4), becomes, changing the variable t in $(2\pi t)/T$:

$$f(q) = \frac{1}{2}\int_0^{2\pi} (a(q)\dot{q}|\dot{q})\, dt - \omega^2 \int_0^{2\pi} V(q)\, dt \tag{3.1}$$

where $\omega = T/2\pi$.

Then the research of the T-periodic solutions of (1.1) is reduced to the research of the critical points of (3.1) in H^1.

The main result of this section is the following:

THEOREM 3.1. *Assume condition* (2.3) *holds and moreover*:

(3.2) *V is subquadratic at infinity, that is there exists $\alpha \in]0, 2[$ and $R \in \mathbf{R}_+$ such that*

$$(\nabla V(q)|q) - \alpha V(q) \leqslant 0 \qquad \textit{for any } q \in \mathbf{R}^N, \quad |q| > R;$$

(3.3) *there exists* $\beta \in]0, 2-\alpha[$ *such that*

$$a'(q)q + \beta a(q) \quad \textit{is positive semidefinite};$$

(3.4) $V(q) \to +\infty$ as $|q| \to +\infty$;

(3.5) $V(0) = 0$ *is the minimum of* V *and* $V'(q) \neq 0$ *for any* $q \neq 0$;

(3.6) a_{ij} *and* V *are twice differentiable at the origin and* $V''(0)$ *has all positive eigenvalues*.

For any $k \in \mathbb{N}$, $k \neq 0$, *let* $T(k) = 2\pi[(k^2+1)\nu/\lambda]^{\frac{1}{2}}$, *where* ν *is the largest eigenvalue of* $\{a_{ij}(0)\}$, λ *the first eigenvalue of the Hessian matrix* $V''(0)$. *Then for any* $T > T(k)$, *problem* (1.1) *possesses at least* kN T*-periodic distinct* (¹) *solutions.*

Before proving this theorem a S^1 version of a result contained in [1] needs to be recalled and a preliminar lemma must be stated too.

THEOREM 3.2. *Let* H *be a real Hilbert space on which a unitary representation* G *of the* S^1 *group acts. Suppose that* $f \in C^1(H, \mathbb{R})$ *verifies the following assumptions*:

(3.7) f *is invariant under the action of* G;

(3.8) f *satisfies the* (P.S.) *condition*;

(3.9) *there exist two closed subspaces* V *and* W *of* H *with* codim $W < \infty$ *and there exist two real constants* $c_0 > c_\infty$ *and* $\varrho \in \mathbb{R}_+$ *such that*

i) $f(q) < c_0 < f(0)$ *for each* $q \in \partial B_\varrho \cap V$;

ii) $f(q) \geqslant c_\infty$ *for each* $q \in W$;

(3.10) $f(q) > c_0$ *for each* $q \in \mathrm{Fix}\,(S^1)$ *such that* $f'(q) = 0$.

(¹) $q_1(t)$ and $q_2(t)$ will be said distinct iff q_1 cannot be obtained by q_2 by a time translation.

Then there exist at least

$$\tfrac{1}{2}(\dim V - \operatorname{codim} W)$$

orbits of critical points with critical values in $[c_\infty, c_0]$.

PROOF. The claim follows from theorem 2.4 of [1], by suitable modifications contained in Theorem 1.4 of [3].

LEMMA 3.3. *Suppose that* (2.3), (3.2), (3.3) *and* (3.4) *hold. Then* f *verifies the* (P.S.) *condition.*

PROOF. - Let $\{q_n\}$ be a sequence of H^1 such that:

$$\{f(q_n)\} \text{ is bounded} \tag{3.11}$$

$$\|f'(q_n)\|\,\|q_n\| \to 0 \quad \text{as } n \to \infty . \tag{3.12}$$

Those statements imply that there exist two real constants M_1 and M_2 such that:

$$\omega^2 \int_0^{2\pi} V(q_n)\,dt \leqslant M_1 + \frac{1}{2}\int_0^{2\pi} (a(q_n)\dot q_n|\dot q_n)\,dt \tag{3.13}$$

and

$$\langle f'(q_n), q_n\rangle = \int_0^{2\pi} (a(q_n)\dot q_n|\dot q_n)\,dt + \frac{1}{2}\int_0^{2\pi} (a'(q_n)q_n\dot q_n|\dot q_n)\,dt - \tag{3.14}$$

$$- \omega^2 \int_0^{2\pi} (\nabla V(q_n)|q_n)\,dt \leqslant M_2 .$$

By (3.2), (3.13) and (3.14) it follows that

$$\int_0^{2\pi} (a(q_n)\dot q_n|\dot q_n)\,dt + \frac{1}{2}\int_0^{2\pi} (a'(q_n)q_n\dot q_n|\dot q_n)\,dt \leqslant \alpha M_1 + M_2 + \frac{\alpha}{2}\int_0^{2\pi} (a(q_n)\dot q_n|\dot q_n)\,dt$$

and hence, by (3.3)

$$\alpha M_1 + M_2 \geqslant \frac{2-\alpha}{2}\int_0^{2\pi} (a(q_n)\dot q_n|\dot q_n)\,dt + \frac{1}{2}\int_0^{2\pi} (a'(q_n)q_n\dot q_n|\dot q_n)\,dt \geqslant \tag{3.15}$$

$$\geqslant \frac{2-\alpha-\beta}{2}\int_0^{2\pi} (a(q_n)\dot q_n|\dot q_n)\,dt .$$

Now (2.3) and (3.15) imply that

$$\int_0^{2\pi} |\dot{q}_n|^2 \, dt \quad \text{is bounded},$$

and, by (3.13),

$$\int_0^{2\pi} V(q_n) \, dt \quad \text{is bounded, too}.$$

The last two conditions, in addition with (3.4), imply that

$$\|q_n\| \quad \text{is bounded}. \tag{3.16}$$

Consider, now, the decomposition

$$H^1 = H^+ \oplus \mathbf{R}^N \tag{3.17}$$

where

$$H^+ = \left\{ q \in H^1 \middle| \int_0^{2\pi} q \, dt = 0 \right\}. \tag{3.18}$$

Then, for each $n \in \mathbf{N}$

$$q_n = q_n^+ + q_n^0 \qquad \text{where } q_n^+ \in H^+, \quad q_n^0 \in \mathbf{R}^N$$

and, by (3.16)

$$\|q_n^+\| \quad \text{is bounded}.$$

Arguing as in the proof of lemma 1.8 of [6] a subsequence of $\{q_n^+\}$, strongly convergent in H^1, can be found; whence $\{q_n\}$ itself has a H^1 strongly convergent subsequence.

Proof of theorem 3.1. Consider the following subspaces of H^1:

$$W = \bigoplus_{n \geqslant 1} M_{\lambda_n}$$

$$W_k = \bigoplus_{n \leqslant k} M_{\lambda_n} \qquad k \in \mathbf{N}, \quad k \neq 0$$

where M_{λ_n} denotes the eigenspace corresponding to the eigenvalue λ_n of the operator $q \to -\ddot{q}$ in H^1.

To reach the claim it is enaugh to show that for any fixed $k \in \mathbb{N}$, $k \neq 0$, there exist $c_\infty < c_0 < 0$ and $\varrho \in \mathbb{R}_+$ such that

$$(3.19) \qquad f(q) \geqslant c_\infty \quad \text{for each } q \in W$$

$$(3.20) \qquad f(q) \leqslant c_0 \quad \text{for each } q \in W_k\,, \quad \|q\| = \varrho$$

and

$$(3.21) \qquad f(q) > c_0 \quad \text{for any } q \in \mathbb{R}^N \text{ s.t. } f'(q) = 0\,.$$

In order to do so, first remark that, by virtue of (3.2), there exists a real constant $c_1 \in \mathbb{R}_+$ such that

$$(3.22) \qquad V(q) \leqslant c_1 |q|^\alpha \quad \text{if } |q| \geqslant R\,.$$

Then, let q be in W; by (2.3) and (3.22), $c_2 \in \mathbb{R}$ and $c_\infty \in \mathbb{R}$ exist such that

$$(3.23) \qquad f(q) \geqslant \frac{\mu}{2} |\dot{q}|_2^2 - \omega^2 c_1 |q|_\alpha^\alpha \geqslant \lambda_1 \frac{\mu}{2} |q|_2^2 - \omega^2 c_2 |q|_2^\alpha \geqslant c_\infty\,.$$

Now, let q be in W_k; using the Taylor expantion of f, it follows that

$$(3.24) \qquad f(q) = \frac{1}{2} \left\{ \int_0^{2\pi} (a(0)\dot{q}|\dot{q})\, dt - \omega^2 \int_0^{2\pi} (V''(0)q|q)\, dt \right\} + \\ + o(\|q\|^2) \leqslant \frac{1}{2} (\nu |\dot{q}|_2^2 - \omega^2 \lambda |q|_2^2) + o(\|q\|^2)$$

where ν is the largest eigenvalue of $\{a_{ij}(0)\}$ and λ the first eigenvalue of $V''(0)$.

Taking

$$\omega^2 > \nu/\lambda(k^2 + 1)$$

and $\varrho \in \mathbb{R}_+$ small enaugh, a $c_0 \in \mathbb{R}_-$, $c_0 > c_\infty$, can be found, such that

$$(3.25) \qquad f(q) \leqslant c_0 \quad \text{for any } q \in W_k\,, \quad \|q\| = \varrho\,.$$

Furthermore, by virtue of (3.5), $q = 0$ is the only element of $\mathbf{R}^N$ such that $f'(q) = 0$ and $f(0) = 0 > c_0$. Hence (3.21) holds.

The functional f has been proved to satisfy (3.19), (3.20), (3.21) and the (P.S.) condition, then theorem 3.2 holds and thus f has at least

$$\tfrac{1}{2}(\dim W_k - \operatorname{codim} W) = kN$$

orbits of critical points.

4. The case of a bounded potential.

The object of the present section is to look for the T-periodic solutions of problem (1.1) in the case of a bounded time-dependent potential. The action functional related to this problem is

$$f(q) = \frac{1}{2}\int_0^{2\pi} (a(q)\dot{q}|\dot{q})\,dt - \omega^2\int_0^{2\pi} V(q,t)\,dt\,. \tag{4.1}$$

THEOREM 4.1. *Suppose the condition* (2.3) *holds in addition to the following further hypotheses*:

(4.2) *V is T-periodic in the variable t;*

(4.3) *there exists $c \in \mathbf{R}$ such that*

$$\lim_{|q|\to\infty} V(q,t) = c \qquad \textit{uniformly with respect to } t;$$

(4.4) *$V(q,t) < c$, for any $q \in \mathbf{R}^N$, $t \in \mathbf{R}$;*

(4.5) *$\lim_{|q|\to\infty} V'(q,t) = 0$, uniformly with respect to t;*

(4.6) *$\lim_{|q|\to\infty} a'_{ij}(q) = 0$ for any $i, j = 1, \ldots, N$.*

Then there exists at least one T-periodic solution of problem (1.1).

Before proving the theorem a remark and a preliminar lemma need to be stated.

REMARK 4.2. The (P.S.) condition cannot be satisfied by f at the level $c_0 = -2\pi\omega^2 c$ because some divergent sequences of $\mathbb{R}^N$ elements verify (3.11) and (3.12), by virtue of (4.3) and (4.5).

LEMMA 4.3. *Suppose the hypothesis of theorem* 4.1 *hold*; *then the* (P.S.) *condition is satisfied by f in* $\mathbb{R} - \{c_0\}$.

PROOF. Let c' be in $\mathbb{R}$, $c' \neq c_0$, and $\{q_n\}$ be a sequence in H^1 such that

$$f(q_n) \to c' \quad \text{as } n \to \infty \tag{4.7}$$

and

$$\|f'(q_n)\|\,\|q_n\| \to 0 \quad \text{as } n \to \infty\,. \tag{4.8}$$

By (2.3), (4.4) and (4.7), it follows that

$$\{|\dot{q}_n|\} \quad \text{is bounded}\,.$$

Arguing by contradiction, suppose that $\|q_n\|$ is not bounded; then, as $L^\infty \hookrightarrow L^2$, $\{|q_n|_\infty\}$ has to be unbounded too, and thus, by (4.5) and (4.8):

$$\lim_{n\to\infty}\int_0^{2\pi}\left(\left(a(q_n) + \frac{1}{2}\,a'(q_n)q_n^+\right)\dot{q}_n|\dot{q}_n\right)dt = 0\,.$$

Hence, by (4.6),

$$\lim_{n\to\infty}\int_0^{2\pi}(a(q_n)\dot{q}_n|\dot{q}_n)\,dt = 0\,.$$

In view of (4.3), this implies that $f(q_n) \to c_0$, in contradiction with (4.7).

Because of its boundedness, $\{\|q_n\|\}$ has then a weakly convergent subsequence. Arguing as in lemma 3.3, that convergence can be proved to be strong in H^1.

PROOF OF THEOREM 4.1. Here it will be used the Rabinowitz saddle point theorem (see theorem 1.2 of [12]).

Consider the decomposition

$$H^1 = H^+ \oplus \mathbb{R}^N$$

where H^+ is as in (3.18); the first step to reach the claim is to establish that there exists $R^* \in \mathbb{R}^+$ such that

$$\sup_{\partial(B_{R^*} \cap \mathbb{R}^N)} f \leqslant \inf_{H^+} f\,. \tag{4.9}$$

Let q be in H^+; then

$$f(q) \geqslant \frac{\mu}{2}\,\frac{1}{\lambda_1 + 1}\,\|q\|^2 - \omega^2 \int_0^{2\pi} V(q, t)\,dt\,.$$

Taking $K \in \mathbb{R}_+$, in view of (4.4), there exists $\eta \in \mathbb{R}_+$, $\eta \neq 0$, such that

$$-\,V(q) \geqslant -\,c + \eta \qquad \text{if } \|q\| \leqslant K$$

and then

$$f(q) \geqslant c_0 + 2\pi\eta\omega^2 \qquad \text{if } \|q\| \leqslant K\,,$$

where $c_0 = -\,2\pi\omega^2 c$.

Moreover, if $\|q\| \geqslant K$

$$f(q) \geqslant \frac{\mu}{2}\,\frac{1}{\lambda_1 + 1}\,K + c_0$$

and hence, gathering the last two statements, the existence of $\varepsilon \in \mathbb{R}_+$ such that

$$f(q) \geqslant c_0 + \varepsilon \qquad \text{for each } q \in H^+ \tag{4.10}$$

has been showed.

Furthermore, by (4.3) there exists $R^* \in \mathbb{R}_+$ such that $q \in \mathbb{R}^N$, $\|q\| \geqslant R^*$ implies

$$V(q, t) > c - \varepsilon/(2\pi\omega^2) \qquad \text{for each } t \in \mathbb{R}$$

and thus

$$f(q) < c_0 + \varepsilon \quad \text{for each } q \in \mathbb{R}^N, \quad \|q\| = R^* \tag{4.11}$$

which, jointly with (4.10), implies (4.9).

The second step of this proof consists in showing that

$$c_0 < \inf_{H^+} f \tag{4.12}$$

so that (P.S.) condition holds in $[\inf_{H^+} f, +\infty[$, (see remark 4.2 and lemma 4.3).

Let ϱ be in $\mathbb{R}_+$ large enaugh; since (4.4) holds, there exists $\eta_1 \in \mathbb{R}_+$ such that $|q|_\infty \leqslant \varrho$ implies that

$$-\omega^2 \int_0^{2\pi} V(q, t)\, dt > c_0 + \eta_1$$

and whence

$$f(q) \geqslant c_0 + \eta_1 \quad \text{for any } q \in H^+, \quad |q|_\infty \leqslant \varrho\,. \tag{4.13}$$

Let q be in H^+ with $|q|_\infty \geqslant \varrho$. Then, by the imbedding $H^1 \hookrightarrow C^0$, there exists $k \in \mathbb{R}_+$ such that

$$|\dot{q}|_2 \geqslant k\varrho$$

and then $\eta_2 \in \mathbb{R}_+$, $\eta_2 = \mu\varrho k/2$, exists such that

$$f(q) \geqslant \eta_2 + c_0 \quad \text{for any } q \in H^+, \quad |q|_\infty > \varrho\,. \tag{4.14}$$

By (4.13) and (4.14), taking $\eta \in \mathbb{R}_+$, $\eta = \min\{\eta_1, \eta_2\}$, it follows that

$$f(q) \geqslant c_0 + \eta \quad \text{for any } q \in H^+$$

which implies (4.12).

Then the functional f satisfies (4.9) and the (P.S.) condition in $[\inf_{H^+} f, +\infty[$, so the hypotheses of the Rabinowitz saddle point theorem are verified and hence there exists at least one critical value c^* such that

$$c^* \geqslant \inf_{H^+} f$$

to which a nontrivial solution of problem (1.1) corresponds.

5. Forced oscillations.

This section is devoted to the study of the forced Lagrangian system

$$\frac{d}{dt}\frac{\partial \mathfrak{L}}{\partial \xi}(q,\dot{q}) - \frac{\partial \mathfrak{L}}{\partial q}(q,\dot{q}) = g(t) \tag{5.1}$$

where $g \in L^2(\mathbb{R}, \mathbb{R}^N)$ is a T-periodic forcing term.

The existence of at least one T-periodic solution of (5.1) will be established.

The action functional related to this problem is

$$f(q) = \frac{1}{2}\int_0^{2\pi} (a(q)\dot{q}|\dot{q})\,dt - \omega^2\int_0^{2\pi} V(q)\,dt - \omega^2\int_0^{2\pi}(g|q)\,dt\,. \tag{5.2}$$

THEOREM 5.1. *Assume that* (2.3), (3.2), (3.3) *and* (3.4) *hold. Then problem* (5.1) *admits at least one nontrivial solution.*

PROOF. Arguing as in the proof of lemma 3.3, it is easy to show that the functional f satisfies the (P.S.) condition.

Then, consider H^1 decomposed as in (3.17); in order to reach the claim, it is enaugh to show that there exists $R^* \in \mathbb{R}_+$ such that

$$\sup_{\partial(B_{R^*}\cap\mathbb{R}^N)} f \leqslant \inf_{H^+} f\,. \tag{5.3}$$

Let q be in H^+. By (3.22) and (2.3), there exist two real constants c_2 and c_3 such that

$$f(q) \geqslant \lambda_1\mu/2|q|_2^2 - c_2\omega^2|q|_2^\alpha - \omega^2|g|_2|q|_2 \geqslant c_3 \tag{5.4}$$

Let R^* be a positive real number and $q \in \partial(B_{R^*}\cap\mathbb{R}^N)$. Then there exists $c_4 \in \mathbb{R}$ such that

$$f(q) \leqslant - c_4 - \omega^2 R^*|g|_2\,. \tag{5.5}$$

Choosing R^* large eanugh, (5.4) and (5.5) imply (5.3).

Since f satisfies (5.3) and (P.S.) condition, the Rabinowitz saddle point theorem holds (see [12]) and then at least one nontrivial solution of problem (5.1) exists.

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