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On Outer Automorphisms of Černikov p-Groups.

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0. Introduction.

As is well known, every finite p-group that is not cyclic of order p, has non inner p-automorphisms. This theorem, proved by Gaschütz in [1], was later made more precise by Schmid and then extended by Menegazzo and Stonehewer. In [2] in fact, Schmid proves that, apart from some exceptions, Out G has a normal p-subgroup (always in the hypothesis that G is a finite p-group) while in [3] Menegazzo and Stonehewer prove an analogous theorem to that one of Gaschütz in the case of infinite nilpotent p-groups. Even in the case that G is infinite the normal p-subgroups of Out G have been studied and in [4], Marconi has reached an analogous result to the one obtained by Schmid. In this paper the problem of the existence of outer p-automorphisms is studied in the hypothesis that G is an infinite Černikov p-group, obtaining an affirmative answer for a certain class of such groups. To be more precise, if G is a Černikov p-group, indicating with G_0 its finite residual and with Fit G its Fitting subgroup, we have the following

THEOREM. – Let G be a non nilpotent Černikov p-group. If Fit $G > G_0$ and $G_0 \cap Z(G)$ is divisible then G has outer p-automorphisms.

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Even the case Fit $G = G_0$ is examined obtaining

THEOREM. – Let G be a non nilpotent Černikov p-group and assume Fit $G = G_0$ and $Z(G)$ divisible. Then G has non inner p-automorphisms or $H^1(G/G_0, G_0) = 0$ and the natural image of G/G_0 in Aut G_0 is a Sylow p-subgroup of Aut G_0.

The last section of this work is devoted to the construction of some examples which show what can happen if Fit $G = G_0$, $Z(G)$ is divisible and the image of G/G_0 in Aut G_0 is a Sylow p-subgroup.

1. Preliminaries.

If G is a Černikov p-group we shall indicate from now on with G_0 its finite residual that is an artinian divisible abelian group and with Fit G the Fitting subgroup of G. It is worth while remembering that G/G_0 is a finite group so that $|G| \leqslant \aleph_0$, while Fit $G = C_G(G_0)$ is nilpotent and its centralizer in G coincides with $Z(\text{Fit}\, G)$. In the proof of theorem 2.1, we shall use the results about nilpotent p-groups cited in the introduction, which are here below listed for the readers' use.

THEOREM 1.1 (Gaschütz [1]). If G is a finite p-group that is not cyclic of order p, then G has a non inner p-automorphism.

THEOREM 1.2 (Schmid [2]). Let G be a finite non abelian p-group. Then p divides the order of $C_{\text{Out}\, G}(Z(G))$.

THEOREM 1.3 (Menegazzo-Stonehewer [3]). Let G be a nilpotent p-group. If G is neither cyclic of order p nor isomorphic to a direct product of k quasi-cyclic p-groups with $k < p - 1$, then G has an outer automorphism of order p.

THEOREM 1.4 (Marconi [4]). Let H be an infinite nilpotent p-group Then $O_p(\text{Out}\, H) = 1$ if and only if one of the following conditions holds:

i) H is elementary abelian

ii) H is divisible and p is odd

iii) H is the central product of $\Omega_1(G)$ and of a quasi cyclic p-group with $\Omega_1(G)$ extra special of exponent $p \neq 2$.

First of all we want to prove that a Černikov p-group always has outer automorphisms, a fact which comes easily from the following theorem

THEOREM 1.5 (Pettet [5]). Let G be periodic and $H \leqslant G$ a Černikov group such that $|G: N_G(H)|$ is finite. If $C_{\mathrm{Aut}\, G}(H)$ is finite or countable and $C_{\mathrm{Inn}\, G}(H)$ is Černikov, then G is Černikov and $G_0 = H_0^G$.

COROLLARY 1.1. Let G be an infinite Černikov p-group. Then $|\mathrm{Aut}\, G| > \aleph_0$. In particular $\mathrm{Out}\, G \neq 1$.

PROOF. If $|\mathrm{Aut}\, G| = \aleph_0$ then, with the same notations of Theorem 1.5 let $H = 1$. H and G satisfy the hypotheses of Theorem I.5 so $G_0 = H_0^G = 1$, a contradiction. So $|\mathrm{Aut}\, G| > \aleph_0$ and, therefore, $\mathrm{Out}\, G \neq 1$. #

The proof of theorem 2.1 is based in great part on the following fact concerning the cohomology groups of $G/\mathrm{Fit}\, G$.

LEMMA 1.1. Let G be a Černikov p-group, G_0 its finite residual, $F = \mathrm{Fit}\, G$. Suppose $G_0 \cap Z(G)$ divisible. If $H^1(G/F, Z(F)) = 0$ then $H^m(G/F, Z(F)) = 0 \ \forall m > 0$.

PROOF. Let $K = G/F$ and $A = Z(F)$. F is nilpotent so $A \geqslant G_0$ and therefore we can write $A = G_0 \oplus L_1$ where L_1 is finite. Also $Z(G) = D \oplus L_2$ with D divisible and L_2 finite. Let $p^n = \max\{|L_1|, |K|\}$ and consider the following short exact sequence in G-Mod (and therefore in K-Mod)

$$0 \to A[p^n] \to A \xrightarrow{j} G_0 \to 0$$

where j is the multication by p^n. We have also the related long exact sequence

$$0 \to H^0(K, A[p^n]) \to H^0(K, A) \to H^0(K, G_0) \to$$
$$\to H^1(K, A[p^n]) \to \ldots \to H^m(K, A[p^n]) \to H^m(K, A) \to \ldots .$$

For every K-module we have $H^0(K, M) = \{m \in M: m^x = m \ \forall x \in K\}$, so that we can rewrite this sequence as follows

$$0 \to A[p^n] \cap Z(G) \to Z(G) \to G_0 \cap Z(G) \xrightarrow{\theta} H^1(K, A[p^n]) \to$$
$$\to 0 \to H^1(K, G_0) \to H^2(K, A[p^n]) \to \ldots \to H^m(K, A[p^n]) \to$$
$$\to H^m(K, A) \to H^m(K, G_0) \to H^{m+1}(K, A[p^n]) \to H^{m+1}(K, A) \to \ldots$$

because $H^1(G/F, Z(F)) = 0$. Now θ is surjective and $G_0 \cap Z(G)$ is divisible, so $H^1(K, A[p^n]) = 0$ because it is a finite group. Then, by [1], $H^m(K, A[p^n]) = 0 \ \forall m > 0$ so that, as it is easy to see, $H^1(K, G_0) = 0$ and $H^m(K, A)$ is isomorphic to $H^m(K, G_0) \ \forall m > 0$. Now consider the exact sequence

$$0 \to G_0[p^n] \to G_0 \xrightarrow{j} G_0 \to 0$$

where j is the multiplication by p^n, and the related cohomology sequence

$$0 \to G_0[p^n] \cap Z(G) \to G_0 \cap Z(G) \to G_0 \cap Z(G) \to H^1(K, G_0[p^n]) \to$$
$$\to H^1(K, G_0) \xrightarrow{j} H^1(K, F_0) \to H^2(K, G_0[p^n]) \to \ldots \to H^m(K, G_0[p^n]) \to$$
$$\to H^m(K, G_0) \xrightarrow{j} H^m(K, G_0) \to H^{m+1}(K, G_0[p^n]) \to H^{m+1}(K, G_0) \to \ldots .$$

As before we can see that $H^m(K, G_0[p^n]) = 0 \ \forall m > 0$ so that, $\forall m > 1$, we have

$$0 \to H^m(K, G_0) \xrightarrow{j} H^m(K, G_0) \to 0 .$$

But j is the trivial morphism because the exponent of $H^m(K, G_0)$ divides $|K|$ and therefore $H^m(K, G_0) = 0 = H^m(K, A)$ as claimed. #

2. Main theorems.

By theorem 1.3 we can limit ourselves to the case in which G is non nilpotent. The principal result obtained is the following

THEOREM 2.1. Let G be a non nilpotent Černikov p-group, G_0 its finite residual. If Fit. $G > G_0$ and $G_0 \cap Z(G)$ is divisible then G has outer p-automorphisms.

PROOF. Consider the extension $e: 1 \to F \to G \to K \to 1$ where $F = \mathrm{Fit}\, G = C_G(G_0)$ and $K = G/F$. F is characteristic in G so $\mathrm{Out}\, e = \mathrm{Out}\, G$. The Wells sequence (Wells [6]) associated to e is

$$0 \to H^1(K, Z(F)) \to \mathrm{Out}\, G \to N_{\mathrm{Out}\, F}(D)/D \to H^2(K, Z(F)) .$$

Here D is the image of K in $\mathrm{Out}\, F$ obtained by the natural morphism $\chi: K \to \mathrm{Out}\, F$ associated to the extension e. $K \simeq D$ because $C_G(F) = Z(F) \leqslant F$. If $H^1(K, Z(F)) \neq 0$ then it is easy to construct a non inner p-automorphism of G choosing an outer derivation $\delta: K \to Z(F)$ and setting $x^\alpha = x(xF)^\delta$. It is well know that α is an outer p-automorphism of G. Then we may assume $H^1(K, Z(F)) = 0$. By lemma 1.1 we have $H^2(K, Z(F)) = 0$ so that the Wells sequence becomes $\mathrm{Out}\, G \simeq N_{\mathrm{Out}\, F}(D)/D$. Our purpose is now to prove that $N_{\mathrm{Out}\, F}(D)/D$ has non trivial p-subgroups. The first step is to show that $O_p(\mathrm{Out}\, F) \neq 1$ using Theorem 1.3. Surely F doesn't satisfy conditions i) or ii) of that theorem. Furthermore, G being non nilpotent, $\mathrm{rg}\, G_0 \geqslant p - 1$ so that $\mathrm{rg}\, G_0 > 1$ and F doesn't satisfy condition iii). Two cases are to be examined:

a) $O_p(\mathrm{Out}\, F) \leqslant D$.

We can write $F = BZ(F)$ with B a finite characteristic subgroup such that F/B divisible. If B is abelian so is F.

$$C = C_{\mathrm{Aut}\, F}(F/G_0, G_0) \simeq \mathrm{Hom}\, (F/G_0, G_0) \neq 1$$

is a normal p-subgroup of $\mathrm{Aut}\, F = \mathrm{Out}\, F$ so it is contained in D. But this is impossible because the only element in D centralizing G_0 is 1. Then B cannot be abelian. By Theorem 1.2 there exist an outer p-automorphism α of B centralizing $Z(B) \geqslant B \cap Z(G)$. We can extend this automorphism α to an automorphism β of F setting $x^\beta = x^\alpha$ if $x \in B$, $x^\beta = x$ if $x \in Z(G) \setminus B$. β is well defined, it is outer and has the same period of α. This implies that $H = C_{\mathrm{Out}\, F}(Z(F))$ has non trivial p-subgroups. If $\alpha \in C_{\mathrm{Aut}\, F}(Z(F))$ there exist an integer n such that α^n is the identity on $F/Z(F)$, that is $\alpha^n \in C_{\mathrm{Aut}\, F}(Z(F), F/Z(F)) \simeq H^1(F/Z(F), Z(F))$ that is a p-group of finite exponent. So $C_{\mathrm{Aut}\, F}(Z(F))$ is periodic and therefore H is finite. D acts on H by conjugation, then it normalizes a non trivial p-Sylow subgroup of H, say P. D is strictly contained in PD because $D \cap H = 1$ and therefore $N_{PD}(D) > D$. This implies that $N_{\mathrm{Out}\, F}(D)/D$ has non trivial p-subgroups.

b) $O_p(\mathrm{Out}\,F) \not\subset D$.

Let $T = O_p(\mathrm{Out}\,F)D$. T is a Černikov p-group so D is strictly contained in its normalizer and, for this reason, $N_{\mathrm{Aut}\,F}(D)/D$ has non trivial p-subgroups. #

We are then left to examine the case in which $G_0 = \mathrm{Fit}\,G$. In these hypotheses the existence of outer p-automorphisms in no longer certain. We have in fact

THEOREM 2.2. Let G be a non nilpotent Černikov p-group, G_0 its finite residual and assume $\mathrm{Fit}\,G = C_G(G_0) = G_0$, $H^1(K, Z(F)) = 0$ and $Z(G)$ divisible. Then G has outer p-automorphisms if and only if the natural image of G/G_0 in $\mathrm{Aut}\,G_0$ is not a Sylow p-subgroup of $\mathrm{Aut}\,G_0$.

PROOF. – As in the proof of Theorem 2.1 we obtain $\mathrm{Out}\,G \simeq N_{\mathrm{Aut}\,G_0}(D)/D$. If D is not a Sylow p-subgroup of $\mathrm{Aut}\,G_0$, then there exists a p-subgroup P of $\mathrm{Aut}\,G_0$ such that $D < P$. P is finite so $D < N_P(D)$, hence $N_{\mathrm{Aut}\,G_0}(D)/D$ has non trivial p-subgroups. On the other hand, if G has an outer p-automorphism then $\exists \alpha \in N_{\mathrm{Aut}\,G_0}(D)/D$ such that $\alpha^p = 1$, then the group $R = \langle\alpha\rangle D$ is a p-group, $R > D$ and, therefore, D cannot be a Sylow p-subgroup of $\mathrm{Aut}\,G_0$. #

COROLLARY 2.1. Let G be a non nilpotent Černikov p-group. Suppose $C_G(G_0) = G_0$, $Z(G)$ divisible and that the image of G/G_0 in $\mathrm{Aut}\,G_0$ is a Sylow p-subgroup of $\mathrm{Aut}\,G_0$. Then G has outer p-automorphisms if and only if $H^1(G/G_0, G_0) \neq 0$.

3. Examples.

Corollary 2.1, though establishing a necessary and sufficient condition for the existence of outer p-automorphisms, doesnt allow to establish the existence of Černikov p-groups for which this condition is verified. In this section we shall construct some examples which prove how, if a group satisfies the hypotheses of corollary 2.1, we can have either $H^1(G/G_0, G_0) = 0$ or $H^1(G/G_0, G_0) \neq 0$. From here onwards we shall indicate with R_p and Q_p respectively the ring of p-adic integer and its field of fractions. Let also remember that if $G_0 = (\mathbb{Z}(p^\infty))^n$, then $\mathrm{Aut}\,G \simeq GL(n, R_p)$. The results about the structure of Sylow p-subgroups of $GL(n, Q_p)$ we shall use, have been proved by Vol'vacev in [7].

REMARK. If $p = 2$ there are no Černikov 2-groups satisfying the hypotheses of Corollary 2.1. In fact, if α is the element of Aut G_0 sending every element a of G_0 in its inverse a^{-1}, α belongs to the centre of Aut G_0 so, if D (the image of G/G_0 in Aut G_0) is a Sylow 2-subgroup of Aut G_0 then it contains α. Hence there is an element g of G such that $a^g = a^{-1}\ \forall a \in G_0$. Then $Z(G)$ cannot be divisible because

$$Z(G) \leqslant C_{G_0}(g) = \Omega_1(G_0) .$$

EXAMPLE 1. Let $p \geqslant 3$. Let C be the companion matrix of the polynomial $1 + t + t^2 + ... + t^{p-1}$ and set $A = (1\ 1\ 0 \ldots 0)$.

Consider $X = \begin{pmatrix} C & 0 \\ A & 1 \end{pmatrix}$ where 0 is a column of $p-1$ zeroes. If $B_i = \sum_{j=0}^{i-1} C^j$ we have $X^i = \begin{pmatrix} C^i & 0 \\ AB_i & 1 \end{pmatrix}$. The Sylow p-subgroups of $GL(p, Q_p)$ have order p because $p \geqslant 3$, hence $\langle X \rangle$ is a Sylow p-subgroup of $GL(p, R_p)$. Consider the group $G = G_0\langle x \rangle$ where $G_0 = (\mathbb{Z}(p^\infty))^p$ the direct sum of p copies of $\mathbb{Z}(p^\infty)$ and x is the automorphism represented by the matrix X. An easy calculation shows that G satisfies the hypotheses of Corollary 2.1. We claim that $H^1(G/G_0, G_0) = 0$. Let $\sigma, \tau\colon G_0 \to G_0$ be the morphisms defined by

$$a^\sigma = [a, x] \qquad \text{and } a^\tau = \prod_{i=0}^{p-1} a^{x^i} \quad \forall a \in G_0 .$$

We know that

$$H^1(G/G_0, G_0) \cong \operatorname{Ker} \tau / \operatorname{Im} \sigma , \qquad \operatorname{Im} \sigma \cong G_0/Z(G) \cong (\mathbb{Z}(p^\infty))^{p-1} .$$

More difficult is to find Ker τ. The matrix associated to τ is $Y = 1 + X + X^2 + \ldots + X^{p-1}$ that is $Y = \begin{pmatrix} 0 & 0 \\ B & p \end{pmatrix}$ for some $B \in R_p^{p-1}$. We claim that the first element of B is $p-2$. Infact we have $B = A(\sum_{i=1}^{p-2} B_i) = (\sum_{i=1}^{p-1}\sum_{j=0}^{i-1} C^j) = A(\sum_{i=0}^{p-2} (p-i-1) C^i)$. The elements of place $(1, 1)$ and $(2, 1)$ of the matrix $\sum_{i=0}^{p-2} (p-i-1) C^i$ are, respectively, $p-1$ and -1 so that the first element of B is $p-2$ as claimed.

Let $a = (a_1, \dots, a_p)$ be an element of G_0, $a_i \in \mathbb{Z}(p^\infty)$. By a direct calculation we see that $a^\tau = \left(0, 0, \dots, (p-2)a_1 + \sum_{i=2}^{p-1} \lambda_i a_i + p a_p\right)$ $\lambda_i \in R_p$. But $p-2$ is a unit in R_p so we have

$$\operatorname{Ker} \tau = \left\{(a_1, \dots, a_p);\ a_1 = \frac{-1}{p-2}\left[\sum_{i=2}^{p-1} \lambda_i a_i + p a_p\right]\right\}.$$

Define

$$A_i = \left\{\left(\frac{-\lambda_i}{p-2} a, 0, \dots, \underset{i\text{-th place}}{a}, \dots, 0\right);\ a \in \mathbb{Z}(p^\infty)\right\}.$$

A_i is, obviously, a divisible subgroup of G_0 of rank 1. Furthermore $A_i \cap \sum_{j \neq i} A_j = 0$ so that $\operatorname{Ker} \tau$ is the direct sum of the subgroups A_i and, therefore, is divisible of rank $p-1$. Hence $H^1(G/G_0, G_0) = 0$ and G has no outer p-automorphisms.

Example 2. Let $p > 3$. With the same notations of example 1, let $E = \begin{pmatrix} C & 0 \\ A & 1 \end{pmatrix}$ and $X = \begin{pmatrix} E & 0 \\ 0 & 1 \end{pmatrix}$. X is an element of $GL(p+1, R_p)$.

$\langle X \rangle$, as in example 1, is a Sylow p-subgroup of $GL(p+1, R_p)$ so the group $G = G_0\langle x \rangle$ (where $G_0 = (\mathbb{Z}(p^\infty))^{p+1}$ and x is the automorphism induced by X) satisfies the hypotheses of corollary 2.1. Using the same arguments of example 1 we can see that $\operatorname{Im} \sigma$ is a divisible group of rank $p-1$.

If $a = (a_1, \dots, a_{p+1}) \in G_0$, then

$$a^\tau = \left(0, 0, \dots, (p-2)a_1 + \sum_{i=1}^{p} \lambda_i a_i,\ p a_{p+1}\right).$$

So $\operatorname{Ker} \tau = \left(\bigoplus_{i=2}^{p} A_i\right) \oplus B$ where B is cyclic of order p. Then, in this case, $H^1(G/G_0, G_0) \neq 0$ and G has non inner p-automorphisms.

Example 3. In this example we will construct a group G such that the image of G/G_0 is a Sylow p-subgroup of $GL(n, R_p)$ but not of $GL(n, Q_p)$, as it was in the previous examples. Let $p = 3$ and

$$X = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad X \in GL(4, Q_3) \text{ and } X^3 = I.$$

$\langle X\rangle$ is not a Sylow 3-subgroup of $GL(4, Q_3)$ because they are elementary abelian of order 9. Suppose there exists $Y \in GL(4, R_3)$ s.t. $Y^3 = 1$ and $|\langle X, Y\rangle| = 9$. Set $G_0 = (\mathbb{Z}(3^\infty))^4$. Let x and y be the automorphisms of G_0 induced by X and Y. $C_{G_0}(x) = \{(0, 0, a, b): a, b \in \mathbb{Z}(3^\infty)\}$. $C_{G_0}(x)^y = C_{G_0^y}(x^y) = C_{G_0}(x)$ and therefore Y has the form $Y = \begin{pmatrix} L & 0 \\ M & N \end{pmatrix}$ $L, M, N \in M(2, R)$.

From this point on we set $S = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$ and $T = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$. Using the relation $x^y = x$ we deduce that $L^{-1}SL = S$ and a routine calculation proves that the only possibilities are $L = I, S, S^2$. If $L = S^2$ the first block of Y^2 is S, so we can reduce our discussion to the cases $L = I$ or $L = S$. Note that $N^3 = I$ and that x acts as the identity on the last two components of G_0 so we may assume $N = S$ or $N = I$. Four cases are to be examined:

1) $Y = \begin{pmatrix} S & 0 \\ M & S \end{pmatrix}$

$M = \begin{pmatrix} m & n \\ r & s \end{pmatrix}$ $xy = yx \Leftrightarrow TS + M = MS + ST \Leftrightarrow (m, n, r, s)$ is a solution, in R_3, of the equations

$$m + n = 1 \qquad m - 2n = 0$$
$$r + s = 1 \qquad r - 2s = 0\,.$$

But these equations have no solutions in R_3.

2) $Y = \begin{pmatrix} S & 0 \\ M & I \end{pmatrix}$

$xy = yx \Leftrightarrow TS + M = MS + T \Leftrightarrow M(S - I) = T(S - I) \Leftrightarrow M = T$ and this gives $x = y$

3) $Y = \begin{pmatrix} I & 0 \\ M & S \end{pmatrix}$

$xy = yx \Leftrightarrow T + M = MS + ST \Leftrightarrow (m, n, r, s)$ is a solution of the following equations

$$m + n = -1 \qquad m - 2n = 1$$
$$r + s = -1 \qquad r - 2s = 1\,.$$

But the solution of these equations is not in R_3.

$$4)\ Y = \begin{pmatrix} I & 0 \\ M & I \end{pmatrix}$$

$xy = yx \Leftrightarrow T + M = MS + T \Leftrightarrow M(S - I) = 0 \Leftrightarrow M = 0.$

This proves that $\langle X \rangle$ is a Sylow 3-subgroup of $GL(4, R_3)$. Now, as in example 2, we deduce that $H^1(G/G_0, G_0)$ is cyclic of order 3 so that G has outer 3-automorphisms.

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