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SOME MATHEMATICAL CONSIDERATIONS OF RECEPTOR FIELDS

by Rolando E. PEINADO, Alfred L. OCHS and Eduardo BARAÑANO

The mechanism by which the nervous system encodes and transmits information has long been a central physiological question, albeit, a question which remains unanswered. The nervous system is apparently a computing mechanism and many analogies to existing computing machines have been attempted. However, there has been little further insight since work by the late John von NEUMANN [4].

While commenting on the many similarities, von NEUMANN also pointed out several significant differences. Nerve cells are "all or nothing" binary devices, but, the brain does not seem to use digital arithmetic. Its operation is more analog in nature, although far removed from any digital-analog hybrid computer of existing design. Another fundamental difference, recognized by von NEUMANN, is the serial nature of computing machines and the parallel nature of the nervous system. Construction of the first true parallel processor, the ILLIAC-IV, is only now nearing completion. By way of contrast, the human optic nerve has approximately one million channels while the ILLIAC-IV has 64. The theory of matrices groups, and semi-groups will be useful in describing parallel processes in the nervous system.

Unfortunately, von NEUMANN's insights did not set the direction for research on neurological coding. Later workers pointed out the similarities but overlooked many differences. A considerable amount of work has been devoted to describing how a single neuron encodes its information. In spite of many attempts by laboratories through the world, little insight into the coding process has come forth. The usual hypothesis is that the information content of a nerve's firing pattern is some how represented in the frequency of the impulses or the interval between them. Random or markovian statistical models have been used in an attempt to decipher the code. Perhaps, the failures to find this code point to a deficiency in the question. We suggest that a single neuron need not carry an unambiguous message. Rather, its impulses carry information only in the context of the firing pattern of its neighboring cells. In other words, the nervous system uses a mechanism of parallel processing. This point of view is supported by anatomical observations. In virtually every synaptic layer of every organism, there are as many lateral connections as there are ascending or descending contacts.

As it is clearly impossible to simultaneously examine the number of cells necessary to prove or disprove this question, we must seek an alternative approach. As an aid to prediction, we have programmed a simple computer model of processes observed in the nervous system. We trust that these predictions can ultimately be tested by conventional physiological techniques.

A computer model of information transfer will ultimately prove to be inadequate as it computes only the particular case incorporated by its programmer. A more elegant approach would be to develop a formal mathematical analysis of parallel information transfer processes. We suggest that the algebra of semigroups is particularly applicable to this problem.

The system we chose to model is the receptor field transform in the visual system of the common cat (2,3). For this discussion there is nothing unique about this preparation and our model could just as well describe visual transforms reported in other species. Moreover, similar transform processes have been reported for hearing and touch BEKÉSY [1]. A receptor field is the name given to a particular pattern of light presented to the eye which will influence a single nerve cell under observation. This nerve cell can be anywhere in the visual system, from the retina to the brain ; and the influence can be excitatory, inhibitory, or both. In general, the firing pattern of the cell will have a temporal relation to the stimulus as well as a spatial one.

In the cat, the receptor field of the retina ganglion cells, whose processes from the optic nerve to the brain, are of two complementary types. These are both circular in organization. In one type of field, the central region gives rise to a response only when a stimulus is presented (an "on response") while the surrounding annulus produces a response when the stimulus is removed (an "off response"). In the second type of ganglion cell, the on and off regions are interchanged. Also, and most importantly for the purposes of this paper, the two regions are mutually antagonistic at the same point in time. That is, when a spot of light in one region causes excitation of the ganglion cell, a spot in the other region will cause inhibition of that cell.

At this point in our investigation, we found it expedient to simplify the problem to a rather considerable extent. We here consider the receptors a binary devices which respond to light in an all or nothing fashion. Thus, all reference to brightness and contrast is lost. Our most fundamental restriction is to remove all reference to time. In effect, we consider the state of the nervous system as it would be immediately after an image is presented. This restriction will obviously require a redefinition of the on and off regions as these names can only refer to actions in time.

Consider the firing pattern of an on center ganglion cell just after presentation of the visual stimulus. Light in the center of the field will cause excitation and light in the surround will, because of its antagonistic action, cause inhibition of the ganglion cell. Thus, in the model, we call this an excitatory center cell, or simply, an E cell. In a complementary fashion, the firing pattern of an off center (on surround) ganglion cell will become an inhibitory cell, or, an I cell. These ganglion cells, at least as we idealized them, are nerve cells with a certain number of spatially distributed inputs, both excitatory and inhibitory, and a

threshold which determines the state of its binary output.

For a given neuron in a  $k$ -layer of cells and with rectangular coordinates  $(i, j)$ , we define the function

$$N_k(i, j) = [A \sum_{d=N_1}^{N_2} R(d, \theta) + S \sum_{d=N_3}^{N_4} R(d, \theta)] - T_k(i, j)$$

where  $A$  and  $S$  are numbers used as weighting factors ;  $R(d, \theta)$  is the output state of a particular cell in the  $k$ -layer, identified by its polar coordinates  $(d, \theta)$  taking each  $(i, j)$  as a center ; and  $T_k(i, j)$  is the threshold.

A receptor is a cell for which, in the 1-layer

$$n_1(i, j) = A R(d, \theta) - T_1(i, j)$$

and  $R(d, \theta)$  is the input signal.

By neural networks, we mean a matrix  $m \times n$ , whose entries are  $n_k(i, j)$  cells. A receptor field is a 1-neural network whose cells are receptors.

Let us define a function  $P_k$  from  $k$ -neural network to the set  $\{0, 1\}$  as follows.

$$P_k[n_k(i, j)] = \begin{cases} 1, & \text{if } n_k(i, j) > 0, \\ 0, & \text{if } n_k(i, j) \leq 0. \end{cases}$$

For each  $k$ -neural network, we can form the  $P_k$  matrix of zeros and ones that replaces each  $n_k(i, j)$  for  $P_k[n_k(i, j)]$ .

We remark that  $n_k(i, j)$  depends on values of  $A$ ,  $S$  and  $T_k$ . Thus for a fixed  $k$ , by varying these parameters, we obtain possible different  $k$ -neural networks. Thus for a fixed  $k$  using the properties of matrices it is easy to show that :

THEOREM. - Under the usual addition of matrices, and subject to the following relations :

$$1 + 0 = 0 + 1 = 1, \quad 1 + 1 = 1, \quad 0 + 0 = 0,$$

the set of all  $P_k$ -matrices for a semigroup for a fixed  $k$ .

In these semigroups, we will seek internal characteristics that determine the invariants of patterns of the information transfer as has been observed by the results obtained by the computer program.

Let us remark that the aforementioned semigroups are subsemigroups of the semigroups of all binary relations on a set. Hence, we will investigate this semigroup to learn more about the structure of our semigroup.

The logic of the computer program is as follows : We store in memory three  $36 \times 36$  arrays. The first array holds the state of the retinal receptors. Each element can be either 0 or 1 (having not seen or seen light). The second and

third arrays hold the output of our calculations and represent the state of E and I ganglion cells at that location, stored as a 0 or 1. Mounted on a magnetic tape unit is an address file. For each ganglion cell there is a list of neighbor receptor cell addresses.

The array is not square in the usual sense, rather it is composed of  $36 \times 36$  close packed hexagons. We chose this only because one can approximate a circle in such a system. The receptor under the ganglion cell is the zeroth neighbor ( $N = 0$ ). The outer circle of 36 cells is the sixth neighbor. There are 131 addresses in each list and a separate list for each ganglion cell. Whenever a neighbor is found to be outside of the array, it is arbitrarily given the address (0, 0). This cell will thus determine the state of the exterior, either light or dark. Usually it would be dark.

To do the receptor field calculations the ganglion arrays are computed one cell at a time. This, of course, is how one simulates a parallel machine. The address list of a particular cell is loaded into memory and the following calculation is performed.

E cell :

$$0 < A (E) \sum_{j=1}^{19} R_j - \sum_{j=20}^{131} R_j - T (E)$$

I cell :

$$0 < - A (I) \sum_{j=1}^{19} R_j + \sum_{j=20}^{131} R_j - T (I)$$

Where  $R_j$  are the receptor cells in the address file, A determines the antagonism between the cells in the center and the surround, and T the threshold. If the condition is true, the corresponding ganglion cell is considered to fire and set to 1, otherwise it is set = 0.

In the calculations presented below, the inner field was composed of the zeroth, first and second rings of neighbor receptors. The outer field was taken to be the fourth to the sixth rings.

By appropriately selecting the threshold (E) and the antagonism factor (A), some interesting condensations of the data arise. In Figure 1, a straight line maps into two clusters of E cells which defined the ends of that line. These clusters have vector properties which would be useful in removing ambiguities arising from more than one line being presented. Note, the values given for A and T are octal numbers.

An angle maps into equally interesting firing patterns. The E and I cells both form a vector which bisects the angle. The I cells form a cluster on the inside of the angle and point inward. The length of the I cell vector is roughly proportional to the inverse of the angle.

The symmetries of a hexagonal array introduce some artifacts which are most apparent in the  $90^\circ$  angle. In the receptor array, the vertical and horizontal lines

are different in detail and, not surprisingly, the outputs are not symmetrical. The angle vectors do not exactly bisect the angle and the end vectors have a different shape. This problem is minimized by using a figure which is more than one cell wide.

The form of the transform will vary by changing  $T$  and  $A$ . These variations are illustrated in Figure 2 for the  $60^\circ$  angle with a variation of  $T$  and with  $A$  held constant.

The possibility of modifying an information transfer system is very suggestive. We would like to consider an analogy with adaptive control systems. For example, a high performance aircraft must operate over a wide variety of altitudes and speeds, both sub and supersonic. Here, the control system adapts itself so that the aircraft "feels the same" to the pilot. By analogy, perhaps the nervous system has its incoming data under adaptive control. The transfer could then be optimized to the particular information the nervous system would want to consider. In a sense, this ability could be a part of "paying attention" to a particular object. This suggestion is not only consistent with psychophysical observations on attention, but is also, consistent with the anatomy. Some neurons have been observed to carry information to the retina from higher centers. It has been thought that these were involved in negative feedback of some sort, perhaps for brightness adaptation. It is equally possible that they could serve a system of adaptive information transfer.

We should distinguish our model from "perceptron" machines. At this word is generally used, perceptrons are pattern recognition devices. At some later time, it may be desirable to follow our receptor field model with a perceptron. It would be interesting to see if the recognition ability were enhanced by an appropriate manipulation of the incoming visual array.

As we don't know the optimum input to a perceptron, we also don't know the optimum input to a brain. Perhaps our choice of interesting transforms has little to do with nature. Only recourse to experiment can answer that question. If our model has physiological virtue, it would be to suggest such experiments.

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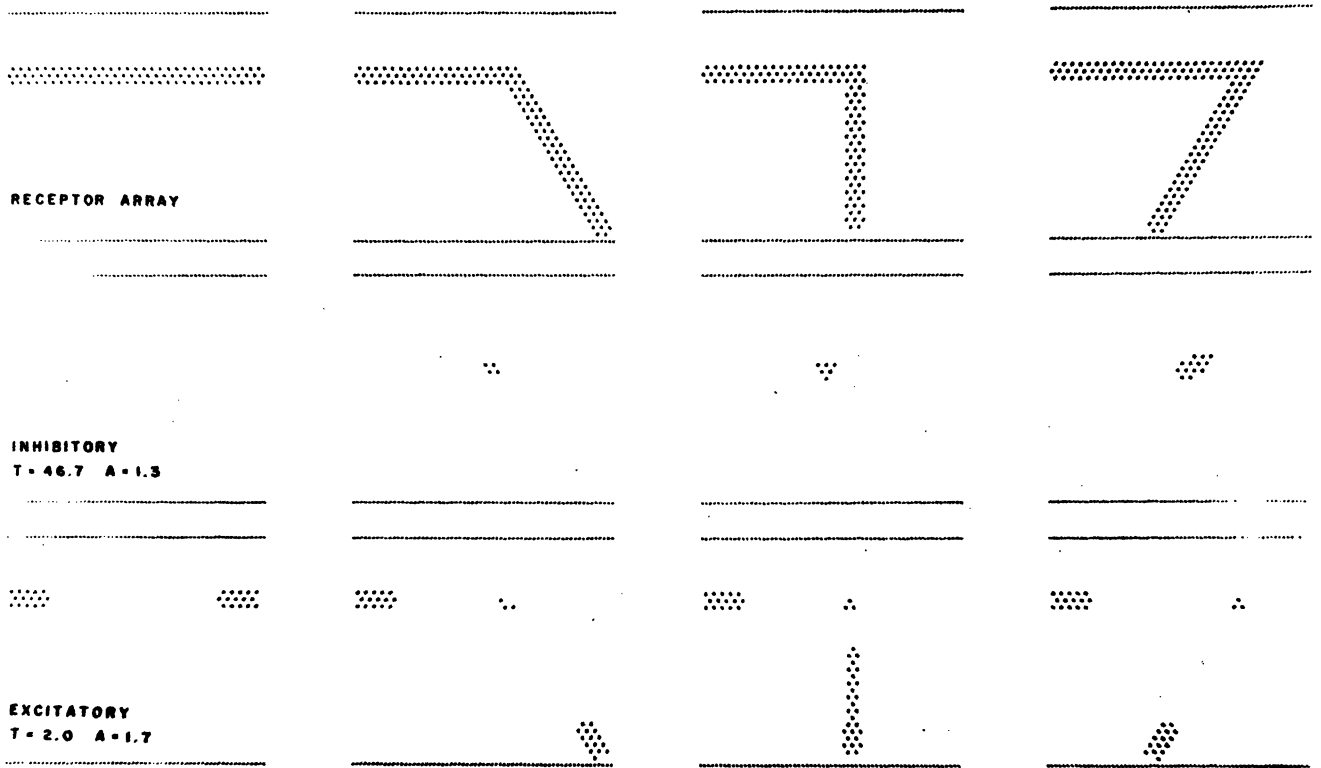


Fig. 1

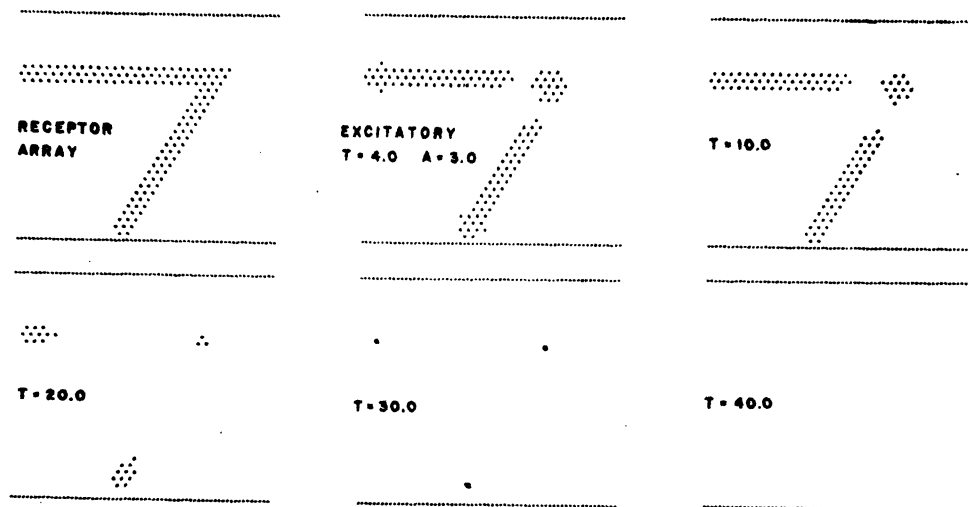


Fig. 2